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Introduction to Science and the Realm of Physics, Physical Quantities, and Units

class="introduction"

Galaxies are as immense as atoms are small. Yet the same laws of physics describe both, and all the rest of nature—an indication of the underlying unity in the universe. The laws of physics are surprisingly few in number, implying an underlying simplicity to nature’s apparent complexity. (credit: NASA, JPL-Caltech, P. Barmby, Harvard-Smithsonian Center for

Astrophysics
)

What is your first reaction when you hear the word “physics”? Did you imagine working through difficult equations or memorizing formulas that seem to have no real use in life outside the physics classroom? Many people come to the subject of physics with a bit of fear. But as you begin your exploration of this broad-ranging subject, you may soon come to realize that physics plays a much larger role in your life than you first thought, no matter your life goals or career choice.

Think now about all of the technological devices that you use on a regular basis. Computers, smart phones, GPS systems, MP3 players, and satellite radio might come to mind. Next, think about the most exciting modern technologies that you have heard about in the news, such as trains that levitate above tracks, “invisibility cloaks” that bend light around them, and microscopic robots that fight cancer cells in our bodies. All of these groundbreaking advancements, commonplace or unbelievable, rely on the principles of physics. Aside from playing a significant role in technology, professionals such as engineers, pilots, physicians, physical therapists, electricians, and computer programmers apply physics concepts in their daily work. For example, a pilot must understand how wind forces affect a flight path and a physical therapist must understand how the muscles in the body experience forces as they move and bend. As you will learn in this text, physics principles are propelling new, exciting technologies, and these principles are applied in a wide range of careers.

Kinesiology (the study of the mechanics of body movements) is an important of study in **athletics**. Wires, pulleys, and weights are used in **traction systems**. **Food** is the source of energy in the body. Work transform it into heat. Humans have a constant body **temperature** despite the body heat and the surrounding temperature. Physics is also applied in **physical therapy**. The study of fluids and pressure is applied to blood **flow and respiration**. **Hearing** is the perception of sound. You heard about **ultrasonic scanners**. They are a direct application of the study of sounds. We will discuss **electrical safety**. Bioelectricity is the mode of transport of information within the **nervous system**. The physics of optics is the basics study to understand vision. **X-rays** are part of the electromagnetic system.

In this text, you will begin to explore the history of the formal study of physics, beginning with natural philosophy and the ancient Greeks, and leading up through a review of Sir Isaac Newton and the laws of physics that bear his name. You will also be introduced to the standards scientists use when they study physical quantities and the interrelated system of measurements most of the scientific community uses to communicate in a single mathematical language. Finally, you will study the limits of our ability to be accurate and precise, and the reasons scientists go to painstaking lengths to be as clear as possible regarding their own limitations.

Physics: An Introduction

- Explain the difference between a principle and a law.
- Explain the difference between a model and a theory.

The flight formations of migratory birds such as Canada geese are governed by the laws of physics. (credit: David Merrett)

The physical universe is enormously complex in its detail. Every day, each of us observes a great variety of objects and phenomena. Over the centuries, the curiosity of the human race has led us collectively to explore and catalog a tremendous wealth of information. From the flight of birds to the colors of flowers, from lightning to gravity, from quarks to clusters of galaxies, from the flow of time to the mystery of the creation of the universe, we have asked questions and assembled huge arrays of facts. In the face of all these details, we have discovered that a surprisingly small and unified set of physical laws can explain what we observe. As humans, we make generalizations and seek order. We have found that nature is remarkably cooperative—it exhibits the *underlying order and simplicity* we so value.

Science and the Realm of Physics

Science consists of the theories and laws that are the general truths of nature as well as the body of knowledge they encompass. Scientists are continually trying to expand this body of knowledge and to perfect the expression of the laws that describe it. **Physics** is concerned with describing the interactions of energy, matter, space, and time, and it is especially interested in what fundamental mechanisms underlie every phenomenon. The concern for describing the basic phenomena in nature essentially defines the *realm of physics*.

Physics aims to describe the function of everything around us, from the movement of tiny charged particles to the motion of people, cars, and spaceships. In fact, almost everything around you can be described quite accurately by the laws of physics. Consider a smart phone ([link]). Physics describes how electricity interacts with the various circuits inside the device. This knowledge helps engineers select the appropriate materials and circuit layout when building the smart phone. Next, consider a GPS system. Physics describes the relationship between the speed of an object, the distance over which it travels, and the time it takes to travel that distance. When you use a GPS device in a vehicle, it utilizes these physics equations to determine the travel time from one location to another.

The Apple "iPhone" is a common

smart phone with a GPS function. Physics describes the way that electricity flows through the circuits of this device. Engineers use their knowledge of physics to construct an iPhone with features that consumers will enjoy. One specific feature of an iPhone is the GPS function. GPS uses physics equations to determine the driving time between two locations on a map. (credit: @gletham GIS, Social, Mobile Tech Images)

Applications of Physics

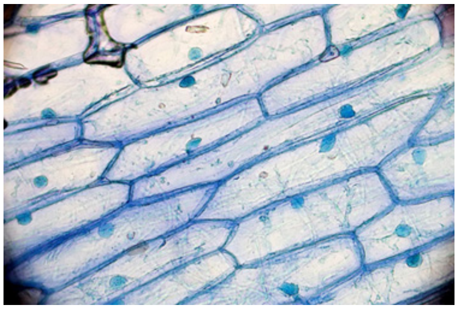
You need not be a scientist to use physics. On the contrary, knowledge of physics is useful in everyday situations as well as in nonscientific professions. It can help you understand how microwave ovens work, why metals should not be put into them, and why they might affect pacemakers. (See [link] and [link].) Physics allows you to understand the hazards of radiation and rationally evaluate these hazards more easily. Physics also explains the reason why a black car radiator helps remove heat in a car engine, and it explains why a white roof helps keep the inside of a house cool. Similarly, the operation of a car's ignition system as well as the transmission of electrical signals through our body's nervous system are much easier to understand when you think about them in terms of basic physics.

Physics is the foundation of many important disciplines and contributes directly to others. Chemistry, for example—since it deals with the interactions of atoms and molecules—is rooted in atomic and molecular physics. Most branches of engineering are applied physics. In architecture, physics is at the heart of structural stability, and is involved in the acoustics, heating, lighting, and cooling of buildings. Parts of geology rely heavily on physics, such as radioactive dating of rocks, earthquake analysis, and heat transfer in the Earth. Some disciplines, such as biophysics and geophysics, are hybrids of physics and other disciplines.

Physics has many applications in the biological sciences. On the microscopic level, it helps describe the properties of cell walls and cell membranes ([link] and [link]). On the macroscopic level, it can explain the heat, work, and power associated with the human body. Physics is involved in medical diagnostics, such as x-rays, magnetic resonance imaging (MRI), and ultrasonic blood flow measurements. Medical therapy sometimes directly involves physics; for example, cancer radiotherapy uses ionizing radiation. Physics can also explain sensory phenomena, such as how musical instruments make sound, how the eye detects color, and how lasers can transmit information.

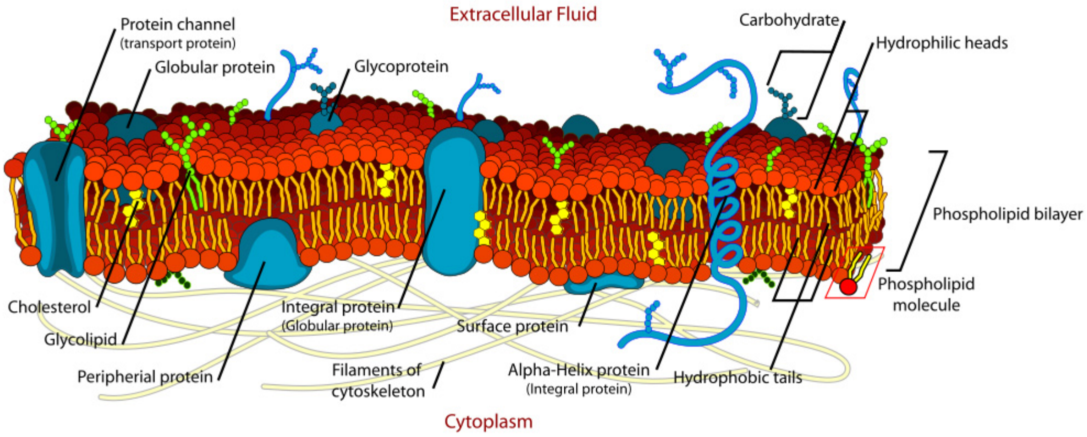
The laws of physics help us understand how common appliances work. For example, the laws of physics can help explain how microwave ovens heat up food, and they also help us understand why it is dangerous to place metal objects in a microwave oven. (credit: MoneyBlogNewz)

These two applications of physics have more in common than meets the eye. Microwave ovens use electromagnetic waves to heat food. Magnetic resonance imaging (MRI) also uses electromagnetic waves to yield an image of the brain, from which the exact location of tumors can be determined. (credit: Rashmi Chawla, Daniel Smith, and Paul E. Marik)



Physics, chemistry,

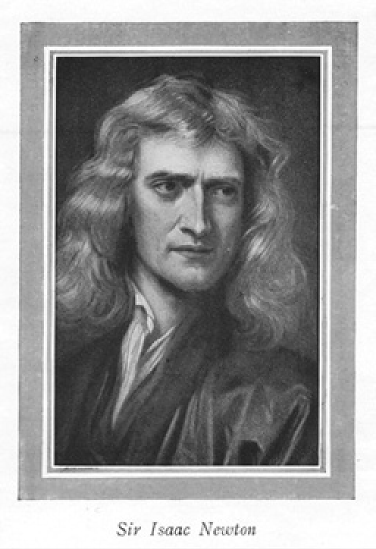
and biology help describe the properties of cell walls in plant cells, such as the onion cells seen here. (credit: Umberto Salvagnin)



An artist’s rendition of the the structure of a cell membrane. Membranes form the boundaries of animal cells and are complex in structure and function. Many of the most fundamental properties of life, such as the firing of nerve cells, are related to membranes. The disciplines of biology, chemistry, and physics all help us understand the membranes of animal cells. (credit: Mariana Ruiz)

Models, Theories, and Laws; The Role of Experimentation

The laws of nature are concise descriptions of the universe around us; they are human statements of the underlying laws or rules that all natural processes follow. Such laws are intrinsic to the universe; humans did not create them and so cannot change them. We can only discover and understand them. Their discovery is a very human endeavor, with all the elements of mystery, imagination, struggle, triumph, and disappointment inherent in any creative effort. (See [link] and [link].) The cornerstone of discovering natural laws is observation; science must describe the universe as it is, not as we may imagine it to be.



Isaac Newton (1642–1727) was very reluctant to publish his revolutionary work and had to be convinced to do so. In his later years, he stepped down from his academic post and became exchequer of the Royal Mint. He took this post seriously, inventing reeding (or creating ridges) on the edge of coins to prevent unscrupulous people from trimming the silver off of them before using them as currency. (credit: Arthur Shuster and Arthur E. Shipley: *Britain's Heritage of Science*. London, 1917.)

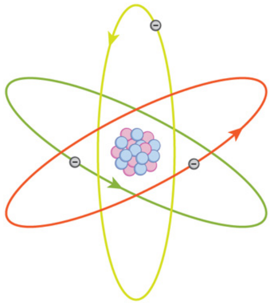
Marie Curie (1867–1934) sacrificed

monetary assets to help finance her early research and damaged her physical well-being with radiation exposure. She is the only person to win Nobel prizes in both physics and chemistry. One of her daughters also won a Nobel Prize. (credit: Wikimedia Commons)

A **model** is a representation of something that is often too difficult (or impossible) to display directly. While a model is justified with experimental proof, it is only accurate under limited situations. An example is the planetary model of the atom in which electrons are pictured as orbiting the nucleus, analogous to the way planets orbit the Sun. (See [link].) We cannot observe electron orbits directly, but the mental image helps explain the observations we can make, such as the emission of light from hot gases (atomic spectra). Physicists use models for a variety of purposes. For example, models can help physicists analyze a scenario and perform a calculation, or they can be used to represent a situation in the form of a computer simulation. A **theory** is an explanation for patterns in nature that is supported by scientific evidence and verified multiple times by various groups of researchers. Some theories include models to help visualize phenomena, whereas others do not. Newton’s theory of gravity, for example, does not require a model or mental image, because we can observe the objects directly with our own senses. The kinetic theory of gases, on the other hand, is a model in which a gas is viewed as being composed of atoms and molecules. Atoms and molecules are too small to be observed directly with our senses—thus, we picture them mentally to understand what our instruments tell us about the behavior of gases.

A **law** uses concise language to describe a generalized pattern in nature that is supported by scientific evidence and repeated experiments. Often, a law can be expressed in the form of a single mathematical equation. Laws and theories are similar in that they are both scientific statements that result from a tested hypothesis and are supported by scientific evidence. However, the designation *law* is reserved for a concise and very general statement that describes phenomena in nature, such as the law that energy is conserved during any process, or Newton’s second law of motion, which relates force, mass, and acceleration by the simple equation $\mathbf{F} = m\mathbf{a}$. A theory, in contrast, is a less concise statement of observed phenomena. For example, the Theory of Evolution and the Theory of Relativity cannot be expressed concisely enough to be considered a law. The biggest difference between a law and a theory is that a theory is much more complex and dynamic. A law describes a single action, whereas a theory explains an entire group of related phenomena. And, whereas a law is a postulate that forms the foundation of the scientific method, a theory is the end result of that process.

Less broadly applicable statements are usually called principles (such as Pascal’s principle, which is applicable only in fluids), but the distinction between laws and principles often is not carefully made.



What is a model? This planetary model of the atom shows electrons orbiting the nucleus. It is a drawing that we use to form a mental image of the atom that we cannot see directly with our eyes because it is too small.

Note:

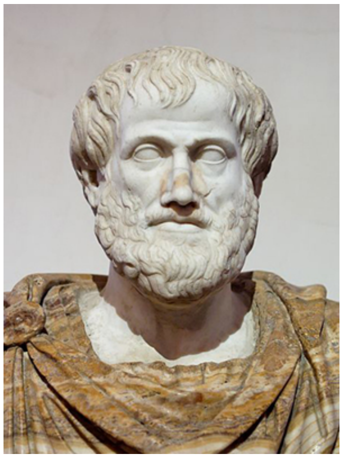
The Scientific Method

As scientists inquire and gather information about the world, they follow a process called the **scientific method**. This process typically begins with an observation and question that the scientist will research. Next, the scientist typically performs some research about the topic and then devises a hypothesis. Then, the scientist will test the hypothesis by performing an experiment. Finally, the scientist analyzes the results of the experiment and draws a conclusion. Note that the scientific method can be applied to many situations that are not limited to science, and this method can be modified to suit the situation.

Consider an example. Let us say that you try to turn on your car, but it will not start. You undoubtedly wonder: Why will the car not start? You can follow a scientific method to answer this question. First off, you may perform some research to determine a variety of reasons why the car will not start. Next, you will state a hypothesis. For example, you may believe that the car is not starting because it has no engine oil. To test this, you open the hood of the car and examine the oil level. You observe that the oil is at an acceptable level, and you thus conclude that the oil level is not contributing to your car issue. To troubleshoot the issue further, you may devise a new hypothesis to test and then repeat the process again.

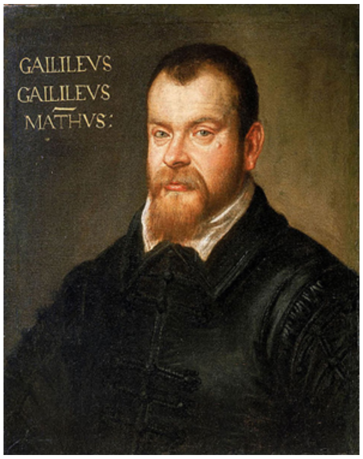
The Evolution of Natural Philosophy into Modern Physics

Physics was not always a separate and distinct discipline. It remains connected to other sciences to this day. The word *physics* comes from Greek, meaning nature. The study of nature came to be called “natural philosophy.” From ancient times through the Renaissance, natural philosophy encompassed many fields, including astronomy, biology, chemistry, physics, mathematics, and medicine. Over the last few centuries, the growth of knowledge has resulted in ever-increasing specialization and branching of natural philosophy into separate fields, with physics retaining the most basic facets. (See [link], [link], and [link].) Physics as it developed from the Renaissance to the end of the 19th century is called **classical physics**. It was transformed into modern physics by revolutionary discoveries made starting at the beginning of the 20th century.

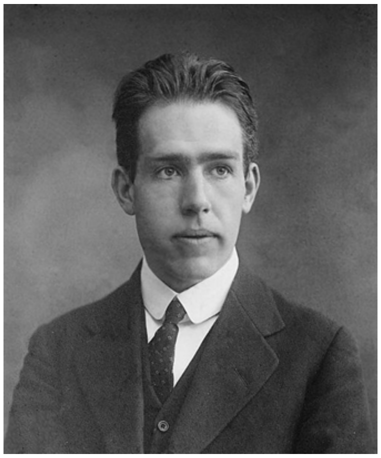


Over the centuries, natural philosophy has evolved into more specialized disciplines, as illustrated by the contributions of some of the greatest minds in history. The Greek philosopher **Aristotle** (384–322 B.C.) wrote on a broad range of topics including physics, animals, the soul, politics, and poetry. (credit: Jastrow

(2006)/Ludovisi Collection)



Galileo Galilei (1564–1642) laid the foundation of modern experimentation and made contributions in mathematics, physics, and astronomy. (credit: Domenico Tintoretto)



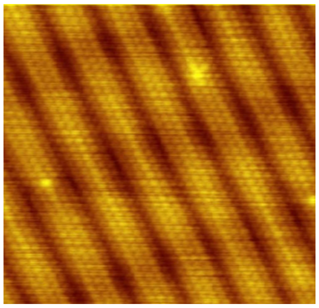
Niels Bohr (1885–1962) made fundamental contributions to the development of quantum mechanics, one part of modern physics. (credit: United States Library of Congress Prints and Photographs Division)

Classical physics is not an exact description of the universe, but it is an excellent approximation under the following conditions: Matter must be moving at speeds less than about 1% of the speed of light, the objects dealt with must be large enough to be seen with a microscope, and only weak gravitational fields, such as the field generated by the Earth, can be involved. Because humans live under such circumstances, classical physics seems intuitively reasonable, while many aspects of modern physics seem bizarre. This is why models are so useful in modern physics—they let us conceptualize phenomena we do not ordinarily experience. We can relate to models in human terms and visualize what happens when objects move at high speeds or imagine what objects too small to observe with our senses might be like. For example, we can understand an atom’s properties because we can picture it in our minds, although we have never seen an atom with our eyes. New tools, of course, allow us to better picture phenomena we cannot see. In fact, new instrumentation has allowed us in recent years to actually “picture” the atom.

Note:

Limits on the Laws of Classical Physics

For the laws of classical physics to apply, the following criteria must be met: Matter must be moving at speeds less than about 1% of the speed of light, the objects dealt with must be large enough to be seen with a microscope, and only weak gravitational fields (such as the field generated by the Earth) can be involved.



Using a scanning tunneling microscope (STM), scientists can see the individual atoms that compose this sheet of gold. (credit: Erwinrossen)

Some of the most spectacular advances in science have been made in modern physics. Many of the laws of classical physics have been modified or rejected, and revolutionary changes in technology, society, and our view of the universe have resulted. Like science fiction, modern physics is filled with fascinating objects beyond our normal experiences, but it has the advantage over science fiction of being very real. Why, then, is the majority of this text devoted to topics of classical physics? There are two main reasons: Classical physics gives an extremely accurate description of the universe under a wide range of everyday circumstances, and knowledge of classical physics is necessary to understand modern physics.

Modern physics itself consists of the two revolutionary theories, relativity and quantum mechanics. These theories deal with the very fast and the very small, respectively. **Relativity** must be used whenever an object is traveling at greater than about 1% of the speed of light or experiences a strong gravitational field such as that near the Sun. **Quantum mechanics** must be used for objects smaller than can be seen with a microscope. The combination of these two theories is *relativistic quantum mechanics*, and it describes the behavior of small objects traveling at high speeds or experiencing a strong gravitational field. Relativistic quantum mechanics is the best universally applicable theory we have. Because of its mathematical complexity, it is used only when necessary, and the other theories are used whenever they will produce sufficiently accurate results. We will find, however, that we can do a great deal of modern physics with the algebra and trigonometry used in this text.

Exercise:

Check Your Understanding

Problem:

A friend tells you he has learned about a new law of nature. What can you know about the information even before your friend describes the law? How would the information be different if your friend told you he had learned about a scientific theory rather than a law?

Solution:

Without knowing the details of the law, you can still infer that the information your friend has learned conforms to the requirements of all laws of nature: it will be a concise description of the universe around us; a statement of the underlying rules that all natural processes follow. If the information had been a theory, you would be able to infer that the information will be a large-scale, broadly applicable generalization.

Note:
PhET Explorations: Equation Grapher
Learn about graphing polynomials. The shape of the curve changes as the constants are adjusted. View the curves for the individual terms (e.g. $y = bx$) to see how they add to generate the polynomial curve.
https://phet.colorado.edu/sims/equation-grapher/equation-grapher_en.html

Summary

- Science seeks to discover and describe the underlying order and simplicity in nature.
- Physics is the most basic of the sciences, concerning itself with energy, matter, space and time, and their interactions.
- Scientific laws and theories express the general truths of nature and the body of knowledge they encompass. These laws of nature are rules that all natural processes appear to follow.

Conceptual Questions

Exercise:

Problem:

Models are particularly useful in relativity and quantum mechanics, where conditions are outside those normally encountered by humans. What is a model?

Exercise:

Problem: How does a model differ from a theory?

Exercise:

Problem:

If two different theories describe experimental observations equally well, can one be said to be more valid than the other (assuming both use accepted rules of logic)?

Exercise:

Problem: What determines the validity of a theory?

Exercise:

Problem:

Certain criteria must be satisfied if a measurement or observation is to be believed. Will the criteria necessarily be as strict for an expected result as for an unexpected result?

Exercise:

Problem:

Can the validity of a model be limited, or must it be universally valid? How does this compare to the required validity of a theory or a law?

Exercise:

Problem:

Classical physics is a good approximation to modern physics under certain circumstances. What are they?

Exercise:

Problem: When is it *necessary* to use relativistic quantum mechanics?

Exercise:

Problem:

Can classical physics be used to accurately describe a satellite moving at a speed of 7500 m/s? Explain why or why not.

Glossary

classical physics

physics that was developed from the Renaissance to the end of the 19th century

physics

the science concerned with describing the interactions of energy, matter, space, and time; it is especially interested in what fundamental mechanisms underlie every phenomenon

model

representation of something that is often too difficult (or impossible) to display directly

theory

an explanation for patterns in nature that is supported by scientific evidence and verified multiple times by various groups of researchers

law

a description, using concise language or a mathematical formula, a generalized pattern in nature that is supported by scientific evidence and repeated experiments

scientific method
a method that typically begins with an observation and question that the scientist will research; next, the scientist typically performs some research about the topic and then devises a hypothesis; then, the scientist will test the hypothesis by performing an experiment; finally, the scientist analyzes the results of the experiment and draws a conclusion

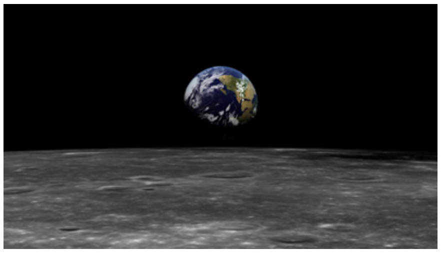
modern physics
the study of relativity, quantum mechanics, or both

relativity
the study of objects moving at speeds greater than about 1% of the speed of light, or of objects being affected by a strong gravitational field

quantum mechanics
the study of objects smaller than can be seen with a microscope

Physical Quantities and Units

- Perform unit conversions both in the SI and English units.
- Explain the most common prefixes in the SI units and be able to write them in scientific notation.

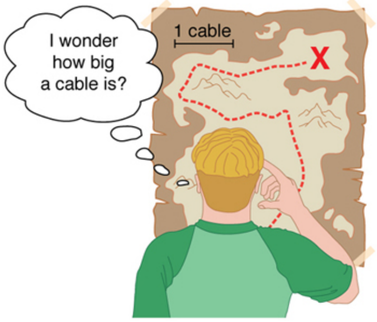


The distance from Earth to the Moon may seem immense, but it is just a tiny fraction of the distances from Earth to other celestial bodies. (credit: NASA)

The range of objects and phenomena studied in physics is immense. From the incredibly short lifetime of a nucleus to the age of the Earth, from the tiny sizes of sub-nuclear particles to the vast distance to the edges of the known universe, from the force exerted by a jumping flea to the force between Earth and the Sun, there are enough factors of 10 to challenge the imagination of even the most experienced scientist. Giving numerical values for physical quantities and equations for physical principles allows us to understand nature much more deeply than does qualitative description alone. To comprehend these vast ranges, we must also have accepted units in which to express them. And we shall find that (even in the potentially mundane discussion of meters, kilograms, and seconds) a profound simplicity of nature appears—all physical quantities can be expressed as combinations of only four fundamental physical quantities: length, mass, time, and electric current.

We define a **physical quantity** either by *specifying how it is measured* or by *stating how it is calculated* from other measurements. For example, we define distance and time by specifying methods for measuring them, whereas we define *average speed* by stating that it is calculated as distance traveled divided by time of travel.

Measurements of physical quantities are expressed in terms of **units**, which are standardized values. For example, the length of a race, which is a physical quantity, can be expressed in units of meters (for sprinters) or kilometers (for distance runners). Without standardized units, it would be extremely difficult for scientists to express and compare measured values in a meaningful way. (See [link].)


I wonder how big a cable is?
1 cable
X

Distances given in unknown units are maddeningly useless.

There are two major systems of units used in the world: **SI units** (also known as the metric system) and **English units** (also known as the customary or imperial system). **English units** were historically used in nations once ruled by the British Empire and are still widely used in the United States. Virtually every other country in the world now uses SI units as the standard; the metric system is also the standard system agreed upon by scientists and mathematicians. The acronym “SI” is derived from the French *Système International*.

SI Units: Fundamental and Derived Units

[link] gives the fundamental SI units that are used throughout this textbook. This text uses non-SI units in a few applications where they are in very common use, such as the measurement of blood pressure in millimeters of mercury (mm Hg). Whenever non-SI units are discussed, they will be tied to SI units through conversions.

Length	Mass	Time	Electric Current
meter (m)	kilogram (kg)	second (s)	ampere (A)

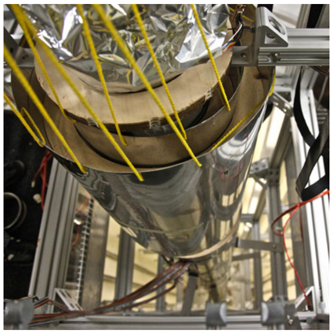
Fundamental SI Units

It is an intriguing fact that some physical quantities are more fundamental than others and that the most fundamental physical quantities can be defined *only* in terms of the procedure used to measure them. The units in which they are measured are thus called **fundamental units**. In this textbook, the fundamental physical quantities are taken to be length, mass, time, and electric current. (Note that electric current will not be introduced until much later in this text.) All other physical quantities, such as force and electric charge, can be expressed as algebraic combinations of length, mass, time, and current (for example, speed is length divided by time); these units are called **derived units**.

Units of Time, Length, and Mass: The Second, Meter, and Kilogram

The Second

The SI unit for time, the **second**(abbreviated s), has a long history. For many years it was defined as 1/86,400 of a mean solar day. More recently, a new standard was adopted to gain greater accuracy and to define the second in terms of a non-varying, or constant, physical phenomenon (because the solar day is getting longer due to very gradual slowing of the Earth’s rotation). Cesium atoms can be made to vibrate in a very steady way, and these vibrations can be readily observed and counted. In 1967 the second was redefined as the time required for 9,192,631,770 of these vibrations. (See [link].) Accuracy in the fundamental units is essential, because all measurements are ultimately expressed in terms of fundamental units and can be no more accurate than are the fundamental units themselves.



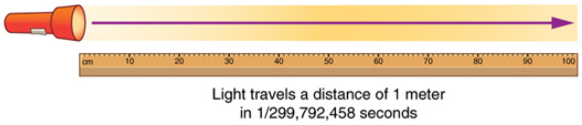
An atomic clock such as this one uses the vibrations of cesium atoms to keep time to a precision of better than a microsecond per year. The fundamental unit of time, the second, is based on such clocks. This image is looking down from the top of an atomic fountain nearly 30 feet tall! (credit: Steve Jurvetson/Flickr)

The Meter

The SI unit for length is the **meter** (abbreviated m); its definition has also changed over time to become more accurate and precise. The meter was first defined in 1791 as 1/10,000,000 of the distance from the equator to the North Pole. This measurement was improved in 1889 by redefining the meter to be the distance between two engraved lines on a platinum-iridium bar now kept near Paris. By 1960, it had become possible to define the meter even more accurately in terms of the wavelength of light, so it was again redefined as 1,650,763.73 wavelengths of orange light emitted by krypton atoms. In 1983, the meter was given its present definition (partly for greater accuracy) as the distance light travels in a vacuum in 1/299,792,458 of a second. (See [link].) This change defines the speed of light to be exactly 299,792,458 meters per second. The length of the meter will change if the speed of light is someday measured with greater accuracy.

The Kilogram

The SI unit for mass is the **kilogram** (abbreviated kg); it is defined to be the mass of a platinum-iridium cylinder kept with the old meter standard at the International Bureau of Weights and Measures near Paris. Exact replicas of the standard kilogram are also kept at the United States' National Institute of Standards and Technology, or NIST, located in Gaithersburg, Maryland outside of Washington D.C., and at other locations around the world. The determination of all other masses can be ultimately traced to a comparison with the standard mass.



The meter is defined to be the distance light travels in 1/299,792,458 of a second in a vacuum. Distance traveled is speed multiplied by time.

Metric Prefixes

SI units are part of the **metric system**. The metric system is convenient for scientific and engineering calculations because the units are categorized by factors of 10. [link] gives metric prefixes and symbols used to denote various factors of 10.

Metric systems have the advantage that conversions of units involve only powers of 10. There are 100 centimeters in a meter, 1000 meters in a kilometer, and so on. In nonmetric systems, such as the system of U.S. customary units, the relationships are not as simple—there are 12 inches in a foot, 5280 feet in a mile, and so on. Another advantage of the metric system is that the same unit can be used over extremely large ranges of values simply by using an appropriate metric prefix. For example, distances in meters are suitable in construction, while distances in kilometers are appropriate for air travel, and the tiny measure of nanometers are convenient in optical design. With the metric system there is no need to invent new units for particular applications.

Note:
The Quest for Microscopic Standards for Basic Units

Prefix	Symbol	Value[footnote] See Appendix A for a discussion of powers of 10.	Example (some are approximate)

Prefix	**Symbol**	**Value[footnote] See Appendix A for a discussion of powers of 10.**	**Example (some are approximate)**			
exa	E	10^{18}	exameter	Em	10^{18} m	distance light travels in a century
peta	P	10^{15}	petasecond	Ps	10^{15} s	30 million years
tera	T	10^{12}	terawatt	TW	10^{12} W	powerful laser output
giga	G	10^{9}	gigahertz	GHz	10^{9} Hz	a microwave frequency
mega	M	10^{6}	megacurie	MCi	10^{6} Ci	high radioactivity
kilo	k	10^{3}	kilometer	km	10^{3} m	about 6/10 mile
hecto	h	10^{2}	hectoliter	hL	10^{2} L	26 gallons
deka	da	10^{1}	dekagram	dag	10^{1} g	teaspoon of butter
—	—	10^{0} (=1)				
deci	d	10^{-1}	deciliter	dL	10^{-1} L	less than half a soda
centi	c	10^{-2}	centimeter	cm	10^{-2} m	fingertip thickness
milli	m	10^{-3}	millimeter	mm	10^{-3} m	flea at its shoulders

Prefix	**Symbol**	**Value[footnote] See Appendix A for a discussion of powers of 10.**	**Example (some are approximate)**			
micro	µ	10^{-6}	micrometer	µm	10^{-6} m	detail in microscope
nano	n	10^{-9}	nanogram	ng	10^{-9} g	small speck of dust
pico	p	10^{-12}	picofarad	pF	10^{-12} F	small capacitor in radio
femto	f	10^{-15}	femtometer	fm	10^{-15} m	size of a proton
atto	a	10^{-18}	attosecond	as	10^{-18} s	time light crosses an atom

Metric Prefixes for Powers of 10 and their Symbols

Unit Conversion and Dimensional Analysis

Let us consider a simple example of how to convert units. Let us say that we want to convert 80 meters (m) to kilometers (km).

The first thing to do is to list the units that you have and the units that you want to convert to. In this case, we have units in *meters* and we want to convert to *kilometers*.

Next, we need to determine a **conversion factor** relating meters to kilometers. A conversion factor is a ratio expressing how many of one unit are equal to another unit. For example, there are 12 inches in 1 foot, 100 centimeters in 1 meter, 60 seconds in 1 minute, and so on. In this case, we know that there are 1,000 meters in 1 kilometer.

Now we can set up our unit conversion. We will write the units that we have and then multiply them by the conversion factor so that the units cancel out, as shown:
Equation:

$$80\ \cancel{\text{m}} \times \frac{1\ \text{km}}{1000\ \cancel{\text{m}}} = 0.080\ \text{km}.$$

Note that the unwanted m unit cancels, leaving only the desired km unit. You can use this method to convert between any types of unit. This long way applies mainly for other conversions than multiples of

10. For a shortcut with multiple of 10 use the table at: http://johnaltounji.weebly.com/si-educational-page.html

Click Appendix C for a more complete list of conversion factors.

Lengths in meters		**Masses in kilograms (more precise values in parentheses)**		**Times in seconds (more precise values in parentheses)**	
10^{-18}	Present experimental limit to smallest observable detail	10^{-30}	Mass of an electron $\left(9.11 \times 10^{-31}\ \text{kg}\right)$	10^{-23}	Time for light to cross a proton
10^{-15}	Diameter of a proton	10^{-27}	Mass of a hydrogen atom $\left(1.67 \times 10^{-27}\ \text{kg}\right)$	10^{-22}	Mean life of an extremely unstable nucleus
10^{-14}	Diameter of a uranium nucleus	10^{-15}	Mass of a bacterium	10^{-15}	Time for one oscillation of visible light
10^{-10}	Diameter of a hydrogen atom	10^{-5}	Mass of a mosquito	10^{-13}	Time for one vibration of an atom in a solid
10^{-8}	Thickness of membranes in cells of living organisms	10^{-2}	Mass of a hummingbird	10^{-8}	Time for one oscillation of an FM radio wave
10^{-6}	Wavelength of visible light	1	Mass of a liter of water (about a quart)	10^{-3}	Duration of a nerve impulse
10^{-3}	Size of a grain of sand	10^{2}	Mass of a person	1	Time for one heartbeat

Lengths in meters		**Masses in kilograms (more precise values in parentheses)**		**Times in seconds (more precise values in parentheses)**	
1	Height of a 4-year-old child	10^3	Mass of a car	10^5	One day $(8.64 \times 10^4\ \text{s})$
10^2	Length of a football field	10^8	Mass of a large ship	10^7	One year (y) $(3.16 \times 10^7\ \text{s})$
10^4	Greatest ocean depth	10^{12}	Mass of a large iceberg	10^9	About half the life expectancy of a human
10^7	Diameter of the Earth	10^{15}	Mass of the nucleus of a comet	10^{11}	Recorded history
10^{11}	Distance from the Earth to the Sun	10^{23}	Mass of the Moon $(7.35 \times 10^{22}\ \text{kg})$	10^{17}	Age of the Earth
10^{16}	Distance traveled by light in 1 year (a light year)	10^{25}	Mass of the Earth $(5.97 \times 10^{24}\ \text{kg})$	10^{18}	Age of the universe
10^{21}	Diameter of the Milky Way galaxy	10^{30}	Mass of the Sun $(1.99 \times 10^{30}\ \text{kg})$		
10^{22}	Distance from the Earth to the nearest large galaxy (Andromeda)	10^{42}	Mass of the Milky Way galaxy (current upper limit)		
10^{26}	Distance from the Earth to the edges of the known universe	10^{53}	Mass of the known universe (current upper limit)		

Approximate Values of Length, Mass, and Time

Example:
Unit Conversions: A Short Drive Home

Suppose that you drive the 10.0 km from your university to home in 20.0 min. Calculate your average speed (a) in kilometers per hour (km/h) and (b) in meters per second (m/s). (Note: Average speed is distance traveled divided by time of travel.)

Strategy

First we calculate the average speed using the given units. Then we can get the average speed into the desired units by picking the correct conversion factor and multiplying by it. The correct conversion factor is the one that cancels the unwanted unit and leaves the desired unit in its place.

Solution for (a)

(1) Calculate average speed. Average speed is distance traveled divided by time of travel. (Take this definition as a given for now—average speed and other motion concepts will be covered in a later module.) In equation form,

Equation:

$$\text{average speed} = \frac{\text{distance}}{\text{time}}.$$

(2) Substitute the given values for distance and time.

Equation:

$$\text{average speed} = \frac{10.0\ \text{km}}{20.0\ \text{min}} = 0.500\frac{\text{km}}{\text{min}}.$$

(3) Convert km/min to km/h: multiply by the conversion factor that will cancel minutes and leave hours. That conversion factor is $60\ \text{min}/\text{hr}$. Thus,

Equation:

$$\text{average speed} = 0.500\frac{\text{km}}{\text{min}} \times \frac{60\ \text{min}}{1\ \text{h}} = 30.0\frac{\text{km}}{\text{h}}.$$

Discussion for (a)

To check your answer, consider the following:

(1) Be sure that you have properly cancelled the units in the unit conversion. If you have written the unit conversion factor upside down, the units will not cancel properly in the equation. If you accidentally get the ratio upside down, then the units will not cancel; rather, they will give you the wrong units as follows:

Equation:

$$\frac{\text{km}}{\text{min}} \times \frac{1\ \text{hr}}{60\ \text{min}} = \frac{1}{60}\frac{\text{km}\cdot\text{hr}}{\text{min}^2},$$

which are obviously not the desired units of km/h.

(2) Check that the units of the final answer are the desired units. The problem asked us to solve for average speed in units of km/h and we have indeed obtained these units.

(3) Check the significant figures. Because each of the values given in the problem has three significant figures, the answer should also have three significant figures. The answer 30.0 km/hr does indeed have three significant figures, so this is appropriate. Note that the significant figures in the conversion factor are not relevant because an hour is *defined* to be 60 minutes, so the precision of the conversion factor is perfect.

(4) Next, check whether the answer is reasonable. Let us consider some information from the problem—if you travel 10 km in a third of an hour (20 min), you would travel three times that far in an hour. The answer does seem reasonable.

Solution for (b)

There are several ways to convert the average speed into meters per second.

(1) Start with the answer to (a) and convert km/h to m/s. Two conversion factors are needed—one to convert hours to seconds, and another to convert kilometers to meters.

(2) Multiplying by these yields

Equation:

$$\text{Average speed} = 30.0\frac{\text{km}}{\text{h}} \times \frac{1\ \text{h}}{3{,}600\ \text{s}} \times \frac{1{,}000\ \text{m}}{1\ \text{km}},$$

Equation:

$$\text{Average speed} = 8.33\frac{\text{m}}{\text{s}}.$$

Discussion for (b)

If we had started with 0.500 km/min, we would have needed different conversion factors, but the answer would have been the same: 8.33 m/s.

You may have noted that the answers in the worked example just covered were given to three digits. Why? When do you need to be concerned about the number of digits in something you calculate? Why not write down all the digits your calculator produces? The module Accuracy, Precision, and Significant Figures will help you answer these questions.

Note:

Nonstandard Units

Exercise:

Check Your Understanding

Problem:

Some hummingbirds beat their wings more than 50 times per second. A scientist is measuring the time it takes for a hummingbird to beat its wings once. Which fundamental unit should the scientist use to describe the measurement? Which factor of 10 is the scientist likely to use to describe the motion precisely? Identify the metric prefix that corresponds to this factor of 10.

Solution:

The scientist will measure the time between each movement using the fundamental unit of seconds. Because the wings beat so fast, the scientist will probably need to measure in milliseconds, or 10^{-3} seconds. (50 beats per second corresponds to 20 milliseconds per beat.)

Exercise:

Check Your Understanding

Problem:

One cubic centimeter is equal to one milliliter. What does this tell you about the different units in the SI metric system?

Solution:

The fundamental unit of length (meter) is probably used to create the derived unit of volume (liter). The measure of a milliliter is dependent on the measure of a centimeter.

Summary

- Physical quantities are a characteristic or property of an object that can be measured or calculated from other measurements.
- Units are standards for expressing and comparing the measurement of physical quantities. All units can be expressed as combinations of four fundamental units.
- The four fundamental units we will use in this text are the meter (for length), the kilogram (for mass), the second (for time), and the ampere (for electric current). These units are part of the metric system, which uses powers of 10 to relate quantities over the vast ranges encountered in nature.
- The four fundamental units are abbreviated as follows: meter, m; kilogram, kg; second, s; and ampere, A. The metric system also uses a standard set of prefixes to denote each order of magnitude greater than or lesser than the fundamental unit itself.
- Unit conversions involve changing a value expressed in one type of unit to another type of unit. This is done by using conversion factors, which are ratios relating equal quantities of different units.

Conceptual Questions

Exercise:

Problem: Identify some advantages of metric units.

Problems & Exercises

Exercise:

Problem:

The speed limit on some interstate highways is roughly 100 km/h. (a) What is this in meters per second? (b) How many miles per hour is this?

Solution:

a. 27.8 m/s
b. 62.1 mph

Exercise:

Problem:

A car is traveling at a speed of 33 m/s. (a) What is its speed in kilometers per hour? (b) Is it exceeding the 90 km/h speed limit?

Exercise:

Problem:

Show that $1.0\text{ m/s} = 3.6\text{ km/h}$. Hint: Show the explicit steps involved in converting $1.0\text{ m/s} = 3.6\text{ km/h}$.

Solution:

$$\frac{1.0\text{ m}}{\text{s}} = \frac{1.0\text{ m}}{\text{s}} \times \frac{3600\text{ s}}{1\text{ hr}} \times \frac{1\text{ km}}{1000\text{ m}}$$

$$= 3.6\text{ km/h}.$$

Exercise:

Problem:

American football is played on a 100-yd-long field, excluding the end zones. How long is the field in meters? (Assume that 1 meter equals 3.281 feet.)

Exercise:

Problem:

Soccer fields vary in size. A large soccer field is 115 m long and 85 m wide. What are its dimensions in feet and inches? (Assume that 1 meter equals 3.281 feet.)

Solution:

length: 377 ft; $4.53\times 10^{3}\text{ in.}$ width: 280 ft; $3.3\times 10^{3}\text{ in.}$

Exercise:

Problem:

What is the height in meters of a person who is 6 ft 1.0 in. tall? (Assume that 1 meter equals 39.37 in.)

Exercise:

Problem:

Mount Everest, at 29,028 feet, is the tallest mountain on the Earth. What is its height in kilometers? (Assume that 1 kilometer equals 3,281 feet.)

Solution:

8.847 km

Exercise:

Problem: The speed of sound is measured to be 342 m/s on a certain day. What is this in km/h?

Exercise:

Problem:

Tectonic plates are large segments of the Earth’s crust that move slowly. Suppose that one such plate has an average speed of 4.0 cm/year. (a) What distance does it move in 1 s at this speed? (b) What is its speed in kilometers per million years?

Solution:

(a) $1.3\times 10^{-9}\text{ m}$

(b) 40 km/My

Exercise:

Problem:

(a) Refer to [link] to determine the average distance between the Earth and the Sun. Then calculate the average speed of the Earth in its orbit in kilometers per second. (b) What is this in meters per second?

Glossary

physical quantity
a characteristic or property of an object that can be measured or calculated from other measurements

units
a standard used for expressing and comparing measurements

SI units
the international system of units that scientists in most countries have agreed to use; includes units such as meters, liters, and grams

English units
system of measurement used in the United States; includes units of measurement such as feet, gallons, and pounds

fundamental units
units that can only be expressed relative to the procedure used to measure them

derived units
units that can be calculated using algebraic combinations of the fundamental units

second
the SI unit for time, abbreviated (s)

meter
the SI unit for length, abbreviated (m)

kilogram
the SI unit for mass, abbreviated (kg)

metric system
a system in which values can be calculated in factors of 10

order of magnitude
refers to the size of a quantity as it relates to a power of 10

conversion factor
a ratio expressing how many of one unit are equal to another unit

Introduction to One-Dimensional Kinematics class="introduction"

The motion of an American kestrel through the air can be described by the bird’s displacement, speed, velocity, and acceleration. When it flies in a straight line without any change in direction, its motion is said to be one dimensional. (credit: Vince Maidens, Wikimedia Commons)

Objects are in motion everywhere we look. Everything from a tennis game to a space-probe flyby of the planet Neptune involves motion. When you are resting, your heart moves blood through your veins. And even in inanimate objects, there is continuous motion in the vibrations of atoms and molecules. Questions about motion are interesting in and of themselves: *How long will it take for a space probe to get to Mars? Where will a football land if it is thrown at a certain angle?* But an understanding of motion is also key to understanding other concepts in physics. An understanding of acceleration, for example, is crucial to the study of force.

Our formal study of physics begins with **kinematics** which is defined as the *study of motion without considering its causes*. The word “kinematics” comes from a Greek term meaning motion and is related to other English words such as “cinema” (movies) and “kinesiology” (the study of human motion). In this chapter, we examine the simplest type of motion—namely, motion along a straight line, or one-dimensional motion.

Displacement

- Define position, displacement, distance, and distance traveled.
- Explain the relationship between position and displacement.
- Distinguish between displacement and distance traveled.
- Calculate displacement and distance given initial position, final position, and the path between the two.

These cyclists in Vietnam can be described by their position relative to buildings and a canal. Their motion can be described by their change in position, or displacement, in the frame of reference. (credit: Suzan Black, Fotopedia)

Position

In order to describe the motion of an object, you must first be able to describe its **position**—where it is at any particular time. More precisely, you need to specify its position relative to a convenient reference frame. Earth is often used as a reference frame, and we often describe the position of an object as it relates to stationary objects in that reference frame. For example, a rocket launch would be described in terms of the position of the rocket with respect to the Earth as a whole, while a professor's position could be described in terms of where she is in relation to the nearby white board. (See [link].) In other cases, we use reference frames that are not stationary but are in motion relative to the Earth. To describe the position of a person in an airplane, for example, we use the airplane, not the Earth, as the reference frame. (See [link].)

Displacement

If an object moves relative to a reference frame (for example, if a professor moves to the right relative to a white board or a passenger moves toward the rear of an airplane), then the object's position changes. This change in position is known as **displacement**. The word "displacement" implies that an object has moved, or has been displaced.

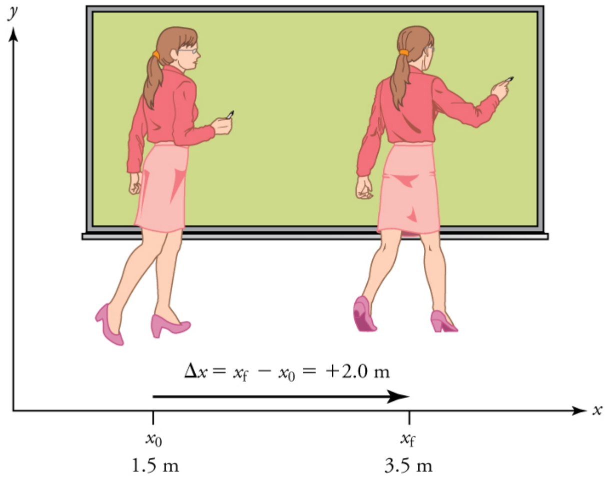
Note:
Displacement
Displacement is the *change in position* of an object:
Equation:

$$\Delta x = x_\text{f} - x_0,$$

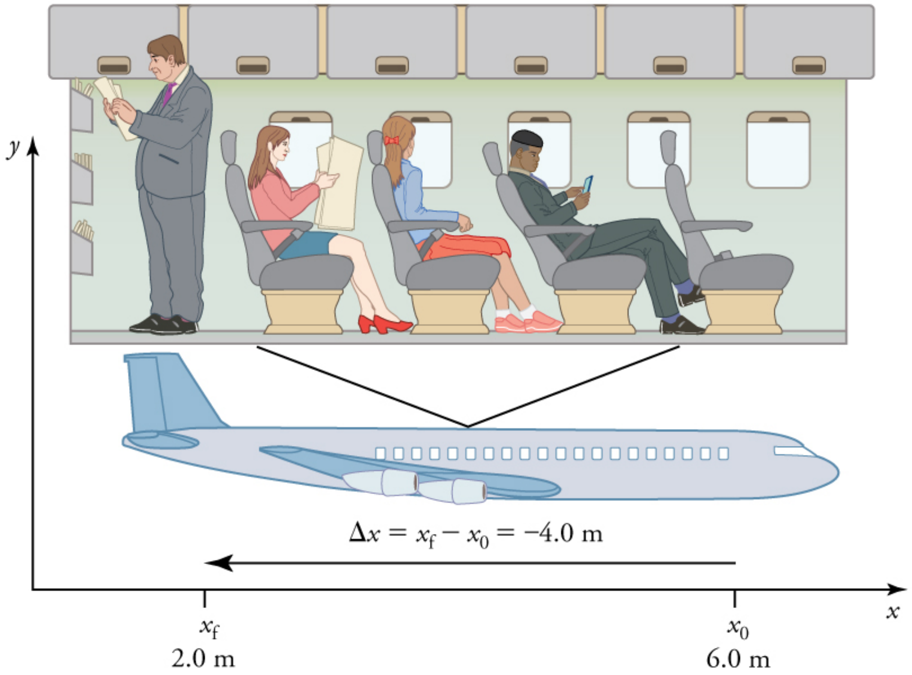
where Δx is displacement, x_f is the final position, and x_0 is the initial position.

In this text the upper case Greek letter Δ (delta) always means "change in" whatever quantity follows it; thus, Δx means *change in position*. Always solve for displacement by subtracting initial position x_0 from final position x_f.

Note that the SI unit for displacement is the meter (m) (see Physical Quantities and Units), but sometimes kilometers and other units of length are used. Keep in mind that when units other than the meter are used in a problem, you may need to convert them into meters to complete the calculation.



A professor paces left and right while lecturing. Her position relative to Earth is given by x. The $+2.0$ m displacement of the professor relative to Earth is represented by an arrow pointing to the right.


y
$\Delta x = x_f - x_0 = -4.0$ m
x
x_f
2.0 m
x_0
6.0 m

A passenger moves from his seat to the back of the plane. His location relative to the airplane is given by x. The -4.0-m displacement of the passenger relative to the plane is represented by an arrow toward the rear of the plane. Notice that the arrow representing his displacement is twice as long as the arrow representing the displacement of the professor (he moves twice as far) in [link].

Note that displacement has a direction as well as a magnitude. The professor's displacement is 2.0 m to the right, and the airline passenger's displacement is 4.0 m toward the rear. In one-dimensional motion, direction can be specified with a plus or minus sign. When you begin a problem, you should select which direction is positive (usually that will be to the right or up, but you are free to select positive as being any direction). The professor's initial position is $x_0 = 1.5\text{ m}$ and her final position is $x_f = 3.5\text{ m}$. Thus her displacement is

Equation:

$$\Delta x = x_f - x_0 = 3.5\text{ m} - 1.5\text{ m} = +2.0\text{ m}.$$

In this coordinate system, motion to the right is positive, whereas motion to the left is negative. Similarly, the airplane passenger's initial position is $x_0 = 6.0\ \text{m}$ and his final position is $x_\text{f} = 2.0\ \text{m}$, so his displacement is

Equation:

$$\Delta x = x_\text{f} - x_0 = 2.0\ \text{m} - 6.0\ \text{m} = -4.0\ \text{m}.$$

His displacement is negative because his motion is toward the rear of the plane, or in the negative x direction in our coordinate system.

Distance

Although displacement is described in terms of direction, distance is not. **Distance** is defined to be *the magnitude or size of displacement between two positions*. Note that the distance between two positions is not the same as the distance traveled between them. **Distance traveled** is *the total length of the path traveled between two positions*. Distance has no direction and, thus, no sign. For example, the distance the professor walks is 2.0 m. The distance the airplane passenger walks is 4.0 m.

Note:

Misconception Alert: Distance Traveled vs. Magnitude of Displacement

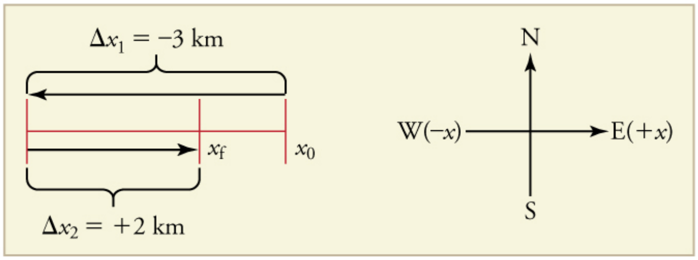
It is important to note that the *distance traveled*, however, can be greater than the magnitude of the displacement (by magnitude, we mean just the size of the displacement without regard to its direction; that is, just a number with a unit). For example, the professor could pace back and forth many times, perhaps walking a distance of 150 m during a lecture, yet still end up only 2.0 m to the right of her starting point. In this case her displacement would be +2.0 m, the magnitude of her displacement would be 2.0 m, but the distance she traveled would be 150 m. In kinematics we nearly always deal with displacement and magnitude of displacement, and almost never with distance traveled. One way to think about this is to assume you marked the start of the motion and the end of the motion. The displacement is simply the difference in the position of the two marks and is independent of the path taken in traveling between the two marks. The distance traveled, however, is the total length of the path taken between the two marks.

Exercise:
Check Your Understanding

Problem:

A cyclist rides 3 km west and then turns around and rides 2 km east. (a) What is her displacement? (b) What distance does she ride? (c) What is the magnitude of her displacement?

Solution:



(a) The rider’s displacement is $\Delta x = x_{\text{f}} - x_0 = -1\text{ km}$. (The displacement is negative because we take east to be positive and west to be negative.)

(b) The distance traveled is $3\text{ km} + 2\text{ km} = 5\text{ km}$.

(c) The magnitude of the displacement is 1 km.

Section Summary

- Kinematics is the study of motion without considering its causes. In this chapter, it is limited to motion along a straight line, called one-dimensional motion.
- Displacement is the change in position of an object.

- In symbols, displacement Δx is defined to be
  Equation:

  $$\Delta x = x_{\text{f}} - x_0,$$

  where x_0 is the initial position and x_{f} is the final position. In this text, the Greek letter Δ (delta) always means “change in” whatever quantity follows it. The SI unit for displacement is the meter (m). Displacement has a direction as well as a magnitude.
- When you start a problem, assign which direction will be positive.
- Distance is the magnitude of displacement between two positions.
- Distance traveled is the total length of the path traveled between two positions.

Conceptual Questions

Exercise:

Problem:

Give an example in which there are clear distinctions among distance traveled, displacement, and magnitude of displacement. Specifically identify each quantity in your example.

Exercise:

Problem:

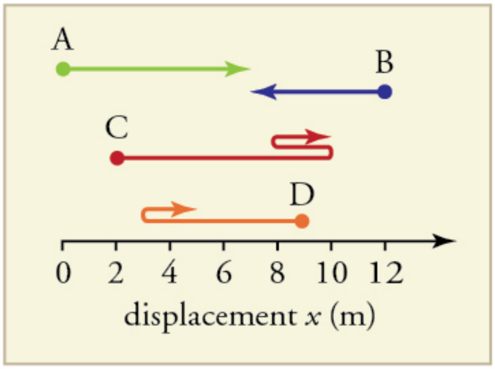
Under what circumstances does distance traveled equal magnitude of displacement? What is the only case in which magnitude of displacement and displacement are exactly the same?

Exercise:

Problem:

Bacteria move back and forth by using their flagella (structures that look like little tails). Speeds of up to $50\ \mu\text{m/s}$ $\left(50 \times 10^{-6}\ \text{m/s}\right)$ have been observed. The total distance traveled by a bacterium is large for its size, while its displacement is small. Why is this?

Problems & Exercises



Exercise:

Problem:

Find the following for path A in [link]: (a) The distance traveled. (b) The magnitude of the displacement from start to finish. (c) The displacement from start to finish.

Solution:

(a) 7 m

(b) 7 m

(c) $+7$ m

Exercise:

Problem:

Find the following for path B in [link]: (a) The distance traveled. (b) The magnitude of the displacement from start to finish. (c) The displacement from start to finish.

Exercise:

Problem:

Find the following for path C in [link]: (a) The distance traveled. (b) The magnitude of the displacement from start to finish. (c) The displacement from start to finish.

Solution:

a) 8m + 2 m + 3 m = 12 m; b) 8 m; c) Δx = 11 m – 2 m = +9 m

Exercise:

Problem:

Find the following for path D in [link]: (a) The distance traveled. (b) The magnitude of the displacement from start to finish. (c) The displacement from start to finish.

Glossary

kinematics
the study of motion without considering its causes

position
the location of an object at a particular time

displacement
the change in position of an object

distance
the magnitude of displacement between two positions

distance traveled
the total length of the path traveled between two positions

Time, Velocity, and Speed

- Explain the relationships between instantaneous velocity, average velocity, instantaneous speed, average speed, displacement, and time.
- Calculate velocity and speed given initial position, initial time, final position, and final time.
- Derive a graph of velocity vs. time given a graph of position vs. time.
- Interpret a graph of velocity vs. time.

The motion of these racing snails can be described by their speeds and their velocities. (credit: tobitasflickr, Flickr)

There is more to motion than distance and displacement. Questions such as, “How long does a foot race take?” and “What was the runner’s speed?” cannot be answered without an understanding of other concepts. In this section we add definitions of time, velocity, and speed to expand our description of motion.

Time

As discussed in Physical Quantities and Units, the most fundamental physical quantities are defined by how they are measured. This is the case with time. Every measurement of time involves measuring a change in some physical quantity. It may be a number on a digital clock, a heartbeat, or the position of the Sun in the sky. In physics, the definition of time is simple—**time** is *change*, or the interval over which change occurs. It is impossible to know that time has passed unless something changes.

The amount of time or change is calibrated by comparison with a standard. The SI unit for time is the second, abbreviated s.

How does time relate to motion? We are usually interested in elapsed time for a particular motion, such as how long it takes an airplane passenger to get from his seat to the back of the plane. To find elapsed time, we note the time at the beginning and end of the motion and subtract the two. For example, a lecture may start at 11:00 A.M. and end at 11:50 A.M., so that the elapsed time would be 50 min. **Elapsed time** Δt is the difference between the ending time and beginning time,

Equation:

$$\Delta t = t_{\text{f}} - t_0,$$

where Δt is the change in time or elapsed time, t_{f} is the time at the end of the motion, and t_0 is the time at the beginning of the motion. (As usual, the delta symbol, Δ, means the change in the quantity that follows it.)

Life is simpler if the beginning time t_0 is taken to be zero, as when we use a stopwatch. If we were using a stopwatch, it would simply read zero at the start of the lecture and 50 min at the end. If $t_0 = 0$, then $\Delta t = t_{\text{f}} \equiv t$.

In this text, for simplicity's sake,

- motion starts at time equal to zero $(t_0 = 0)$
- the symbol t is used for elapsed time unless otherwise specified $(\Delta t = t_{\text{f}} \equiv t)$

Velocity

Your notion of velocity is probably the same as its scientific definition. You know that if you have a large displacement in a small amount of time you have a large velocity, and that velocity has units of distance divided by time, such as miles per hour or kilometers per hour.

Note:
Average Velocity
Average velocity is *displacement (change in position) divided by the time of travel*,
Equation:

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{x_\text{f} - x_0}{t_\text{f} - t_0},$$

where $\bar{v}$ is the *average* (indicated by the bar over the v) velocity, Δx is the change in position (or displacement), and x_f and x_0 are the final and beginning positions at times t_f and t_0, respectively. If the starting time t_0 is taken to be zero, then the average velocity is simply
Equation:

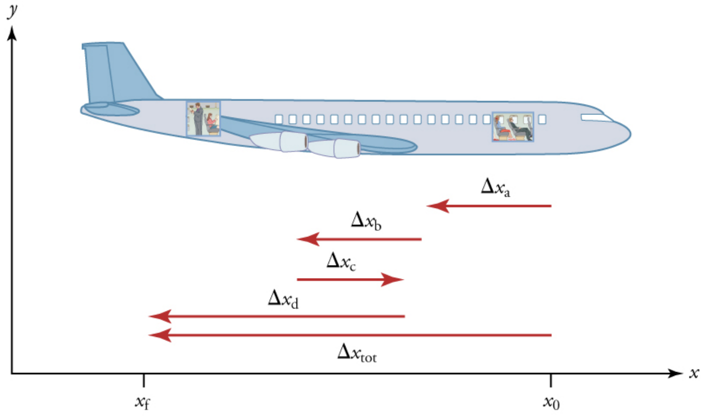
$$\bar{v} = \frac{\Delta x}{t}.$$

Notice that this definition indicates that *velocity is a vector because displacement is a vector*. It has both magnitude and direction. The SI unit for velocity is meters per second or m/s, but many other units, such as km/h and cm/s, are in common use. Suppose, for example, an airplane passenger took 5 seconds to move −4 m (the negative sign indicates that displacement is toward the back of the plane). His average velocity would be
Equation:

$$\bar{v} = \frac{\Delta x}{t} = \frac{-4\text{ m}}{5\text{ s}} = -0.8\text{ m/s}.$$

The minus sign indicates the average velocity is also toward the rear of the plane.

The average velocity of an object does not tell us anything about what happens to it between the starting point and ending point, however. For example, we cannot tell from average velocity whether the airplane passenger stops momentarily or backs up before he goes to the back of the plane. To get more details, we must consider smaller segments of the trip over smaller time intervals.



A more detailed record of an airplane passenger heading toward the back of the plane, showing smaller segments of his trip.

The smaller the time intervals considered in a motion, the more detailed the information. When we carry this process to its logical conclusion, we are left with an infinitesimally small interval. Over such an interval, the average velocity becomes the *instantaneous velocity* or the *velocity at a specific instant*. A car’s speedometer, for example, shows the magnitude (but not the direction) of the instantaneous velocity of the car. (Police give tickets based on instantaneous velocity, but when calculating how long it will take to get from one place to another on a road trip, you need to use average velocity.)

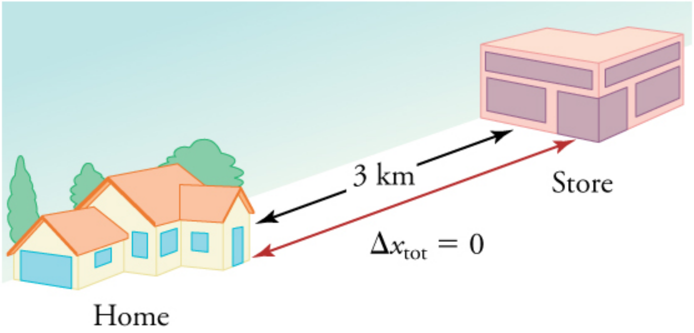
Instantaneous velocity v is the average velocity at a specific instant in time (or over an infinitesimally small time interval).

Speed

In everyday language, most people use the terms “speed” and “velocity” interchangeably. In physics, however, they do not have the same meaning and they are distinct concepts. One major difference is that speed has no direction. Thus *speed is a scalar*. Just as we need to distinguish between instantaneous velocity and average velocity, we also need to distinguish between instantaneous speed and average speed.

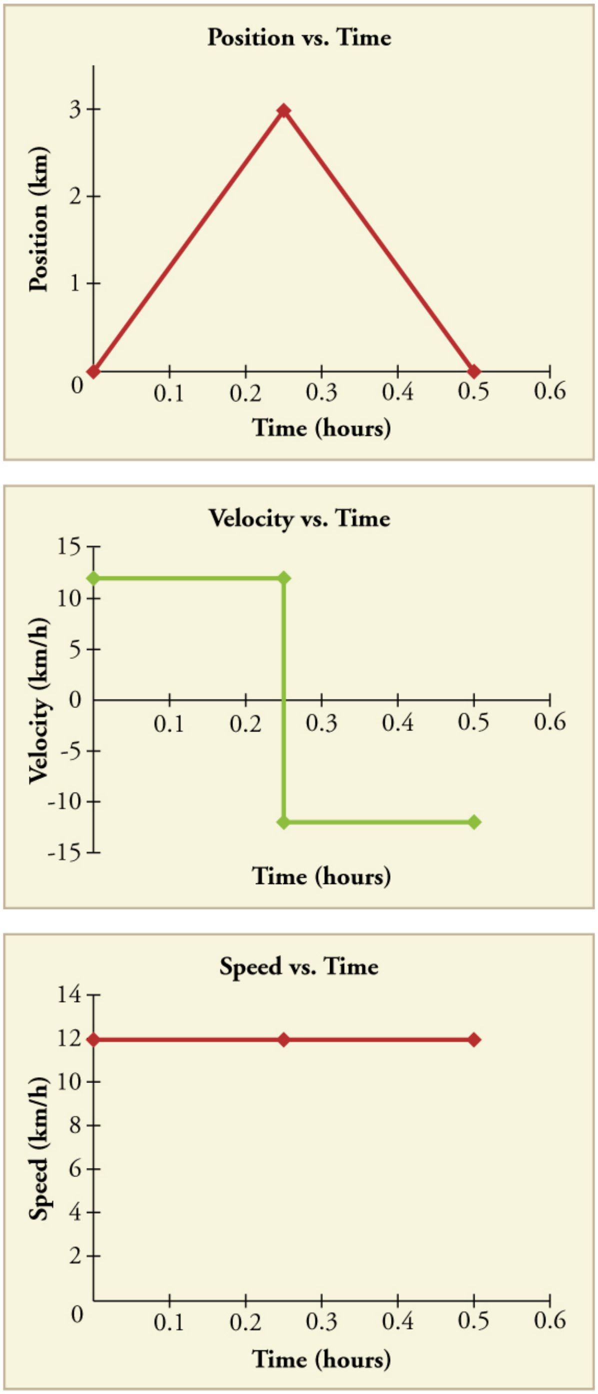
Instantaneous speed is the magnitude of instantaneous velocity. For example, suppose the airplane passenger at one instant had an instantaneous velocity of −3.0 m/s (the minus meaning toward the rear of the plane). At that same time his instantaneous speed was 3.0 m/s. Or suppose that at one time during a shopping trip your instantaneous velocity is 40 km/h due north. Your instantaneous speed at that instant would be 40 km/h—the same magnitude but without a direction. Average speed, however, is very different from average velocity. **Average speed** is the distance traveled divided by elapsed time.

We have noted that distance traveled can be greater than the magnitude of displacement. So average speed can be greater than average velocity, which is displacement divided by time. For example, if you drive to a store and return home in half an hour, and your car’s odometer shows the total distance traveled was 6 km, then your average speed was 12 km/h. Your average velocity, however, was zero, because your displacement for the round trip is zero. (Displacement is change in position and, thus, is zero for a round trip.) Thus average speed is *not* simply the magnitude of average velocity.



During a 30-minute round trip to the store, the total distance traveled is 6 km. The average speed is 12 km/h. The displacement for the round trip is zero, since there was no net change in position. Thus the average velocity is zero.

Another way of visualizing the motion of an object is to use a graph. A plot of position or of velocity as a function of time can be very useful. For example, for this trip to the store, the position, velocity, and speed-vs.-time graphs are displayed in [link]. (Note that these graphs depict a very simplified **model** of the trip. We are assuming that speed is constant during the trip, which is unrealistic given that we'll probably stop at the store. But for simplicity's sake, we will model it with no stops or changes in speed. We are also assuming that the route between the store and the house is a perfectly straight line.)



Position vs. Time

Velocity vs. Time

Speed vs. Time

Position vs. time, velocity vs. time, and speed vs. time on a trip. Note that the velocity for the return trip is negative.

Note:

Making Connections: Take-Home Investigation—Getting a Sense of Speed

If you have spent much time driving, you probably have a good sense of speeds between about 10 and 70 miles per hour. But what are these in meters per second? What do we mean when we say that something is moving at 10 m/s? To get a better sense of what these values really mean, do some observations and calculations on your own:

- calculate typical car speeds in meters per second
- estimate jogging and walking speed by timing yourself; convert the measurements into both m/s and mi/h
- determine the speed of an ant, snail, or falling leaf

Exercise:

Check Your Understanding

Problem:

A commuter train travels from Baltimore to Washington, DC, and back in 1 hour and 45 minutes. The distance between the two stations is approximately 65 km. What is (a) the average velocity of the train, and (b) the average speed of the train in m/s?

Solution:

(a) The average velocity of the train is zero because $x_\text{f} = x_0$; the train ends up at the same place it starts.

(b) The average speed of the train is calculated below. Note that the train travels 65 km one way and 65 km back, for a total distance of 130 km.

Equation:

$$\frac{\text{distance}}{\text{time}} = \frac{130\text{ km}}{105\text{ minutes}}$$

Equation:

$$\frac{130\text{ km}}{105\text{ minutes}} \times \frac{1\text{ minute}}{60\text{ seconds}} = 20.6\text{ m/s}$$

Section Summary

- Time is measured in terms of change, and its SI unit is the second (s). Elapsed time for an event is
  Equation:

  $$\Delta t = t_{\text{f}} - t_0,$$

  where t_{f} is the final time and t_0 is the initial time. The initial time is often taken to be zero, as if measured with a stopwatch; the elapsed time is then just t.
- Average velocity $\bar{v}$ is defined as displacement divided by the travel time. In symbols, average velocity is
  Equation:

  $$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{x_{\text{f}} - x_0}{t_{\text{f}} - t_0}.$$

- The SI unit for velocity is m/s.
- Velocity is a vector and thus has a direction.
- Instantaneous velocity v is the velocity at a specific instant or the average velocity for an infinitesimal interval.
- Instantaneous speed is the magnitude of the instantaneous velocity.
- Instantaneous speed is a scalar quantity, as it has no direction specified.
- Average speed is the total distance traveled divided by the elapsed time. (Average speed is *not* the magnitude of the average velocity.) Speed is a scalar quantity; it has no direction associated with it.

Conceptual Questions

Exercise:

Problem:

Give an example (but not one from the text) of a device used to measure time and identify what change in that device indicates a change in time.

Exercise:

Problem:

There is a distinction between average speed and the magnitude of average velocity. Give an example that illustrates the difference between these two quantities.

Exercise:

Problem:

Does a car’s odometer measure position or displacement? Does its speedometer measure speed or velocity?

Exercise:

Problem:

If you divide the total distance traveled on a car trip (as determined by the odometer) by the time for the trip, are you calculating the average speed or the magnitude of the average velocity? Under what circumstances are these two quantities the same?

Exercise:

Problem:

How are instantaneous velocity and instantaneous speed related to one another? How do they differ?

Problems & Exercises

Exercise:

Problem:

(a) Calculate Earth’s average speed relative to the Sun. (b) What is its average velocity over a period of one year?

Solution:

(a) $3.0 \times 10^4\ \text{m/s}$

(b) 0 m/s

Exercise:

Problem:

A helicopter blade spins at exactly 100 revolutions per minute. Its tip is 5.00 m from the center of rotation. (a) Calculate the average speed of the blade tip in the helicopter’s frame of reference. (b) What is its average velocity over one revolution?

Exercise:

Problem:

The North American and European continents are moving apart at a rate of about 3 cm/y. At this rate how long will it take them to drift 500 km farther apart than they are at present?

Solution:

2×10^7 years

Exercise:

Problem:

Land west of the San Andreas fault in southern California is moving at an average velocity of about 6 cm/y northwest relative to land east of the fault. Los Angeles is west of the fault and may thus someday be at the same latitude as San Francisco, which is east of the fault. How far in the future will this occur if the displacement to be made is 590 km northwest, assuming the motion remains constant?

Exercise:

Problem:

On May 26, 1934, a streamlined, stainless steel diesel train called the Zephyr set the world's nonstop long-distance speed record for trains. Its run from Denver to Chicago took 13 hours, 4 minutes, 58 seconds, and was witnessed by more than a million people along the route. The total distance traveled was 1633.8 km. What was its average speed in km/h and m/s?

Solution:

$34.689\text{ m/s} = 124.88\text{ km/h}$

Exercise:

Problem:

Tidal friction is slowing the rotation of the Earth. As a result, the orbit of the Moon is increasing in radius at a rate of approximately 4 cm/year. Assuming this to be a constant rate, how many years will pass before the radius of the Moon's orbit increases by 3.84×10^6 m (1%)?

Exercise:

Problem:

A student drove to the university from her home and noted that the odometer reading of her car increased by 12.0 km. The trip took 18.0 min. (a) What was her average speed? (b) If the straight-line distance from her home to the university is 10.3 km in a direction 25.0° south of east, what was her average velocity? (c) If she returned home by the same path 7 h 30 min after she left, what were her average speed and velocity for the entire trip?

Solution:

(a) 40.0 km/h

(b) 34.3 km/h, $25^{\circ}\text{ S of E.}$

(c) $\text{average speed} = 3.20\text{ km/h}, \bar{v} = 0.$

Exercise:

Problem:

The speed of propagation of the action potential (an electrical signal) in a nerve cell depends (inversely) on the diameter of the axon (nerve fiber). If the nerve cell connecting the spinal cord to your feet is 1.1 m long, and the nerve impulse speed is 18 m/s, how long does it take for the nerve signal to travel this distance?

Exercise:

Problem:

Conversations with astronauts on the lunar surface were characterized by a kind of echo in which the earthbound person's voice was so loud in the astronaut's space helmet that it was picked up by the astronaut's microphone and transmitted back to Earth. It is reasonable to assume that the echo time equals the time necessary for the radio wave to travel from the Earth to the Moon and back (that is, neglecting any time delays in the electronic equipment). Calculate the distance from Earth to the Moon given that the echo time was 2.56 s and that radio waves travel at the speed of light $(3.00 \times 10^8\ \text{m/s})$.

Solution:

384,000 km

Exercise:

Problem:

A football quarterback runs 15.0 m straight down the playing field in 2.50 s. He is then hit and pushed 3.00 m straight backward in 1.75 s. He breaks the tackle and runs straight forward another 21.0 m in 5.20 s. Calculate his average velocity (a) for each of the three intervals and (b) for the entire motion.

Exercise:

Problem:

The planetary model of the atom pictures electrons orbiting the atomic nucleus much as planets orbit the Sun. In this model you can view hydrogen, the simplest atom, as having a single electron in a circular orbit $1.06 \times 10^{-10}\ \text{m}$ in diameter. (a) If the average speed of the electron in this orbit is known to be $2.20 \times 10^6\ \text{m/s}$, calculate the number of revolutions per second it makes about the nucleus. (b) What is the electron's average velocity?

Solution:

(a) $6.61 \times 10^{15}\ \mathrm{rev/s}$

(b) 0 m/s

Glossary

average speed
distance traveled divided by time during which motion occurs

average velocity
displacement divided by time over which displacement occurs

instantaneous velocity
velocity at a specific instant, or the average velocity over an infinitesimal time interval

instantaneous speed
magnitude of the instantaneous velocity

time
change, or the interval over which change occurs

model
simplified description that contains only those elements necessary to describe the physics of a physical situation

elapsed time
the difference between the ending time and beginning time

Acceleration

- Define and distinguish between instantaneous acceleration, average acceleration, and deceleration.
- Calculate acceleration given initial time, initial velocity, final time, and final velocity.

A plane decelerates, or slows down, as it comes in for landing in St. Maarten. Its acceleration is opposite in direction to its velocity. (credit: Steve Conry, Flickr)

In everyday conversation, to accelerate means to speed up. The accelerator in a car can in fact cause it to speed up. The greater the **acceleration**, the greater the change in velocity over a given time. The formal definition of acceleration is consistent with these notions, but more inclusive.

Note:
Average Acceleration
Average Acceleration is *the rate at which velocity changes,*
Equation:

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{v_{\text{f}} - v_0}{t_{\text{f}} - t_0},$$

where $\bar{a}$ is average acceleration, v is velocity, and t is time. (The bar over the a means *average* acceleration.)

Because acceleration is velocity in m/s divided by time in s, the SI units for acceleration are m/s^2, meters per second squared or meters per second per second, which literally means by how many meters per second the velocity changes every second.

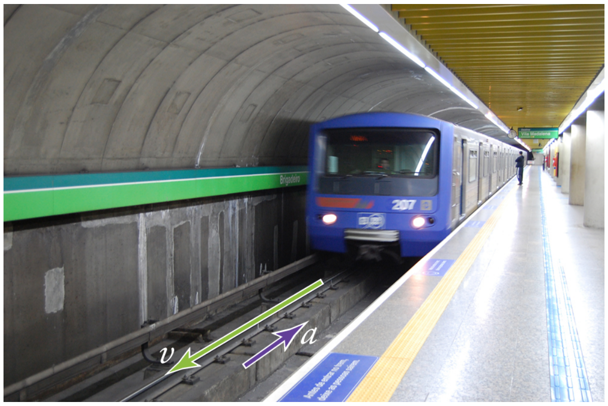
Recall that velocity is a vector—it has both magnitude and direction. This means that a change in velocity can be a change in magnitude (or speed), but it can also be a change in *direction*. For example, if a car turns a corner at constant speed, it is accelerating because its direction is changing. The quicker you turn, the greater the acceleration. So there is an acceleration when velocity changes either in magnitude (an increase or decrease in speed) or in direction, or both.

Note:

Acceleration as a Vector

Acceleration is a vector in the same direction as the *change* in velocity, Δv. Since velocity is a vector, it can change either in magnitude or in direction. Acceleration is therefore a change in either speed or direction, or both.

Keep in mind that although acceleration is in the direction of the *change* in velocity, it is not always in the direction of *motion*. When an object slows down, its acceleration is opposite to the direction of its motion. This is known as **deceleration**.

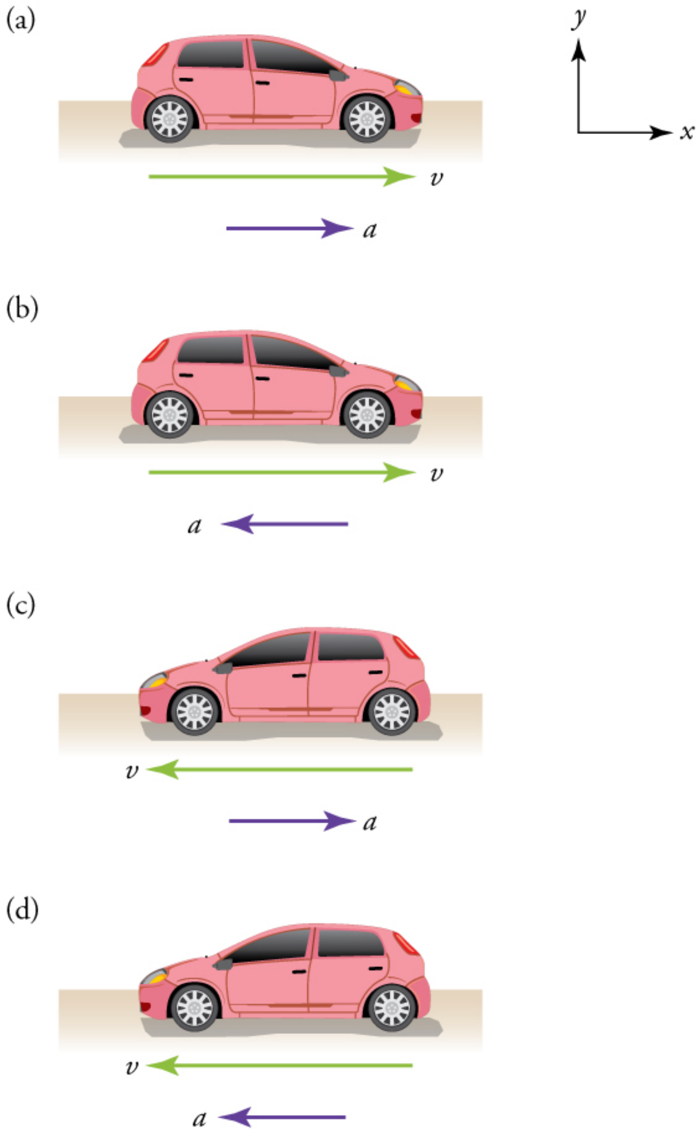


A subway train in Sao Paulo, Brazil, decelerates as it comes into a station. It is accelerating in a direction opposite to its direction of motion. (credit: Yusuke Kawasaki, Flickr)

Note:

Misconception Alert: Deceleration vs. Negative Acceleration

Deceleration always refers to acceleration in the direction opposite to the direction of the velocity. Deceleration always reduces speed. Negative acceleration, however, is acceleration *in the negative direction in the chosen coordinate system*. Negative acceleration may or may not be deceleration, and deceleration may or may not be considered negative acceleration. For example, consider [link].



(a) This car is speeding up as it moves toward the right. It therefore has positive acceleration in our coordinate system. (b) This car is slowing down as it moves toward the right. Therefore, it has negative acceleration in our coordinate system, because its acceleration is toward the left. The car is also decelerating: the direction of its acceleration is opposite to its direction of motion. (c) This car is moving toward the left, but slowing down over time. Therefore, its acceleration is positive in our coordinate system because it is toward the right. However, the car is decelerating because its acceleration is opposite to its motion. (d) This car is speeding up as it moves toward the left. It has negative acceleration because it is accelerating toward the left. However, because its acceleration is in the same direction as its motion, it is speeding up (*not* decelerating).

Example:

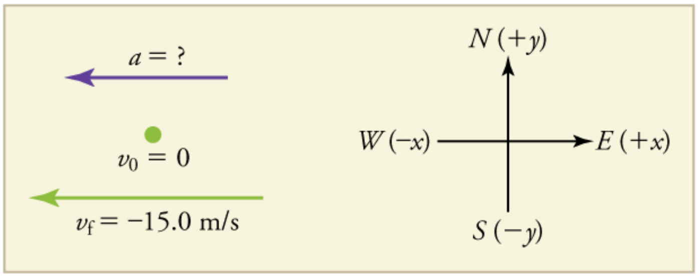
Calculating Acceleration: A Racehorse Leaves the Gate

A racehorse coming out of the gate accelerates from rest to a velocity of 15.0 m/s due west in 1.80 s. What is its average acceleration?

(credit: Jon Sullivan, PD Photo.org)

Strategy

First we draw a sketch and assign a coordinate system to the problem. This is a simple problem, but it always helps to visualize it. Notice that we assign east as positive and west as negative. Thus, in this case, we have negative velocity.



We can solve this problem by identifying Δv and Δt from the given information and then calculating the average acceleration directly from the equation $\bar{a} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_0}{t_f - t_0}$.

Solution

1. Identify the knowns. $v_0 = 0$, $v_f = -15.0\text{ m/s}$ (the negative sign indicates direction toward the west), $\Delta t = 1.80\text{ s}$.
2. Find the change in velocity. Since the horse is going from zero to -15.0 m/s, its change in velocity equals its final velocity: $\Delta v = v_f = -15.0\text{ m/s}$.
3. Plug in the known values (Δv and Δt) and solve for the unknown $\bar{a}$.

Equation:

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{-15.0\text{ m/s}}{1.80\text{ s}} = -8.33\text{ m/s}^2.$$

Discussion

The negative sign for acceleration indicates that acceleration is toward the west. An acceleration of 8.33 m/s^2 due west means that the horse increases its velocity by 8.33 m/s due west each second, that is, 8.33 meters per second per second, which we write as 8.33 m/s^2. This is truly an average acceleration, because the ride is not smooth. We shall see later that an acceleration of this magnitude would require the rider to hang on with a force nearly equal to his weight.

Example:

Exercise:

Check Your Understanding

Problem:

An airplane lands on a runway traveling east. Describe its acceleration.

Solution:

If we take east to be positive, then the airplane has negative acceleration, as it is accelerating toward the west. It is also decelerating: its acceleration is opposite in direction to its velocity.

Note:

PhET Explorations: Moving Man Simulation

Learn about position, velocity, and acceleration graphs. Move the little man back and forth with the mouse and plot his motion. Set the position, velocity, or acceleration and let the simulation move the man for you.

https://archive.cnx.org/specials/e2ca52af-8c6b-450e-ac2f-9300b38e8739/moving-man/

Section Summary

- Acceleration is the rate at which velocity changes. In symbols, **average acceleration** $\bar{a}$ is

  Equation:

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{v_{\mathrm{f}} - v_0}{t_{\mathrm{f}} - t_0}.$$

- The SI unit for acceleration is $\mathrm{m/s^2}$.
- Acceleration is a vector, and thus has a both a magnitude and direction.
- Acceleration can be caused by either a change in the magnitude or the direction of the velocity.
- Instantaneous acceleration a is the acceleration at a specific instant in time.
- Deceleration is an acceleration with a direction opposite to that of the velocity.

Conceptual Questions

Exercise:

Problem:

Is it possible for speed to be constant while acceleration is not zero? Give an example of such a situation.

Exercise:

Problem:

Is it possible for velocity to be constant while acceleration is not zero? Explain.

Exercise:

Problem:

Give an example in which velocity is zero yet acceleration is not.

Exercise:

Problem:

If a subway train is moving to the left (has a negative velocity) and then comes to a stop, what is the direction of its acceleration? Is the acceleration positive or negative?

Exercise:

Problem:

Plus and minus signs are used in one-dimensional motion to indicate direction. What is the sign of an acceleration that reduces the magnitude of a negative velocity? Of a positive velocity?

Problems & Exercises

Exercise:

Problem:

A cheetah can accelerate from rest to a speed of 30.0 m/s in 7.00 s. What is its acceleration?

Solution:

$4.29\ \text{m/s}^2$

Exercise:

Problem: Professional Application

Dr. John Paul Stapp was U.S. Air Force officer who studied the effects of extreme deceleration on the human body. On December 10, 1954, Stapp rode a rocket sled, accelerating from rest to a top speed of 282 m/s (1015 km/h) in 5.00 s, and was brought jarringly back to rest in only 1.40 s! Calculate his (a) acceleration and (b) deceleration. Express each in multiples of g $(9.80\ \text{m/s}^2)$ by taking its ratio to the acceleration of gravity.

Exercise:

Problem:

A commuter backs her car out of her garage with an acceleration of $1.40\ \text{m/s}^2$. (a) How long does it take her to reach a speed of 2.00 m/s? (b) If she then brakes to a stop in 0.800 s, what is her deceleration?

Solution:

(a) 1.43 s

(b) -2.50 m/s^2

Exercise:

Problem:

Assume that an intercontinental ballistic missile goes from rest to a suborbital speed of 6.50 km/s in 60.0 s (the actual speed and time are classified). What is its average acceleration in m/s^2 and in multiples of g (9.80 m/s^2)?

Glossary

acceleration
the rate of change in velocity; the change in velocity over time

average acceleration
the change in velocity divided by the time over which it changes

instantaneous acceleration
acceleration at a specific point in time

deceleration
acceleration in the direction opposite to velocity; acceleration that results in a decrease in velocity

Motion Equations for Constant Acceleration in One Dimension

- Calculate displacement of an object that is not accelerating, given initial position and velocity.
- Calculate final velocity of an accelerating object, given initial velocity, acceleration, and time.
- Calculate displacement and final position of an accelerating object, given initial position, initial velocity, time, and acceleration.

Kinematic equations can help us describe and predict the motion of moving objects such as these kayaks racing in Newbury, England. (credit: Barry Skeates, Flickr)

We might know that the greater the acceleration of, say, a car moving away from a stop sign, the greater the displacement in a given time. But we have not developed a specific equation that relates acceleration and displacement. In this section, we develop some convenient equations for kinematic relationships, starting from the definitions of displacement, velocity, and acceleration already covered.

Notation: *t*, *x*, *v*, *a*

First, let us make some simplifications in notation. Taking the initial time to be zero, as if time is measured with a stopwatch, is a great simplification. Since elapsed time is $\Delta t = t_{\text{f}} - t_0$, taking $t_0 = 0$ means that $\Delta t = t_{\text{f}}$, the final time on the stopwatch. When initial time is taken to be zero, we use the subscript 0 to denote initial values of position and velocity. That is, x_0 *is the initial position* and v_0 *is the initial velocity*. We put no subscripts on the final values. That is, t *is the final time*, x *is the final position*, and v *is the final velocity*. This gives a simpler expression for elapsed time—now, $\Delta t = t$. It also simplifies the expression for displacement, which is now $\Delta x = x - x_0$. Also, it simplifies the expression for change in velocity, which is now $\Delta v = v - v_0$. To summarize, using the simplified notation, with the initial time taken to be zero,

Equation:

$$\begin{array}{rcl}\Delta t & = & t \\ \Delta x & = & x - x_0 \\ \Delta v & = & v - v_0\end{array}$$

where *the subscript 0 denotes an initial value and the absence of a subscript denotes a final value* in whatever motion is under consideration.

We now make the important assumption that *acceleration is constant*. This assumption allows us to avoid using calculus to find instantaneous acceleration. Since acceleration is constant, the average and instantaneous accelerations are equal. That is,

Equation:

$$\bar{a} = a = \text{constant},$$

so we use the symbol a for acceleration at all times. Assuming acceleration to be constant does not seriously limit the situations we can study nor degrade the accuracy of our treatment. For one thing, acceleration *is* constant in a great number of situations. Furthermore, in many other situations we can accurately describe motion by assuming a constant acceleration equal to the average acceleration for that motion. Finally, in motions where acceleration changes drastically, such as a car accelerating to top speed and then braking to a stop, the motion can be considered in separate parts, each of which has its own constant acceleration.

Note:
Solving for Displacement (Δx) and Final Position (x) from Average Velocity when Acceleration (a) is Constant

To get our first two new equations, we start with the definition of average velocity:

Equation:

$$\bar{v} = \frac{\Delta x}{\Delta t}.$$

Substituting the simplified notation for Δx and Δt yields

Equation:

$$\bar{v} = \frac{x - x_0}{t}.$$

Solving for x yields

Equation:

$$x = x_0 + \bar{v}t,$$

where the average velocity is

Equation:

$$\bar{v} = \frac{v_0 + v}{2} \text{ (constant } a\text{)}.$$

The equation $\bar{v} = \frac{v_0+v}{2}$ reflects the fact that, when acceleration is constant, v is just the simple average of the initial and final velocities. For example, if you steadily increase your velocity (that is, with constant acceleration) from 30 to 60 km/h, then your average velocity during this steady increase is 45 km/h. Using the equation $\bar{v} = \frac{v_0 + v}{2}$ to check this, we see that

Equation:

$$\bar{v} = \frac{v_0 + v}{2} = \frac{30 \text{ km/h} + 60 \text{ km/h}}{2} = 45 \text{ km/h},$$

which seems logical.

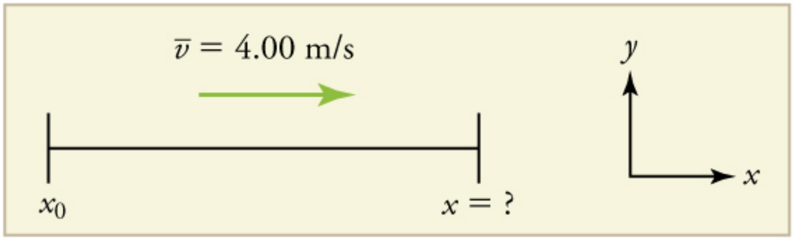
Example:

Calculating Displacement: How Far does the Jogger Run?

A jogger runs down a straight stretch of road with an average velocity of 4.00 m/s for 2.00 min. What is his final position, taking his initial position to be zero?

Strategy

Draw a sketch.



The final position x is given by the equation

Equation:

$$x = x_0 + \bar{v}t.$$

To find x, we identify the values of x_0, $\bar{v}$, and t from the statement of the problem and substitute them into the equation.

Solution

1. Identify the knowns. $\bar{v} = 4.00 \text{ m/s}$, $\Delta t = 2.00 \text{ min}$, and $x_0 = 0 \text{ m}$.
2. Enter the known values into the equation.

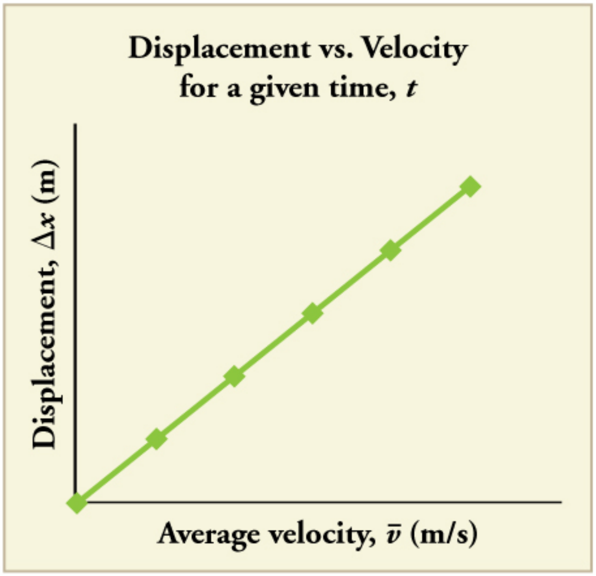
Equation:

$$x = x_0 + \bar{v}t = 0 + (4.00\ \text{m/s})(120\ \text{s}) = 480\ \text{m}$$

Discussion

Velocity and final displacement are both positive, which means they are in the same direction.

The equation $x = x_0 + \bar{v}t$ gives insight into the relationship between displacement, average velocity, and time. It shows, for example, that displacement is a linear function of average velocity. (By linear function, we mean that displacement depends on $\bar{v}$ rather than on $\bar{v}$ raised to some other power, such as $\bar{v}^2$. When graphed, linear functions look like straight lines with a constant slope.) On a car trip, for example, we will get twice as far in a given time if we average 90 km/h than if we average 45 km/h.



There is a linear relationship between displacement and average velocity. For a given time t, an object moving twice as fast as another object will

move twice as far as the other
object.

Note:

Solving for Final Velocity

We can derive another useful equation by manipulating the definition of acceleration.

Equation:

$$a = \frac{\Delta v}{\Delta t}$$

Substituting the simplified notation for Δv and Δt gives us

Equation:

$$a = \frac{v - v_0}{t} \text{ (constant } a\text{)}.$$

Solving for v yields

Equation:

$$v = v_0 + \text{at (constant } a\text{)}.$$

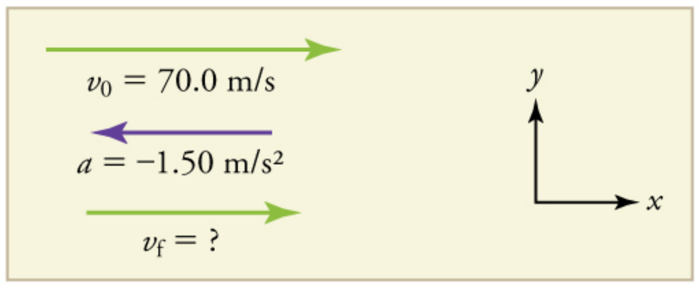
Example:

Calculating Final Velocity: An Airplane Slowing Down after Landing

An airplane lands with an initial velocity of 70.0 m/s and then decelerates at 1.50 m/s^2 for 40.0 s. What is its final velocity?

Strategy

Draw a sketch. We draw the acceleration vector in the direction opposite the velocity vector because the plane is decelerating.



Solution

1. Identify the knowns. $v_0 = 70.0\ \mathrm{m/s}$, $a = -1.50\ \mathrm{m/s}^2$, $t = 40.0\ \mathrm{s}$.
2. Identify the unknown. In this case, it is final velocity, $v_{\mathrm{f}.}$
3. Determine which equation to use. We can calculate the final velocity using the equation $v = v_0 + \mathrm{at}$.
4. Plug in the known values and solve.

Equation:

$$v = v_0 + \mathrm{at} = 70.0\ \mathrm{m/s} + \left(-1.50\ \mathrm{m/s}^2\right)(40.0\ \mathrm{s}) = 10.0\ \mathrm{m/s}$$

Discussion

The final velocity is much less than the initial velocity, as desired when slowing down, but still positive. With jet engines, reverse thrust could be maintained long enough to stop the plane and start moving it backward. That would be indicated by a negative final velocity, which is not the case here.

The airplane lands with an initial velocity of 70.0 m/s and slows to a final velocity of 10.0 m/s before heading for the terminal. Note that the acceleration is negative because its direction is opposite to its velocity, which is positive.

In addition to being useful in problem solving, the equation $v = v_0 + \text{at}$ gives us insight into the relationships among velocity, acceleration, and time. From it we can see, for example, that

- final velocity depends on how large the acceleration is and how long it lasts
- if the acceleration is zero, then the final velocity equals the initial velocity $(v = v_0)$, as expected (i.e., velocity is constant)
- if a is negative, then the final velocity is less than the initial velocity

(All of these observations fit our intuition, and it is always useful to examine basic equations in light of our intuition and experiences to check that they do indeed describe nature accurately.)

Note:

Making Connections: Real-World Connection

The Space Shuttle *Endeavor* blasts off from the Kennedy Space Center in February 2010. (credit: Matthew Simantov, Flickr)

An intercontinental ballistic missile (ICBM) has a larger average acceleration than the Space Shuttle and achieves a greater velocity in the first minute or two of flight (actual ICBM burn times are classified—short-burn-time missiles are more difficult for an enemy to destroy). But the Space Shuttle obtains a greater final velocity, so that it can orbit the earth rather than come directly back down as an ICBM does. The Space Shuttle does this by accelerating for a longer time.

Note:

Solving for Final Position When Velocity is Not Constant ($a \neq 0$)

We can combine the equations above to find a third equation that allows us to calculate the final position of an object experiencing constant acceleration. We start with

Equation:

$$v = v_0 + \text{at}.$$

Adding v_0 to each side of this equation and dividing by 2 gives

Equation:

$$\frac{v_0 + v}{2} = v_0 + \frac{1}{2}\text{at}.$$

Since $\frac{v_0+v}{2} = \bar{v}$ for constant acceleration, then

Equation:

$$\bar{v} = v_0 + \frac{1}{2}\text{at}.$$

Now we substitute this expression for $\bar{v}$ into the equation for displacement, $x = x_0 + \bar{v}t$, yielding

Equation:

$$x = x_0 + v_0 t + \frac{1}{2}at^2 \text{ (constant } a\text{)}.$$

Example:

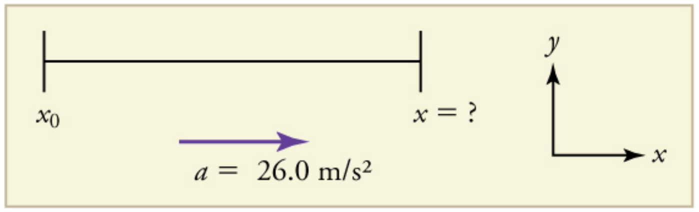
Calculating Displacement of an Accelerating Object: Dragsters

Dragsters can achieve average accelerations of 26.0 m/s^2. Suppose such a dragster accelerates from rest at this rate for 5.56 s. How far does it travel in this time?

U.S. Army Top Fuel pilot Tony "The Sarge" Schumacher begins a race with a controlled burnout. (credit: Lt. Col. William Thurmond. Photo Courtesy of U.S. Army.)

Strategy

Draw a sketch.



We are asked to find displacement, which is x if we take x_0 to be zero. (Think about it like the starting line of a race. It can be anywhere, but we call it 0 and measure all other positions relative to it.) We can use the equation $x = x_0 + v_0t + \frac{1}{2}at^2$once we identify v_0, a, and t from the statement of the problem.

Solution

1. Identify the knowns. Starting from rest means that $v_0 = 0$, a is given as $26.0\ \text{m/s}^2$ and t is given as 5.56 s.
2. Plug the known values into the equation to solve for the unknown x:

Equation:

$$x = x_0 + v_0 t + \frac{1}{2}at^2.$$

Since the initial position and velocity are both zero, this simplifies to

Equation:

$$x = \frac{1}{2}at^2.$$

Substituting the identified values of a and t gives

Equation:

$$x = \frac{1}{2}\left(26.0\ \text{m/s}^2\right)(5.56\ \text{s})^2,$$

yielding

Equation:

$$x = 402\ \text{m}.$$

Discussion

If we convert 402 m to miles, we find that the distance covered is very close to one quarter of a mile, the standard distance for drag racing. So the answer is reasonable. This is an impressive displacement in only 5.56 s, but top-notch dragsters can do a quarter mile in even less time than this.

What else can we learn by examining the equation $x = x_0 + v_0 t + \frac{1}{2}at^2$? We see that:

- displacement depends on the square of the elapsed time when acceleration is not zero. In [link], the dragster covers only one fourth of the total distance in the first half of the elapsed time

- if acceleration is zero, then the initial velocity equals average velocity ($v_0 = \bar{v}$) and $x = x_0 + v_0 t + \frac{1}{2}at^2$ becomes $x = x_0 + v_0 t$

Note:

Solving for Final Velocity when Velocity Is Not Constant ($a \neq 0$)

A fourth useful equation can be obtained from another algebraic manipulation of previous equations.

If we solve $v = v_0 +$ at for t, we get

Equation:

$$t = \frac{v - v_0}{a}.$$

Substituting this and $\bar{v} = \frac{v_0 + v}{2}$ into $x = x_0 + \bar{v}t$, we get

Equation:

$$v^2 = v_0^2 + 2a(x - x_0) \text{ (constant } a\text{).}$$

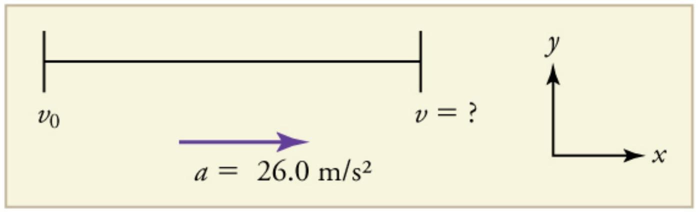
Example:

Calculating Final Velocity: Dragsters

Calculate the final velocity of the dragster in [link] without using information about time.

Strategy

Draw a sketch.



The equation $v^2 = v_0^2 + 2a(x - x_0)$ is ideally suited to this task because it relates velocities, acceleration, and displacement, and no time information is required.

Solution

1. Identify the known values. We know that $v_0 = 0$, since the dragster starts from rest. Then we note that $x - x_0 = 402\ \text{m}$ (this was the answer in [link]). Finally, the average acceleration was given to be $a = 26.0\ \text{m/s}^2$.

2. Plug the knowns into the equation $v^2 = v_0^2 + 2a(x - x_0)$ and solve for v.

Equation:

$$v^2 = 0 + 2\left(26.0\ \text{m/s}^2\right)(402\ \text{m}).$$

Thus

Equation:

$$v^2 = 2.09 \times 10^4\ \text{m}^2/\text{s}^2.$$

To get v, we take the square root:

Equation:

$$v = \sqrt{2.09 \times 10^4\ \text{m}^2/\text{s}^2} = 145\ \text{m/s}.$$

Discussion

145 m/s is about 522 km/h or about 324 mi/h, but even this breakneck speed is short of the record for the quarter mile. Also, note that a square root has two values; we took the positive value to indicate a velocity in the same direction as the acceleration.

An examination of the equation $v^2 = v_0^2 + 2a(x - x_0)$ can produce further insights into the general relationships among physical quantities:

- The final velocity depends on how large the acceleration is and the distance over which it acts
- For a fixed deceleration, a car that is going twice as fast doesn’t simply stop in twice the distance—it takes much further to stop. (This is why

we have reduced speed zones near schools.)

Putting Equations Together

In the following examples, we further explore one-dimensional motion, but in situations requiring slightly more algebraic manipulation. The examples also give insight into problem-solving techniques. The box below provides easy reference to the equations needed.

Note:
Summary of Kinematic Equations (constant a)
Equation:

$$x = x_0 + \bar{v}t$$

Equation:

$$\bar{v} = \frac{v_0 + v}{2}$$

Equation:

$$v = v_0 + \text{at}$$

Equation:

$$x = x_0 + v_0 t + \frac{1}{2}at^2$$

Equation:

$$v^2 = v_0^2 + 2a(x - x_0)$$

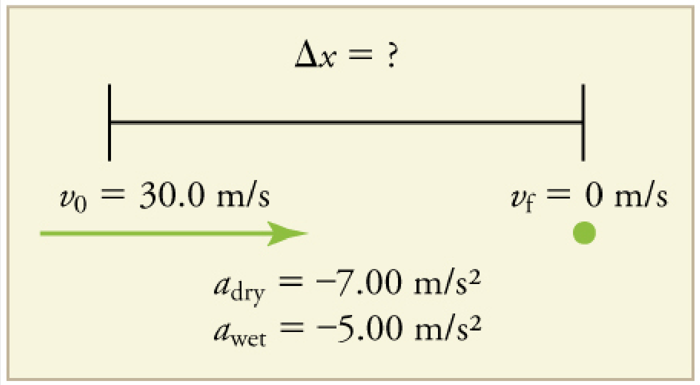
Example:

Calculating Displacement: How Far Does a Car Go When Coming to a Halt?

On dry concrete, a car can decelerate at a rate of $7.00\ \text{m/s}^2$, whereas on wet concrete it can decelerate at only $5.00\ \text{m/s}^2$. Find the distances necessary to stop a car moving at 30.0 m/s (about 110 km/h) (a) on dry concrete and (b) on wet concrete. (c) Repeat both calculations, finding the displacement from the point where the driver sees a traffic light turn red, taking into account his reaction time of 0.500 s to get his foot on the brake.

Strategy

Draw a sketch.



In order to determine which equations are best to use, we need to list all of the known values and identify exactly what we need to solve for. We shall do this explicitly in the next several examples, using tables to set them off.

Solution for (a)

1. Identify the knowns and what we want to solve for. We know that $v_0 = 30.0\ \text{m/s}$; $v = 0$; $a = -7.00\ \text{m/s}^2$ (a is negative because it is in a direction opposite to velocity). We take x_0 to be 0. We are looking for displacement Δx, or $x - x_0$.

2. Identify the equation that will help up solve the problem. The best equation to use is

Equation:

$$v^2 = v_0^2 + 2a(x - x_0).$$

This equation is best because it includes only one unknown, x. We know the values of all the other variables in this equation. (There are other equations that would allow us to solve for x, but they require us to know the stopping time, t, which we do not know. We could use them but it would entail additional calculations.)

3. Rearrange the equation to solve for x.

Equation:

$$x - x_0 = \frac{v^2 - v_0^2}{2a}$$

4. Enter known values.

Equation:

$$x - 0 = \frac{0^2 - (30.0\ \text{m/s})^2}{2\left(-7.00\ \text{m/s}^2\right)}$$

Thus,

Equation:

$$x = 64.3\ \text{m on dry concrete.}$$

Solution for (b)

This part can be solved in exactly the same manner as Part A. The only difference is that the deceleration is $-5.00\ \text{m/s}^2$. The result is

Equation:

$$x_{\text{wet}} = 90.0\ \text{m on wet concrete.}$$

Solution for (c)

Once the driver reacts, the stopping distance is the same as it is in Parts A and B for dry and wet concrete. So to answer this question, we need to calculate how far the car travels during the reaction time, and then add that to the stopping time. It is reasonable to assume that the velocity remains constant during the driver's reaction time.

1. Identify the knowns and what we want to solve for. We know that $\bar{v} = 30.0\ \text{m/s}$; $t_{\text{reaction}} = 0.500\,\text{s}$; $a_{\text{reaction}} = 0$. We take $x_{0-\text{reaction}}$ to be 0. We are looking for x_{reaction}.

2. Identify the best equation to use.

$x = x_0 + \bar{v}t$ works well because the only unknown value is x, which is what we want to solve for.
3. Plug in the knowns to solve the equation.

Equation:

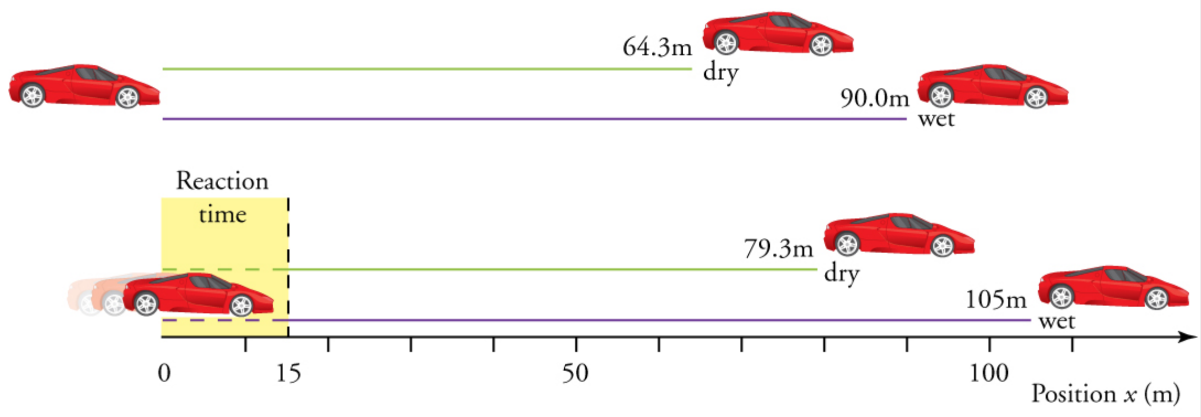
$$x = 0 + (30.0\text{ m/s})(0.500\text{ s}) = 15.0\text{ m}.$$

This means the car travels 15.0 m while the driver reacts, making the total displacements in the two cases of dry and wet concrete 15.0 m greater than if he reacted instantly.
4. Add the displacement during the reaction time to the displacement when braking.

Equation:

$$x_{\text{braking}} + x_{\text{reaction}} = x_{\text{total}}$$

a. 64.3 m + 15.0 m = 79.3 m when dry
b. 90.0 m + 15.0 m = 105 m when wet



The distance necessary to stop a car varies greatly, depending on road conditions and driver reaction time. Shown here are the braking distances for dry and wet pavement, as calculated in this example, for a car initially traveling at 30.0 m/s. Also shown are the total distances traveled from the point where the driver first sees a light turn red, assuming a 0.500 s reaction time.

Discussion

The displacements found in this example seem reasonable for stopping a fast-moving car. It should take longer to stop a car on wet rather than dry pavement. It is interesting that reaction time adds significantly to the displacements. But more important is the general approach to solving problems. We identify the knowns and the quantities to be determined and then find an appropriate equation. There is often more than one way to solve a problem. The various parts of this example can in fact be solved by other methods, but the solutions presented above are the shortest.

With the basics of kinematics established, we can go on to many other interesting examples and applications. In the process of developing kinematics, we have also glimpsed a general approach to problem solving that produces both correct answers and insights into physical relationships. Problem-Solving Basics discusses problem-solving basics and outlines an approach that will help you succeed in this invaluable task.

Note:

Making Connections: Take-Home Experiment—Breaking News

We have been using SI units of meters per second squared to describe some examples of acceleration or deceleration of cars, runners, and trains. To achieve a better feel for these numbers, one can measure the braking deceleration of a car doing a slow (and safe) stop. Recall that, for average acceleration, $\bar{a} = \Delta v / \Delta t$. While traveling in a car, slowly apply the brakes as you come up to a stop sign. Have a passenger note the initial speed in miles per hour and the time taken (in seconds) to stop. From this, calculate the deceleration in miles per hour per second. Convert this to meters per second squared and compare with other decelerations mentioned in this chapter. Calculate the distance traveled in braking.

Exercise:

Check Your Understanding

Problem:

A rocket accelerates at a rate of $20\ \mathrm{m/s}^2$ during launch. How long does it take the rocket to reach a velocity of 400 m/s?

Solution:

To answer this, choose an equation that allows you to solve for time t, given only a, v_0, and v.
Equation:

$$v = v_0 + \mathrm{at}$$

Rearrange to solve for t.
Equation:

$$t = \frac{v - v_0}{a} = \frac{400\ \mathrm{m/s} - 0\ \mathrm{m/s}}{20\ \mathrm{m/s}^2} = 20\ \mathrm{s}$$

Section Summary

- To simplify calculations we take acceleration to be constant, so that $\bar{a} = a$ at all times.
- We also take initial time to be zero.
- Initial position and velocity are given a subscript 0; final values have no subscript. Thus,
  Equation:

$$\begin{array}{rcl} \Delta t & = & t \\ \Delta x & = & x - x_0 \\ \Delta v & = & v - v_0 \end{array}$$

- The following kinematic equations for motion with constant a are useful:
  Equation:

  $$x = x_0 + \bar{v}t$$

  Equation:

  $$\bar{v} = \frac{v_0 + v}{2}$$

  Equation:

  $$v = v_0 + \text{at}$$

  Equation:

  $$x = x_0 + v_0 t + \frac{1}{2}at^2$$

  Equation:

  $$v^2 = v_0^2 + 2a(x - x_0)$$

- In vertical motion, y is substituted for x.

Problems & Exercises

Exercise:

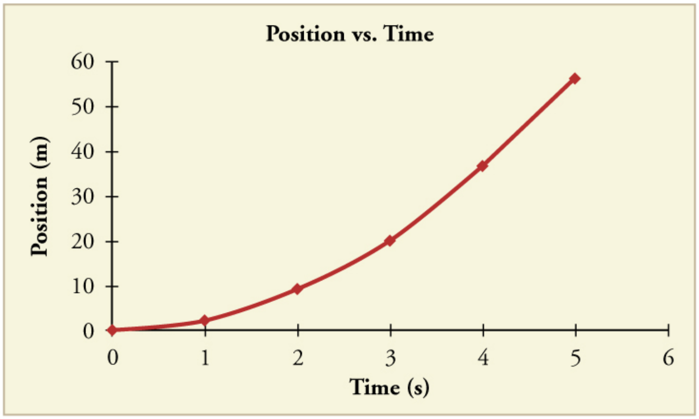
Problem:

An Olympic-class sprinter starts a race with an acceleration of $4.50\ \text{m/s}^2$. (a) What is her speed 2.40 s later? (b) Sketch a graph of her position vs. time for this period.

Solution:

(a) $10.8\ \text{m/s}$

(b)



Exercise:

Problem:

A well-thrown ball is caught in a well-padded mitt. If the deceleration of the ball is $2.10 \times 10^4 \text{ m/s}^2$, and 1.85 ms $(1 \text{ ms} = 10^{-3} \text{ s})$ elapses from the time the ball first touches the mitt until it stops, what was the initial velocity of the ball?

Solution:

38.9 m/s (about 87 miles per hour)

Exercise:

Problem:

A bullet in a gun is accelerated from the firing chamber to the end of the barrel at an average rate of $6.20 \times 10^5 \text{ m/s}^2$ for 8.10×10^{-4} s. What is its muzzle velocity (that is, its final velocity)?

Exercise:

Problem:

(a) A light-rail commuter train accelerates at a rate of 1.35 m/s^2. How long does it take to reach its top speed of 80.0 km/h, starting from rest? (b) The same train ordinarily decelerates at a rate of 1.65 m/s^2. How long does it take to come to a stop from its top speed? (c) In emergencies the train can decelerate more rapidly, coming to rest from 80.0 km/h in 8.30 s. What is its emergency deceleration in m/s^2?

Solution:

(a) 16.5 s

(b) 13.5 s

(c) -2.68 m/s^2

Exercise:

Problem:

While entering a freeway, a car accelerates from rest at a rate of 2.40 m/s^2 for 12.0 s. (a) Draw a sketch of the situation. (b) List the knowns in this problem. (c) How far does the car travel in those 12.0 s? To solve this part, first identify the unknown, and then discuss how you chose the appropriate equation to solve for it. After choosing the equation, show your steps in solving for the unknown, check your units, and discuss whether the answer is reasonable. (d) What is the car's final velocity? Solve for this unknown in the same manner as in part (c), showing all steps explicitly.

Exercise:

Problem:

At the end of a race, a runner decelerates from a velocity of 9.00 m/s at a rate of 2.00 m/s^2. (a) How far does she travel in the next 5.00 s? (b) What is her final velocity? (c) Evaluate the result. Does it make sense?

Solution:

(a) 20.0 m

(b) -1.00 m/s

(c) This result does not really make sense. If the runner starts at 9.00 m/s and decelerates at 2.00 m/s^2, then she will have stopped after 4.50 s. If she continues to decelerate, she will be running backwards.

Exercise:

Problem:Professional Application:

Blood is accelerated from rest to 30.0 cm/s in a distance of 1.80 cm by the left ventricle of the heart. (a) Make a sketch of the situation. (b) List the knowns in this problem. (c) How long does the acceleration take? To solve this part, first identify the unknown, and then discuss how you chose the appropriate equation to solve for it. After choosing the equation, show your steps in solving for the unknown, checking your units. (d) Is the answer reasonable when compared with the time for a heartbeat?

Exercise:

Problem:

In a slap shot, a hockey player accelerates the puck from a velocity of 8.00 m/s to 40.0 m/s in the same direction. If this shot takes 3.33×10^{-2} s, calculate the distance over which the puck accelerates.

Solution:

0.799 m

Exercise:

Problem:

A powerful motorcycle can accelerate from rest to 26.8 m/s (100 km/h) in only 3.90 s. (a) What is its average acceleration? (b) How far does it travel in that time?

Exercise:

Problem:

Freight trains can produce only relatively small accelerations and decelerations. (a) What is the final velocity of a freight train that accelerates at a rate of $0.0500\ \text{m/s}^2$ for 8.00 min, starting with an initial velocity of 4.00 m/s? (b) If the train can slow down at a rate of $0.550\ \text{m/s}^2$, how long will it take to come to a stop from this velocity? (c) How far will it travel in each case?

Solution:

(a) $28.0\ \text{m/s}$

(b) $50.9\ \text{s}$

(c) 7.68 km to accelerate and 713 m to decelerate

Exercise:

Problem:

A fireworks shell is accelerated from rest to a velocity of 65.0 m/s over a distance of 0.250 m. (a) How long did the acceleration last? (b) Calculate the acceleration.

Exercise:

Problem:

A swan on a lake gets airborne by flapping its wings and running on top of the water. (a) If the swan must reach a velocity of 6.00 m/s to take off and it accelerates from rest at an average rate of $0.350\ \text{m/s}^2$, how far will it travel before becoming airborne? (b) How long does this take?

Solution:

(a) $51.4\ \text{m}$

(b) $17.1\ \text{s}$

Exercise:

Problem: **Professional Application:**

A woodpecker's brain is specially protected from large decelerations by tendon-like attachments inside the skull. While pecking on a tree, the woodpecker's head comes to a stop from an initial velocity of 0.600 m/s in a distance of only 2.00 mm. (a) Find the acceleration in m/s^2 and in multiples of g $\left(g = 9.80\ \text{m/s}^2\right)$. (b) Calculate the stopping time. (c) The tendons cradling the brain stretch, making its stopping distance 4.50 mm (greater than the head and, hence, less deceleration of the brain). What is the brain's deceleration, expressed in multiples of g?

Exercise:

Problem:

An unwary football player collides with a padded goalpost while running at a velocity of 7.50 m/s and comes to a full stop after compressing the padding and his body 0.350 m. (a) What is his deceleration? (b) How long does the collision last?

Solution:

(a) -80.4 m/s^2

(b) $9.33 \times 10^{-2} \text{ s}$

Exercise:

Problem:

In World War II, there were several reported cases of airmen who jumped from their flaming airplanes with no parachute to escape certain death. Some fell about 20,000 feet (6000 m), and some of them survived, with few life-threatening injuries. For these lucky pilots, the tree branches and snow drifts on the ground allowed their deceleration to be relatively small. If we assume that a pilot’s speed upon impact was 123 mph (54 m/s), then what was his deceleration? Assume that the trees and snow stopped him over a distance of 3.0 m.

Exercise:

Problem:

Consider a grey squirrel falling out of a tree to the ground. (a) If we ignore air resistance in this case (only for the sake of this problem), determine a squirrel’s velocity just before hitting the ground, assuming it fell from a height of 3.0 m. (b) If the squirrel stops in a distance of 2.0 cm through bending its limbs, compare its deceleration with that of the airman in the previous problem.

Solution:

(a) 7.7 m/s

(b) $-15 \times 10^2 \text{ m/s}^2$. This is about 3 times the deceleration of the pilots, who were falling from thousands of meters high!

Exercise:

Problem:

An express train passes through a station. It enters with an initial velocity of 22.0 m/s and decelerates at a rate of $0.150\ \text{m/s}^2$ as it goes through. The station is 210 m long. (a) How long is the nose of the train in the station? (b) How fast is it going when the nose leaves the station? (c) If the train is 130 m long, when does the end of the train leave the station? (d) What is the velocity of the end of the train as it leaves?

Exercise:

Problem:

Dragsters can actually reach a top speed of 145 m/s in only 4.45 s—considerably less time than given in [link] and [link]. (a) Calculate the average acceleration for such a dragster. (b) Find the final velocity of this dragster starting from rest and accelerating at the rate found in (a) for 402 m (a quarter mile) without using any information on time. (c) Why is the final velocity greater than that used to find the average acceleration? *Hint*: Consider whether the assumption of constant acceleration is valid for a dragster. If not, discuss whether the acceleration would be greater at the beginning or end of the run and what effect that would have on the final velocity.

Solution:

(a) $32.6\ \text{m/s}^2$

(b) $162\ \text{m/s}$

(c) $v > v_{\text{max}}$, because the assumption of constant acceleration is not valid for a dragster. A dragster changes gears, and would have a greater acceleration in first gear than second gear than third gear, etc. The acceleration would be greatest at the beginning, so it would not be accelerating at $32.6\ \text{m/s}^2$ during the last few meters, but substantially less, and the final velocity would be less than 162 m/s.

Exercise:

Problem:

A bicycle racer sprints at the end of a race to clinch a victory. The racer has an initial velocity of 11.5 m/s and accelerates at the rate of $0.500\ \text{m/s}^2$ for 7.00 s. (a) What is his final velocity? (b) The racer continues at this velocity to the finish line. If he was 300 m from the finish line when he started to accelerate, how much time did he save? (c) One other racer was 5.00 m ahead when the winner started to accelerate, but he was unable to accelerate, and traveled at 11.8 m/s until the finish line. How far ahead of him (in meters and in seconds) did the winner finish?

Exercise:

Problem:

In 1967, New Zealander Burt Munro set the world record for an Indian motorcycle, on the Bonneville Salt Flats in Utah, with a maximum speed of 183.58 mi/h. The one-way course was 5.00 mi long. Acceleration rates are often described by the time it takes to reach 60.0 mi/h from rest. If this time was 4.00 s, and Burt accelerated at this rate until he reached his maximum speed, how long did it take Burt to complete the course?

Solution:

104 s

Exercise:

Problem:

(a) A world record was set for the men’s 100-m dash in the 2008 Olympic Games in Beijing by Usain Bolt of Jamaica. Bolt “coasted” across the finish line with a time of 9.69 s. If we assume that Bolt accelerated for 3.00 s to reach his maximum speed, and maintained that speed for the rest of the race, calculate his maximum speed and his acceleration. (b) During the same Olympics, Bolt also set the world record in the 200-m dash with a time of 19.30 s. Using the same assumptions as for the 100-m dash, what was his maximum speed for this race?

Solution:

(a) $v = 12.2\ \text{m/s}$; $a = 4.07\ \text{m/s}^2$

(b) $v = 11.2\ \text{m/s}$

Graphical Analysis of One-Dimensional Motion

- Describe a straight-line graph in terms of its slope and y-intercept.
- Determine average velocity or instantaneous velocity from a graph of position vs. time.
- Determine average or instantaneous acceleration from a graph of velocity vs. time.
- Derive a graph of velocity vs. time from a graph of position vs. time.
- Derive a graph of acceleration vs. time from a graph of velocity vs. time.

A graph, like a picture, is worth a thousand words. Graphs not only contain numerical information; they also reveal relationships between physical quantities. This section uses graphs of position, velocity, and acceleration versus time to illustrate one-dimensional kinematics.

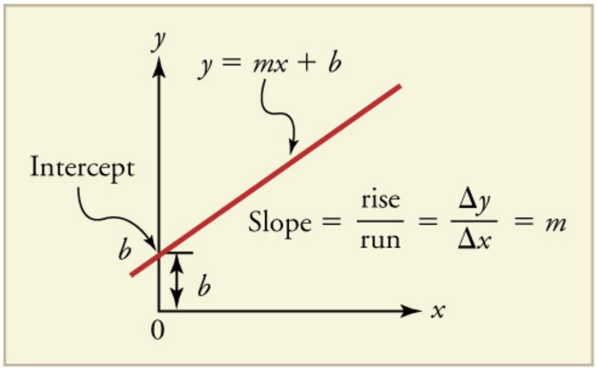
Slopes and General Relationships

First note that graphs in this text have perpendicular axes, one horizontal and the other vertical. When two physical quantities are plotted against one another in such a graph, the horizontal axis is usually considered to be an **independent variable** and the vertical axis a **dependent variable**. If we call the horizontal axis the x-axis and the vertical axis the y-axis, as in [link], a straight-line graph has the general form

Equation:

$$y=\text{mx}+b.$$

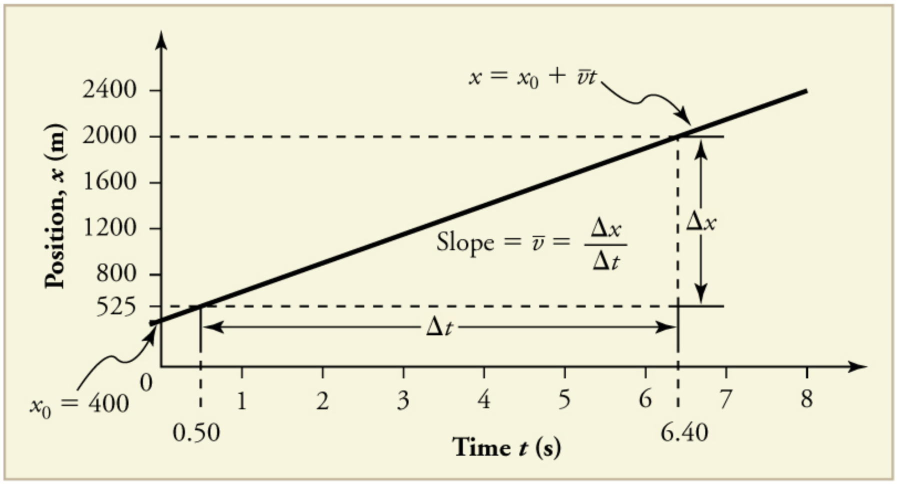
Here m is the **slope**, defined to be the rise divided by the run (as seen in the figure) of the straight line. The letter b is used for the ***y*-intercept**, which is the point at which the line crosses the vertical axis.



A straight-line graph. The equation for a straight line is $y = \text{mx} + b$.

Graph of Position vs. Time (*a* = 0, so *v* is constant)

Time is usually an independent variable that other quantities, such as position, depend upon. A graph of position versus time would, thus, have x on the vertical axis and t on the horizontal axis. [link] is just such a straight-line graph. It shows a graph of position versus time for a jet-powered car on a very flat dry lake bed in Nevada.



Graph of position versus time for a jet-powered car on the Bonneville Salt Flats.

Using the relationship between dependent and independent variables, we see that the slope in the graph above is average velocity $\bar{v}$ and the intercept is position at time zero—that is, x_0. Substituting these symbols into $y = \text{mx} + b$ gives

Equation:

$$x = \bar{v}t + x_0$$

or

Equation:

$$x = x_0 + \bar{v}t.$$

Thus a graph of position versus time gives a general relationship among displacement(change in position), velocity, and time, as well as giving detailed numerical information about a specific situation.

Note:

The Slope of *x* vs. *t*

The slope of the graph of position x vs. time t is velocity v.

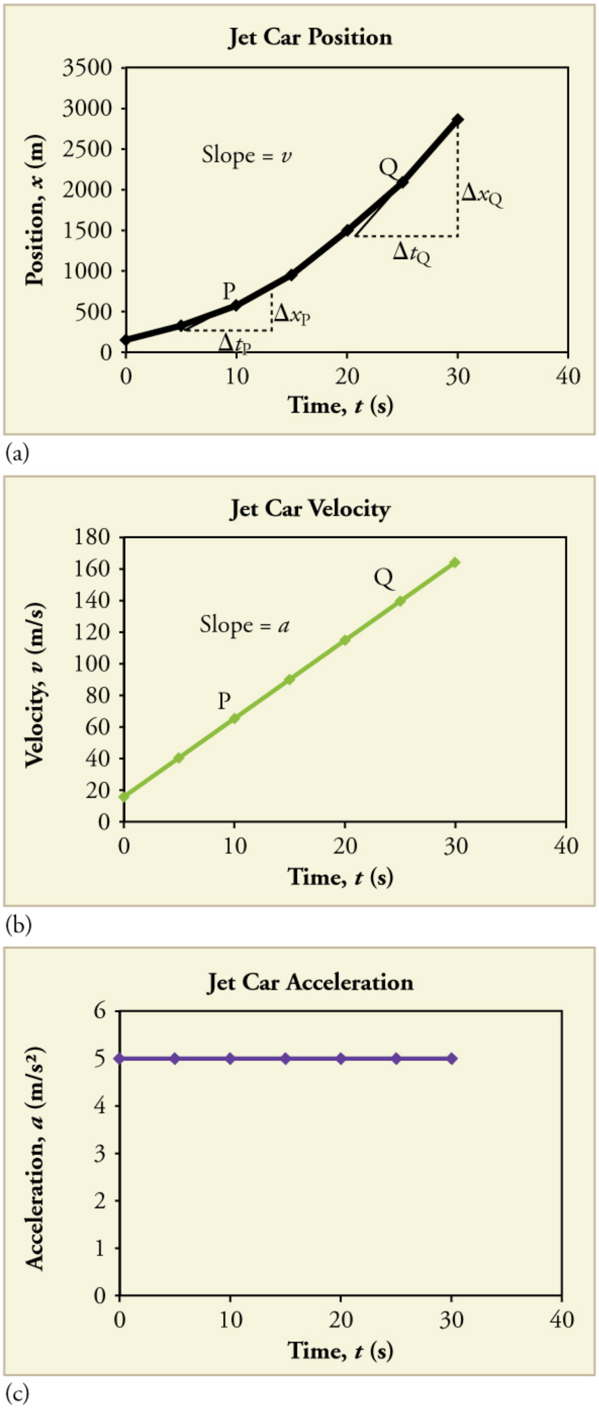
Equation:

$$\text{slope} = \frac{\Delta x}{\Delta t} = v$$

Notice that this equation is the same as that derived algebraically from other motion equations in Motion Equations for Constant Acceleration in One Dimension.

Graphs of Motion when a is constant but $a \neq 0$

The graphs in [link] below represent the motion of the jet-powered car as it accelerates toward its top speed, but only during the time when its acceleration is constant. Time starts at zero for this motion (as if measured with a stopwatch), and the position and velocity are initially 200 m and 15 m/s, respectively.



Graphs of motion of a jet-powered car during the time span when its acceleration is constant. (a) The slope of an x vs. t graph is velocity. This is shown at two points, and the instantaneous velocities obtained are plotted in the next graph. Instantaneous velocity at any point is the slope of the tangent at that point. (b) The slope of the v vs. t graph is constant for this part of the motion, indicating constant acceleration. (c) Acceleration has the constant value of 5.0 m/s^2 over the time interval plotted.

A U.S. Air Force jet car speeds down a track. (credit: Matt Trostle, Flickr)

The graph of position versus time in [link](a) is a curve rather than a straight line. The slope of the curve becomes steeper as time progresses, showing that the velocity is increasing over time. The slope at any point on a position-versus-time graph is the instantaneous velocity at that point. It is found by drawing a straight line tangent to the curve at the point of interest and taking the slope of this straight line. Tangent lines are shown for two points in [link](a). If this is done at every point on the curve and the values are plotted against time, then the graph of velocity versus time shown in [link](b) is obtained. Furthermore, the slope of the graph of velocity versus time is acceleration, which is shown in [link](c).

Carrying this one step further, we note that the slope of a velocity versus time graph is acceleration. Slope is rise divided by run; on a v vs. t graph, rise = change in velocity Δv and run = change in time Δt.

Note:
The Slope of *v* vs. *t*
The slope of a graph of velocity v vs. time t is acceleration a.
Equation:

$$\text{slope} = \frac{\Delta v}{\Delta t} = a$$

Since the velocity versus time graph in [link](b) is a straight line, its slope is the same everywhere, implying that acceleration is constant. Acceleration versus time is graphed in [link](c).

Additional general information can be obtained from [link] and the expression for a straight line, $y = \text{mx} + b$.

In this case, the vertical axis y is V, the intercept b is v_0, the slope m is a, and the horizontal axis x is t. Substituting these symbols yields
Equation:

$$v = v_0 + \text{at}.$$

A general relationship for velocity, acceleration, and time has again been obtained from a graph. Notice that this equation was also derived algebraically from other motion equations in Motion Equations for Constant Acceleration in One Dimension.

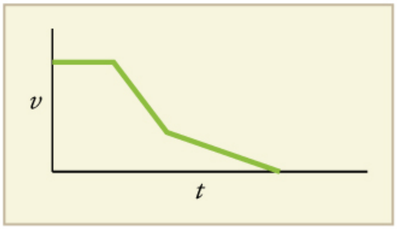
It is not accidental that the same equations are obtained by graphical analysis as by algebraic techniques. In fact, an important way to *discover* physical relationships is to measure various physical quantities and then make graphs of one quantity against another to see if they are correlated in any way. Correlations imply physical relationships and might be shown by smooth graphs such as those above. From such graphs, mathematical relationships can sometimes be postulated. Further experiments are then performed to determine the validity of the hypothesized relationships.

Exercise:

Check Your Understanding

Problem:

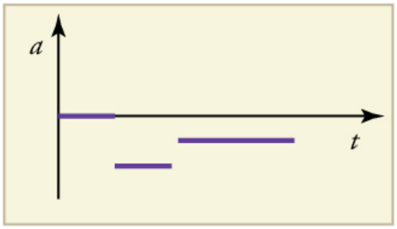
A graph of velocity vs. time of a ship coming into a harbor is shown below. (a) Describe the motion of the ship based on the graph. (b)What would a graph of the ship’s acceleration look like?



Solution:

(a) The ship moves at constant velocity and then begins to decelerate at a constant rate. At some point, its deceleration rate decreases. It maintains this lower deceleration rate until it stops moving.

(b) A graph of acceleration vs. time would show zero acceleration in the first leg, large and constant negative acceleration in the second leg, and constant negative acceleration.



Section Summary

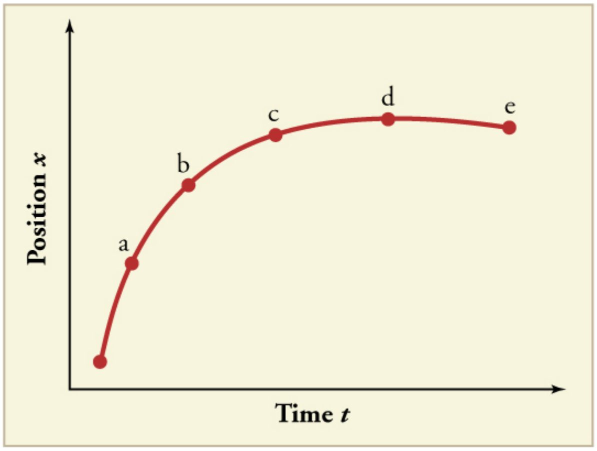
- Graphs of motion can be used to analyze motion.
- Graphical solutions yield identical solutions to mathematical methods for deriving motion equations.
- The slope of a graph of displacement x vs. time t is velocity v.
- The slope of a graph of velocity v vs. time t graph is acceleration a.
- Average velocity, instantaneous velocity, and acceleration can all be obtained by analyzing graphs.

Conceptual Questions

Exercise:

Problem:

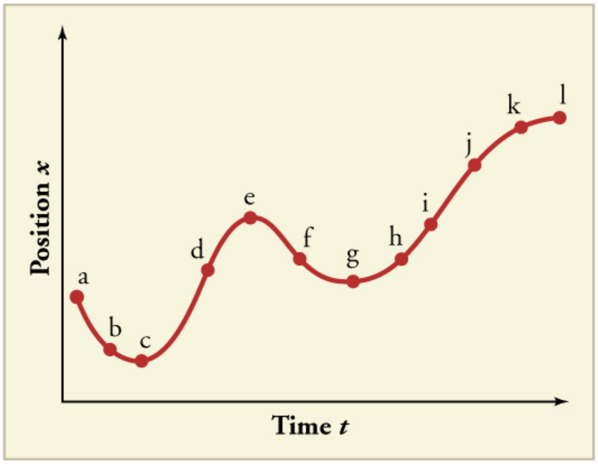
(a) Explain how you can use the graph of position versus time in [link] to describe the change in velocity over time. Identify (b) the time (t_a, t_b, t_c, t_d, or t_e) at which the instantaneous velocity is greatest, (c) the time at which it is zero, and (d) the time at which it is negative.



Exercise:

Problem:

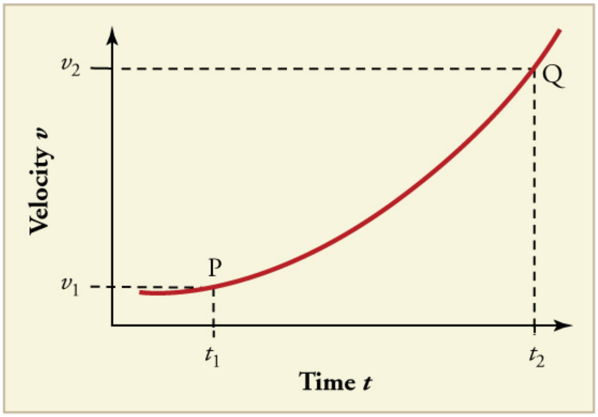
(a) Sketch a graph of velocity versus time corresponding to the graph of position versus time given in [link]. (b) Identify the time or times (t_a, t_b, t_c, etc.) at which the instantaneous velocity is greatest. (c) At which times is it zero? (d) At which times is it negative?



Exercise:

Problem:

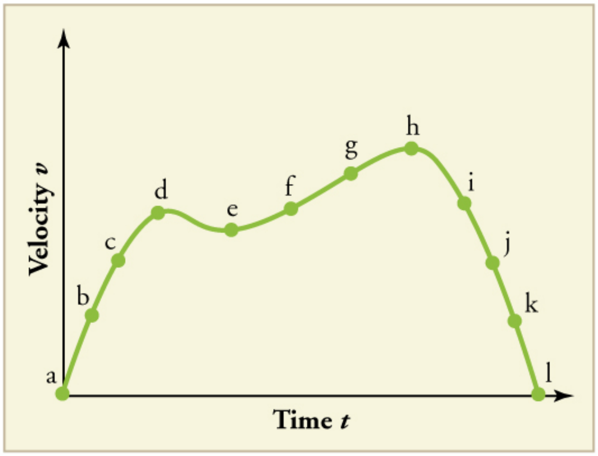
(a) Explain how you can determine the acceleration over time from a velocity versus time graph such as the one in [link]. (b) Based on the graph, how does acceleration change over time?



Exercise:

Problem:

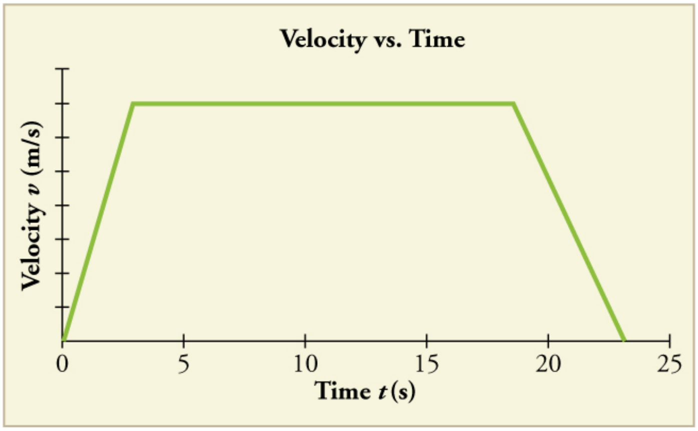
(a) Sketch a graph of acceleration versus time corresponding to the graph of velocity versus time given in [link]. (b) Identify the time or times (t_a, t_b, t_c, etc.) at which the acceleration is greatest. (c) At which times is it zero? (d) At which times is it negative?



Exercise:

Problem:

Consider the velocity vs. time graph of a person in an elevator shown in [link]. Suppose the elevator is initially at rest. It then accelerates for 3 seconds, maintains that velocity for 15 seconds, then decelerates for 5 seconds until it stops. The acceleration for the entire trip is not constant so we cannot use the equations of motion from Motion Equations for Constant Acceleration in One Dimension for the complete trip. (We could, however, use them in the three individual sections where acceleration is a constant.) Sketch graphs of (a) position vs. time and (b) acceleration vs. time for this trip.



Exercise:

Problem:

A cylinder is given a push and then rolls up an inclined plane. If the origin is the starting point, sketch the position, velocity, and acceleration of the cylinder vs. time as it goes up and then down the plane.

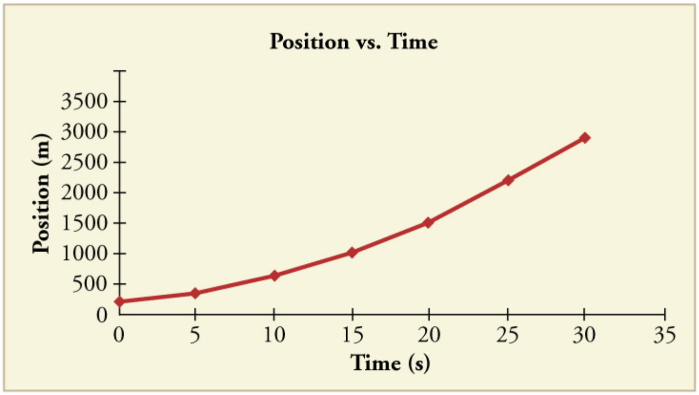
Problems & Exercises

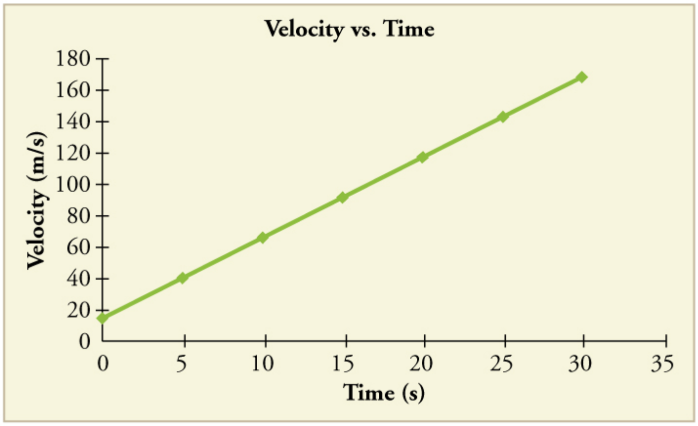
Note: There is always uncertainty in numbers taken from graphs. If your answers differ from expected values, examine them to see if they are within data extraction uncertainties estimated by you.

Exercise:

Problem:

(a) By taking the slope of the curve in [link], verify that the velocity of the jet car is 115 m/s at $t = 20$ s. (b) By taking the slope of the curve at any point in [link], verify that the jet car's acceleration is 5.0 m/s^2.





Solution:

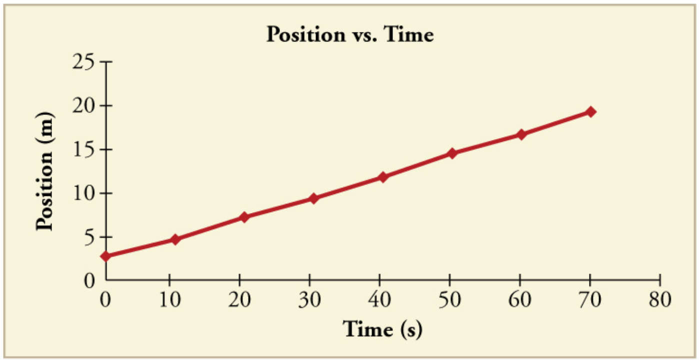
(a) $115\ \text{m/s}$

(b) $5.0\ \text{m/s}^2$

Exercise:

Problem:

Using approximate values, calculate the slope of the curve in [link] to verify that the velocity at $t = 10.0$ s is 0.208 m/s. Assume all values are known to 3 significant figures.



Exercise:

Problem:

Using approximate values, calculate the slope of the curve in [link] to verify that the velocity at $t = 30.0$ s is approximately 0.24 m/s.

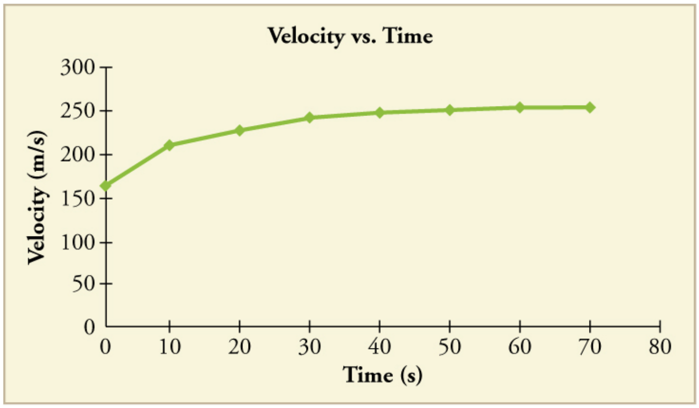
Solution:
Equation:

$$v = \frac{(11.7 - 6.95) \times 10^3 \text{ m}}{(40.0 - 20.0) \text{ s}} = 238 \text{ m/s}$$

Exercise:

Problem:

By taking the slope of the curve in [link], verify that the acceleration is 3.2 m/s^2 at $t = 10$ s.

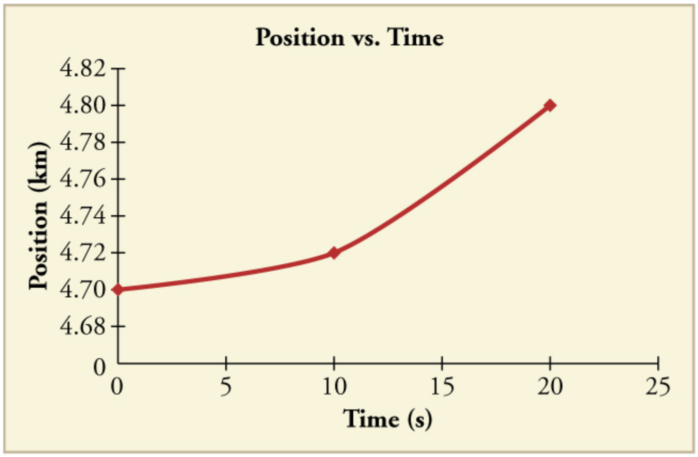


Exercise:

Problem:

Construct the position graph for the subway shuttle train as shown in [link](a). Your graph should show the position of the train, in kilometers, from t = 0 to 20 s. You will need to use the information on acceleration and velocity given in the examples for this figure.

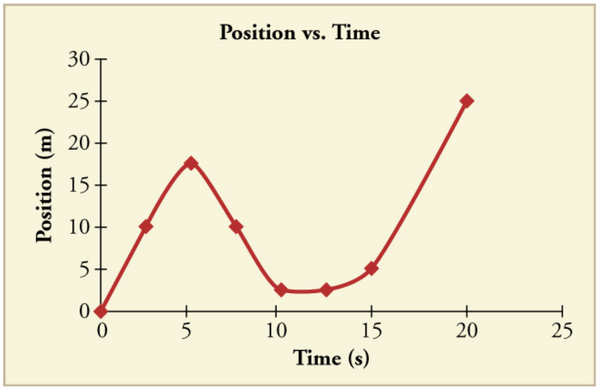
Solution:

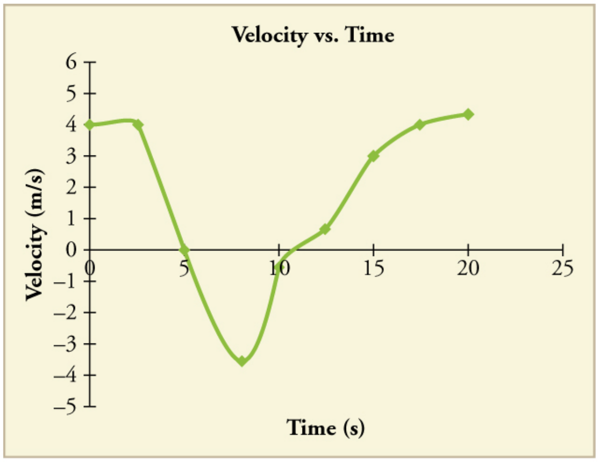


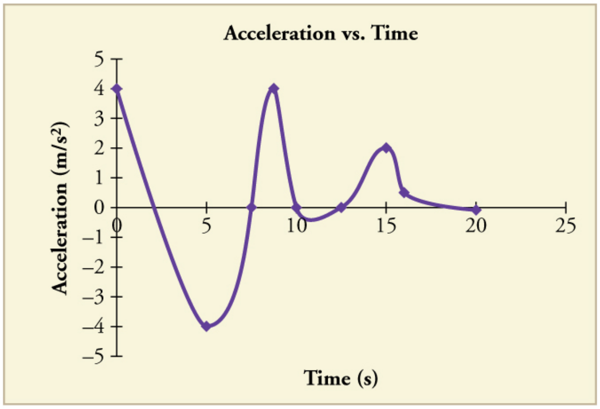
Exercise:

Problem:

(a) Take the slope of the curve in [link] to find the jogger’s velocity at $t = 2.5$ s. (b) Repeat at 7.5 s. These values must be consistent with the graph in [link].



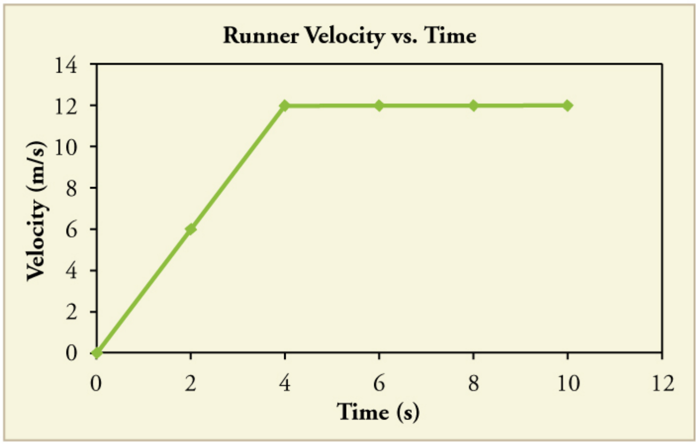




Exercise:

Problem:

A graph of $v(t)$ is shown for a world-class track sprinter in a 100-m race. (See [link]). (a) What is his average velocity for the first 4 s? (b) What is his instantaneous velocity at $t = 5$ s? (c) What is his average acceleration between 0 and 4 s? (d) What is his time for the race?



Solution:

(a) 6 m/s

(b) 12 m/s

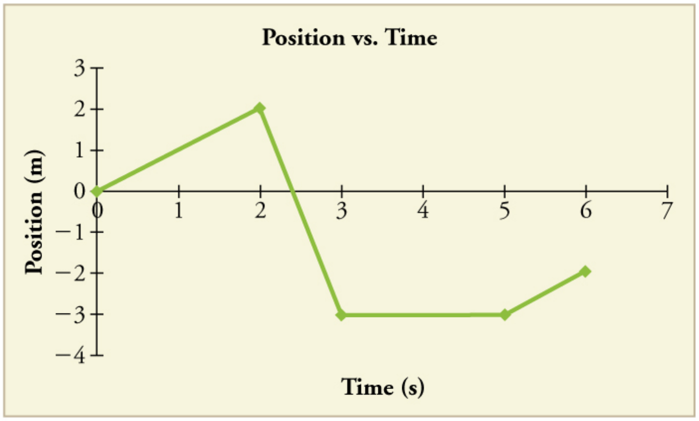
(c) $3\ \text{m/s}^2$

(d) 10 s

Exercise:

Problem:

[link] shows the position graph for a particle for 5 s. Draw the corresponding velocity and acceleration graphs.



Glossary

independent variable
the variable that the dependent variable is measured with respect to; usually plotted along the x-axis

dependent variable
the variable that is being measured; usually plotted along the y-axis

slope

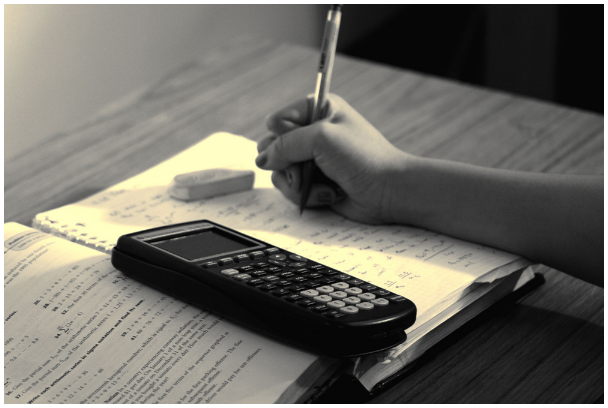
the difference in y-value (the rise) divided by the difference in x-value (the run) of two points on a straight line

y-intercept

the y-value when $x = 0$, or when the graph crosses the y-axis

Problem-Solving Basics for One-Dimensional Kinematics

- Apply problem-solving steps and strategies to solve problems of one-dimensional kinematics.
- Apply strategies to determine whether or not the result of a problem is reasonable, and if not, determine the cause.



Problem-solving skills are essential to your success in Physics. (credit: scui3asteveo, Flickr)

Problem-solving skills are obviously essential to success in a quantitative course in physics. More importantly, the ability to apply broad physical principles, usually represented by equations, to specific situations is a very powerful form of knowledge. It is much more powerful than memorizing a list of facts. Analytical skills and problem-solving abilities can be applied to new situations, whereas a list of facts cannot be made long enough to contain every possible circumstance. Such analytical skills are useful both for solving problems in this text and for applying physics in everyday and professional life.

Problem-Solving Steps

While there is no simple step-by-step method that works for every problem, the following general procedures facilitate problem solving and make it more meaningful. A certain amount of creativity and insight is required as well.

Step 1

Examine the situation to determine which physical principles are involved. It often helps to *draw a simple sketch* at the outset. You will also need to decide which direction is positive and note that on your sketch. Once you have identified the physical principles, it is much easier to find and apply the equations representing those principles. Although finding the correct equation is essential, keep in mind that equations represent physical principles, laws of nature, and relationships among physical quantities. Without a conceptual understanding of a problem, a numerical solution is meaningless.

Step 2

Make a list of what is given or can be inferred from the problem as stated (identify the knowns). Many problems are stated very succinctly and require some inspection to determine what is known. A sketch can also be very useful at this point. Formally identifying the knowns is of particular importance in applying physics to real-world situations. Remember, “stopped” means velocity is zero, and we often can take initial time and position as zero.

Step 3

Identify exactly what needs to be determined in the problem (identify the unknowns). In complex problems, especially, it is not always obvious what needs to be found or in what sequence. Making a list can help.

Step 4

Find an equation or set of equations that can help you solve the problem. Your list of knowns and unknowns can help here. It is easiest if you can find equations that contain only one unknown—that is, all of the other variables are known, so you can easily solve for the unknown. If the equation contains more than one unknown, then an additional equation is needed to solve the problem. In some problems, several unknowns must be determined to get at the one needed most. In such problems it is especially important to keep physical principles in mind to avoid going astray in a sea of equations. You may have to use two (or more) different equations to get the final answer.

Step 5

Substitute the knowns along with their units into the appropriate equation, and obtain numerical solutions complete with units. This step produces the numerical answer; it also provides a check on units that can help you find errors. If the units of the answer are incorrect, then an error has been made. However, be warned that correct units do not guarantee that the numerical part of the answer is also correct.

Step 6

Check the answer to see if it is reasonable: Does it make sense? This final step is extremely important—the goal of physics is to accurately describe nature. To see if the answer is reasonable, check both its magnitude and its sign, in addition to its units. Your judgment will improve as you solve more and more physics problems, and it will become possible for you to make finer and finer judgments regarding whether nature is adequately described by the answer to a problem. This step brings the problem back to its conceptual meaning. If you can judge whether the answer is reasonable, you have a deeper understanding of physics than just being able to mechanically solve a problem.

When solving problems, we often perform these steps in different order, and we also tend to do several steps simultaneously. There is no rigid procedure that will work every time. Creativity and insight grow with experience, and the basics of problem solving become almost automatic. One way to get practice is to work out the text's examples for yourself as you read. Another is to work as many end-of-section problems as possible, starting with the easiest to build confidence and progressing to the more difficult. Once you become involved in physics, you will see it all around you, and you can begin to apply it to situations you encounter outside the classroom, just as is done in many of the applications in this text.

Unreasonable Results

Physics must describe nature accurately. Some problems have results that are unreasonable because one premise is unreasonable or because certain premises are inconsistent with one another. The physical principle applied correctly then produces an unreasonable result. For example, if a person starting a foot race accelerates at 0.40 m/s^2 for 100 s, his final speed will be 40 m/s (about 150 km/h)—clearly unreasonable because the time of 100 s is an unreasonable premise. The physics is correct in a sense, but there is more to describing nature than just manipulating equations correctly. Checking the result of a problem to see if it is reasonable does more than help uncover errors in problem solving—it also builds intuition in judging whether nature is being accurately described.

Use the following strategies to determine whether an answer is reasonable and, if it is not, to determine what is the cause.

Step 1

Solve the problem using strategies as outlined and in the format followed in the worked examples in the text. In the example given in the preceding paragraph, you would identify the givens as the acceleration and time and use the equation below to find the unknown final velocity. That is,

Equation:

$$v = v_0 + \text{at} = 0 + \left(0.40\ \text{m/s}^2\right)(100\ \text{s}) = 40\ \text{m/s}.$$

Step 2

Check to see if the answer is reasonable. Is it too large or too small, or does it have the wrong sign, improper units, …? In this case, you may need to convert meters per second into a more familiar unit, such as miles per hour.
Equation:

$$\left(\frac{40\ \text{m}}{\text{s}}\right)\left(\frac{3.28\ \text{ft}}{\text{m}}\right)\left(\frac{1\ \text{mi}}{5280\ \text{ft}}\right)\left(\frac{60\ \text{s}}{\text{min}}\right)\left(\frac{60\ \text{min}}{1\ \text{h}}\right) = 89\ \text{mph}$$

This velocity is about four times greater than a person can run—so it is too large.

Step 3

If the answer is unreasonable, look for what specifically could cause the identified difficulty. In the example of the runner, there are only two assumptions that are suspect. The acceleration could be too great or the time too long. First look at the acceleration and think about what the number means. If someone accelerates at $0.40\ \text{m/s}^2$, their velocity is increasing by 0.4 m/s each second. Does this seem reasonable? If so, the time must be too long. It is not possible for someone to accelerate at a constant rate of $0.40\ \text{m/s}^2$ for 100 s (almost two minutes).

Section Summary

- *The six basic problem solving steps for physics are:*

Step 1. Examine the situation to determine which physical principles are involved.

Step 2. Make a list of what is given or can be inferred from the problem as stated (identify the knowns).

Step 3. Identify exactly what needs to be determined in the problem (identify the unknowns).

Step 4. Find an equation or set of equations that can help you solve the problem.

Step 5. Substitute the knowns along with their units into the appropriate equation, and obtain numerical solutions complete with units.

Step 6. Check the answer to see if it is reasonable: Does it make sense?

Conceptual Questions

Exercise:

Problem:

What information do you need in order to choose which equation or equations to use to solve a problem? Explain.

Exercise:

Problem:

What is the last thing you should do when solving a problem? Explain.

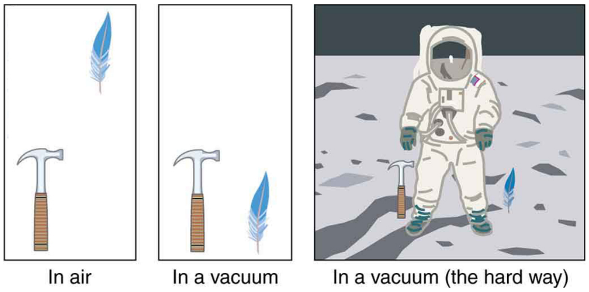
Falling Objects

- Describe the effects of gravity on objects in motion.
- Describe the motion of objects that are in free fall.
- Calculate the position and velocity of objects in free fall.

Falling objects form an interesting class of motion problems. For example, we can estimate the depth of a vertical mine shaft by dropping a rock into it and listening for the rock to hit the bottom. By applying the kinematics developed so far to falling objects, we can examine some interesting situations and learn much about gravity in the process.

Gravity

The most remarkable and unexpected fact about falling objects is that, if air resistance and friction are negligible, then in a given location all objects fall toward the center of Earth with the *same constant acceleration*, *independent of their mass*. This experimentally determined fact is unexpected, because we are so accustomed to the effects of air resistance and friction that we expect light objects to fall slower than heavy ones.


In air
In a vacuum
In a vacuum (the hard way)

A hammer and a feather will fall with the same constant acceleration if air resistance is considered negligible. This is a general characteristic of gravity not unique to Earth, as astronaut David R. Scott demonstrated on the Moon in 1971, where the

acceleration due to gravity is
only 1.67 m/s^2.

In the real world, air resistance can cause a lighter object to fall slower than a heavier object of the same size. A tennis ball will reach the ground after a hard baseball dropped at the same time. (It might be difficult to observe the difference if the height is not large.) Air resistance opposes the motion of an object through the air, while friction between objects—such as between clothes and a laundry chute or between a stone and a pool into which it is dropped—also opposes motion between them. For the ideal situations of these first few chapters, an object *falling without air resistance or friction* is defined to be in **free-fall**.

The force of gravity causes objects to fall toward the center of Earth. The acceleration of free-falling objects is therefore called the **acceleration due to gravity**. The acceleration due to gravity is *constant*, which means we can apply the kinematics equations to any falling object where air resistance and friction are negligible. This opens a broad class of interesting situations to us. The acceleration due to gravity is so important that its magnitude is given its own symbol, *g*. It is constant at any given location on Earth and has the average value

Equation:

$$g = 9.80 \text{ m/s}^2.$$

Although g varies from 9.78 m/s^2 to 9.83 m/s^2, depending on latitude, altitude, underlying geological formations, and local topography, the average value of 9.80 m/s^2 will be used in this text unless otherwise specified. The direction of the acceleration due to gravity is *downward (towards the center of Earth)*. In fact, its direction *defines* what we call vertical. Note that whether the acceleration a in the kinematic equations has the value $+g$ or $-g$ depends on how we define our coordinate system. If we define the upward direction as positive, then $a = -g = -9.80 \text{ m/s}^2$, and if we define the downward direction as positive, then $a = g = 9.80 \text{ m/s}^2$.

One-Dimensional Motion Involving Gravity

The best way to see the basic features of motion involving gravity is to start with the simplest situations and then progress toward more complex ones. So we start by considering straight up and down motion with no air resistance or friction. These assumptions mean that the velocity (if there is any) is vertical. If the object is dropped, we know the initial velocity is zero. Once the object has left contact with whatever held or threw it, the object is in free-fall. Under these circumstances, the motion is one-dimensional and has constant acceleration of magnitude g. We will also represent vertical displacement with the symbol y and use x for horizontal displacement.

Note:
Kinematic Equations for Objects in Free-Fall where Acceleration = -g
Equation:

$$v = v_0 - \mathrm{gt}$$

Equation:

$$y = y_0 + v_0 t - \frac{1}{2} g t^2$$

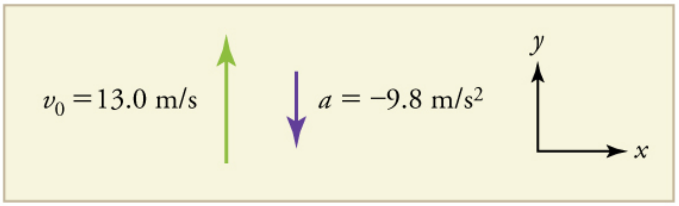
Equation:

$$v^2 = v_0^2 - 2g(y - y_0)$$

Example:
Calculating Position and Velocity of a Falling Object: A Rock Thrown Upward
A person standing on the edge of a high cliff throws a rock straight up with an initial velocity of 13.0 m/s. The rock misses the edge of the cliff as it falls back to earth. Calculate the position and velocity of the rock 1.00 s, 2.00 s, and 3.00 s after it is thrown, neglecting the effects of air resistance.

Strategy

Draw a sketch.



We are asked to determine the position y at various times. It is reasonable to take the initial position y_0 to be zero. This problem involves one-dimensional motion in the vertical direction. We use plus and minus signs to indicate direction, with up being positive and down negative. Since up is positive, and the rock is thrown upward, the initial velocity must be positive too. The acceleration due to gravity is downward, so a is negative. It is crucial that the initial velocity and the acceleration due to gravity have opposite signs. Opposite signs indicate that the acceleration due to gravity opposes the initial motion and will slow and eventually reverse it.

Since we are asked for values of position and velocity at three times, we will refer to these as y_1 and v_1; y_2 and v_2; and y_3 and v_3.

Solution for Position y_1

1. Identify the knowns. We know that $y_0 = 0$; $v_0 = 13.0\ \text{m/s}$; $a = -g = -9.80\ \text{m/s}^2$; and $t = 1.00\ \text{s}$.
2. Identify the best equation to use. We will use $y = y_0 + v_0 t + \frac{1}{2}at^2$ because it includes only one unknown, y (or y_1, here), which is the value we want to find.
3. Plug in the known values and solve for y_1.

Equation:

$$y_1 = 0 + (13.0\ \text{m/s})(1.00\ \text{s}) + \frac{1}{2}\left(-9.80\ \text{m/s}^2\right)(1.00\ \text{s})^2 = 8.10\ \text{m}$$

Discussion

The rock is 8.10 m above its starting point at $t = 1.00$ s, since $y_1 > y_0$. It could be *moving* up or down; the only way to tell is to calculate v_1 and find out if it is positive or negative.

Solution for Velocity v_1

1. Identify the knowns. We know that $y_0 = 0$; $v_0 = 13.0\ \text{m/s}$; $a = -g = -9.80\ \text{m/s}^2$; and $t = 1.00\ \text{s}$. We also know from the solution above that $y_1 = 8.10\ \text{m}$.

2. Identify the best equation to use. The most straightforward is $v = v_0 - \text{gt}$ (from $v = v_0 + \text{at}$, where $a = \text{gravitational acceleration} = -g$).
3. Plug in the knowns and solve.

Equation:

$$v_1 = v_0 - \text{gt} = 13.0\text{ m/s} - \left(9.80\text{ m/s}^2\right)(1.00\text{ s}) = 3.20\text{ m/s}$$

Discussion

The positive value for v_1 means that the rock is still heading upward at $t = 1.00$ s. However, it has slowed from its original 13.0 m/s, as expected.

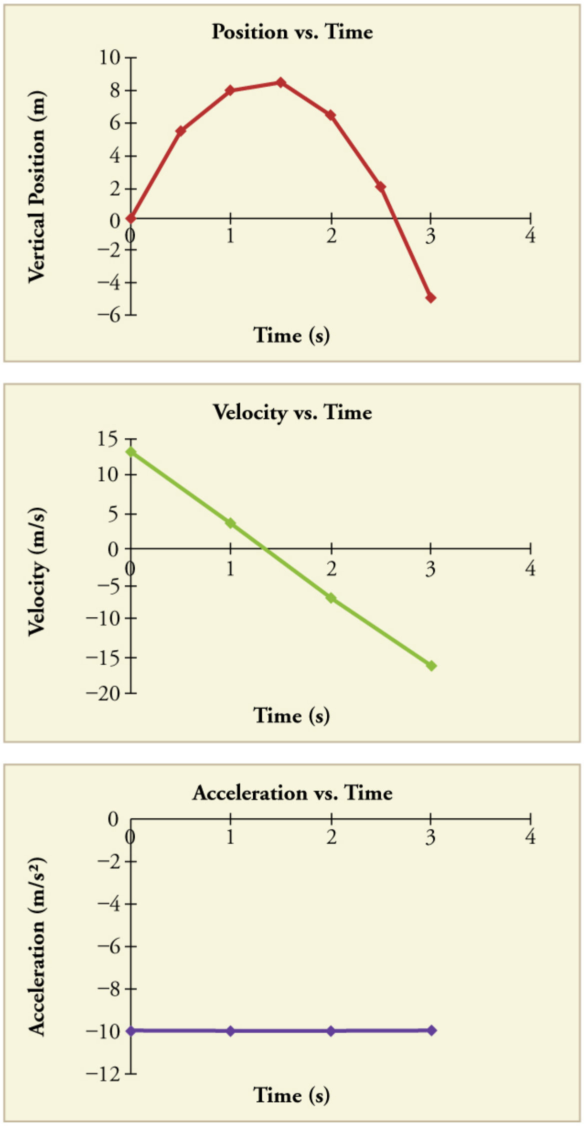
Solution for Remaining Times

The procedures for calculating the position and velocity at $t = 2.00$ s and 3.00 s are the same as those above. The results are summarized in [link] and illustrated in [link].

Time, *t*	**Position, *y***	**Velocity, *v***	**Acceleration, *a***
1.00 s	8.10 m	3.20 m/s	-9.80 m/s^2
2.00 s	6.40 m	-6.60 m/s	-9.80 m/s^2
3.00 s	-5.10 m	-16.4 m/s	-9.80 m/s^2

Results

Graphing the data helps us understand it more clearly.



Vertical position, vertical velocity, and vertical acceleration vs. time for a rock thrown vertically up at the edge of a cliff. Notice that velocity changes linearly with time and that acceleration is constant. *Misconception Alert!* Notice that the position vs. time graph shows vertical position only. It is easy to get the impression that the graph shows some horizontal motion—the shape of the graph looks like the path of a projectile. But this is not the case; the horizontal axis is *time*, not space. The actual path of the rock in space is straight up, and straight down.

Discussion

The interpretation of these results is important. At 1.00 s the rock is above its starting point and heading upward, since y_1 and v_1 are both positive. At 2.00 s, the rock is still above its starting point, but the negative velocity means it is moving downward. At 3.00 s, both y_3 and v_3 are negative, meaning the rock is below its starting point and continuing to move downward. Notice that when the rock is at its highest point (at 1.5 s), its velocity is zero, but its acceleration is still -9.80 m/s^2. Its acceleration is -9.80 m/s^2 for the whole trip—while it is moving up and while it is moving down. Note that the values for y are the positions (or displacements) of the rock, not the total distances traveled. Finally, note that free-fall applies to upward motion as well as downward. Both have the same acceleration—the acceleration due to gravity, which remains constant the entire time. Astronauts training in the famous Vomit Comet, for example, experience free-fall while arcing up as well as down, as we will discuss in more detail later.

Note:

Making Connections: Take-Home Experiment—Reaction Time

A simple experiment can be done to determine your reaction time. Have a friend hold a ruler between your thumb and index finger, separated by about 1 cm. Note the mark on the ruler that is right between your fingers. Have your friend drop the ruler unexpectedly, and try to catch it between your two fingers. Note the new reading on the ruler. Assuming acceleration is that due to gravity, calculate your reaction time. How far would you travel in a car (moving at 30 m/s) if the time it took your foot to go from the gas pedal to the brake was twice this reaction time?

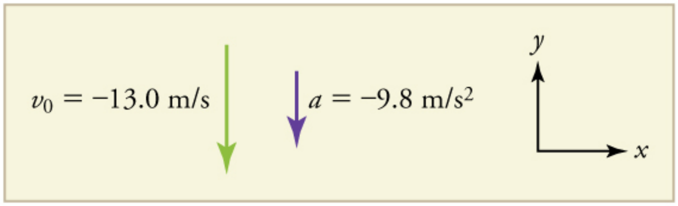
Example:

Calculating Velocity of a Falling Object: A Rock Thrown Down

What happens if the person on the cliff throws the rock straight down, instead of straight up? To explore this question, calculate the velocity of the rock when it is 5.10 m below the starting point, and has been thrown downward with an initial speed of 13.0 m/s.

Strategy

Draw a sketch.



Since up is positive, the final position of the rock will be negative because it finishes below the starting point at $y_0 = 0$. Similarly, the initial velocity is downward and therefore negative, as is the acceleration due to gravity. We expect the final velocity to be negative since the rock will continue to move downward.

Solution

1. Identify the knowns. $y_0 = 0$; $y_1 = -5.10\text{ m}$; $v_0 = -13.0\text{ m/s}$; $a = -g = -9.80\text{ m/s}^2$.

2. Choose the kinematic equation that makes it easiest to solve the problem. The equation $v^2 = v_0^2 + 2a(y - y_0)$ works well because the only unknown in it is v. (We will plug y_1 in for y.)

3. Enter the known values

Equation:

$$v^2 = (-13.0\text{ m/s})^2 + 2\left(-9.80\text{ m/s}^2\right)(-5.10\text{ m} - 0\text{ m}) = 268.96\text{ m}^2/\text{s}^2,$$

where we have retained extra significant figures because this is an intermediate result.

Taking the square root, and noting that a square root can be positive or negative, gives

Equation:

$$v = \pm 16.4\text{ m/s}.$$

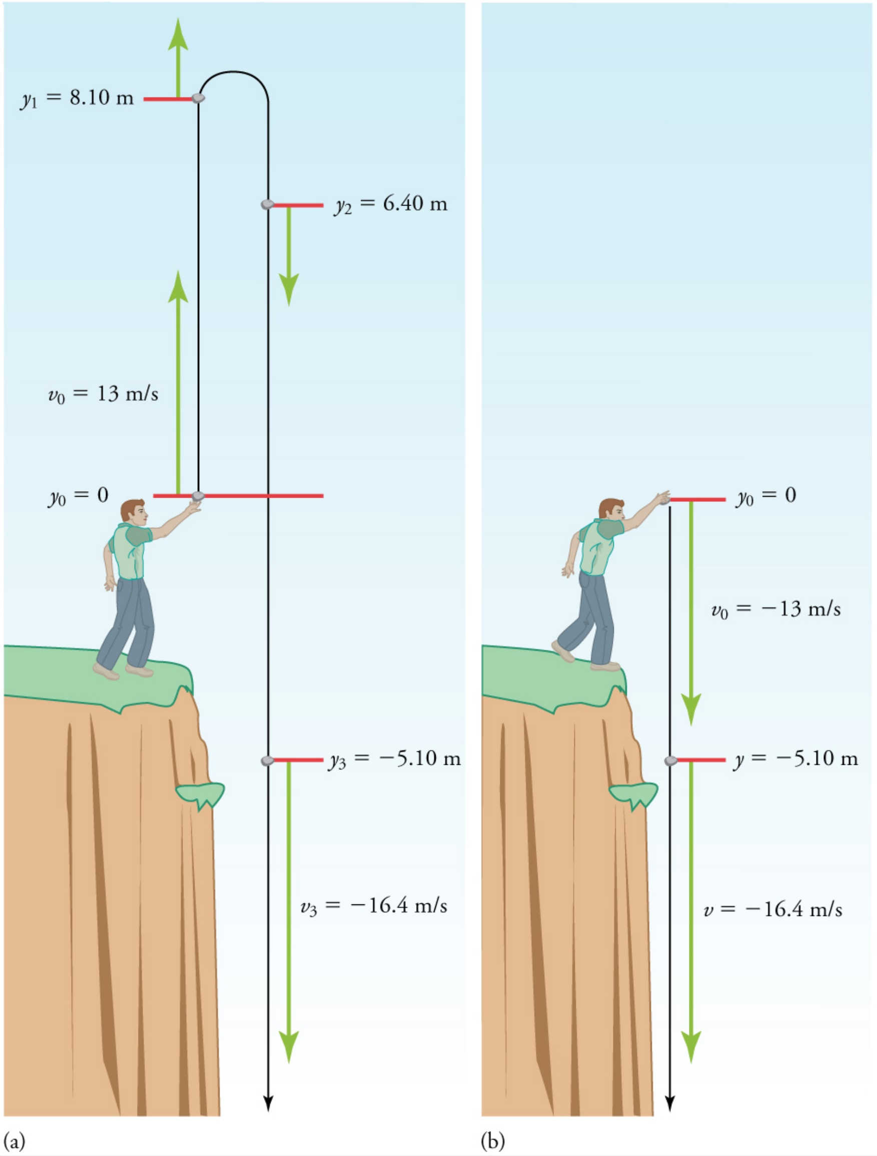
The negative root is chosen to indicate that the rock is still heading down. Thus,

Equation:

$$v = -16.4 \text{ m/s}.$$

Discussion

Note that *this is exactly the same velocity the rock had at this position when it was thrown straight upward with the same initial speed*. (See [link] and [link](a).) This is not a coincidental result. Because we only consider the acceleration due to gravity in this problem, the *speed* of a falling object depends only on its initial speed and its vertical position relative to the starting point. For example, if the velocity of the rock is calculated at a height of 8.10 m above the starting point (using the method from [link]) when the initial velocity is 13.0 m/s straight up, a result of $\pm 3.20 \text{ m/s}$ is obtained. Here both signs are meaningful; the positive value occurs when the rock is at 8.10 m and heading up, and the negative value occurs when the rock is at 8.10 m and heading back down. It has the same *speed* but the opposite direction.



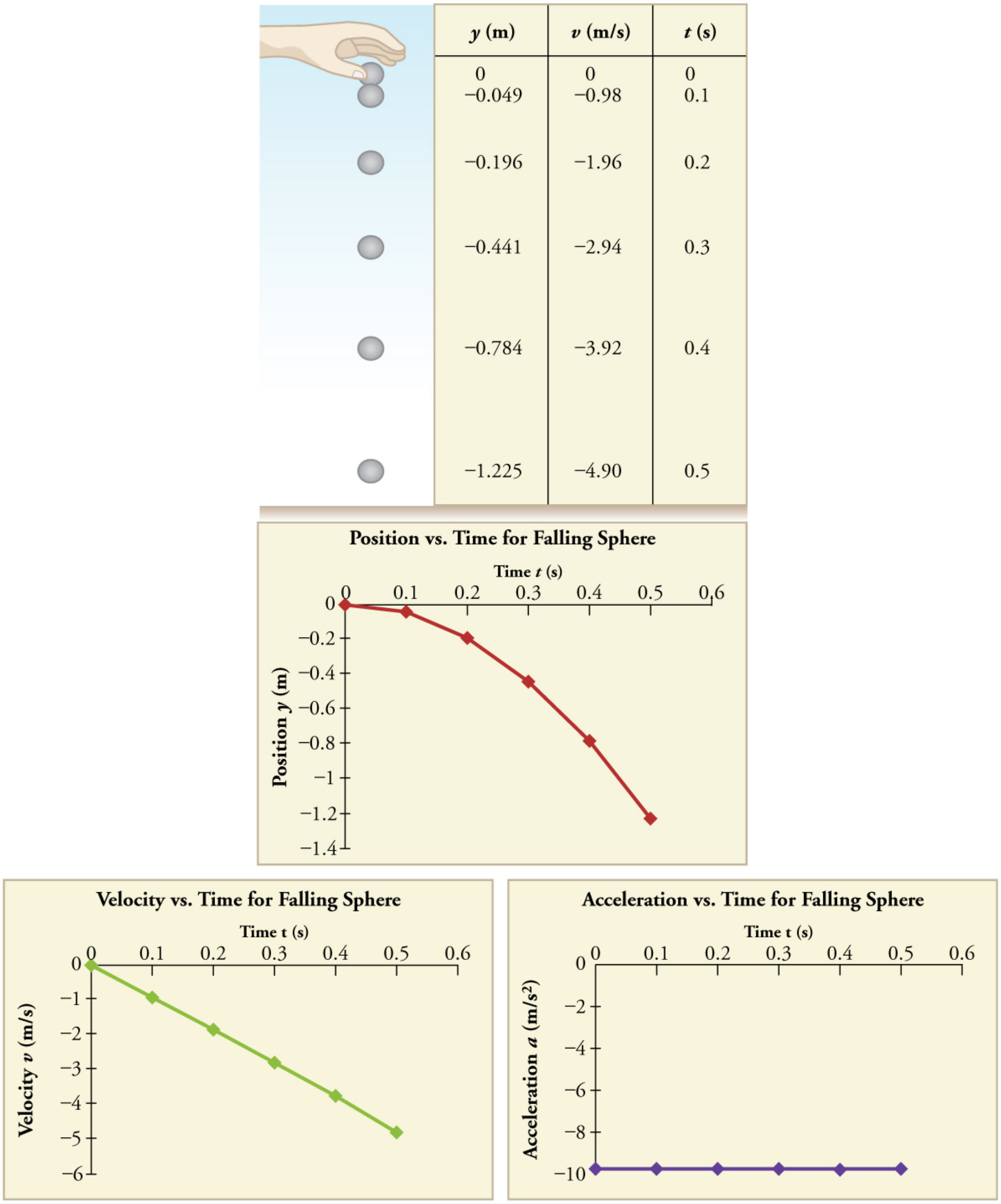
(a) A person throws a rock straight up, as explored in [link]. The arrows are velocity vectors at 0, 1.00, 2.00, and 3.00 s. (b) A person throws a rock straight down from a cliff with the same initial speed as before, as in [link]. Note that at the same distance below the point of release, the rock has the same velocity in both cases.

Another way to look at it is this: In [link], the rock is thrown up with an initial velocity of $13.0\ \text{m/s}$. It rises and then falls back down. When its position is $y=0$ on its way back down, its velocity is $-13.0\ \mathrm{m/s}$. That is, it has the same speed on its way down as on its way up. We would then expect its velocity at a position of $y=-5.10\ \mathrm{m}$ to be the same whether we have thrown it upwards at $+13.0\ \mathrm{m/s}$ or thrown it downwards at $-13.0\ \mathrm{m/s}$. The velocity of the rock on its way down from $y=0$ is the same whether we have thrown it up or down to start with, as long as the speed with which it was initially thrown is the same.

Example:

Find *g* from Data on a Falling Object

The acceleration due to gravity on Earth differs slightly from place to place, depending on topography (e.g., whether you are on a hill or in a valley) and subsurface geology (whether there is dense rock like iron ore as opposed to light rock like salt beneath you.) The precise acceleration due to gravity can be calculated from data taken in an introductory physics laboratory course. An object, usually a metal ball for which air resistance is negligible, is dropped and the time it takes to fall a known distance is measured. See, for example, [link]. Very precise results can be produced with this method if sufficient care is taken in measuring the distance fallen and the elapsed time.



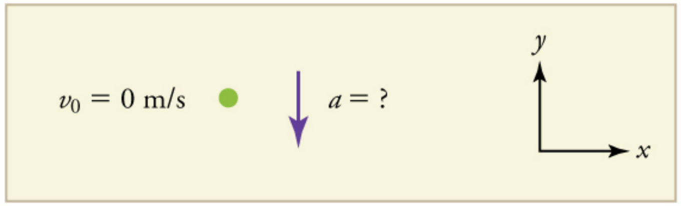
y (m)	v (m/s)	t (s)
0	0	0
−0.049	−0.98	0.1
−0.196	−1.96	0.2
−0.441	−2.94	0.3
−0.784	−3.92	0.4
−1.225	−4.90	0.5

Positions and velocities of a metal ball released from rest when air resistance is negligible. Velocity is seen to increase linearly with time while displacement increases with time squared. Acceleration is a constant and is equal to gravitational acceleration.

Suppose the ball falls 1.0000 m in 0.45173 s. Assuming the ball is not affected by air resistance, what is the precise acceleration due to gravity at this location?

Strategy

Draw a sketch.



We need to solve for acceleration a. Note that in this case, displacement is downward and therefore negative, as is acceleration.

Solution

1. Identify the knowns. $y_0 = 0$; $y = –1.0000$ m; $t = 0.45173$; $v_0 = 0$.
2. Choose the equation that allows you to solve for a using the known values.

Equation:

$$y = y_0 + v_0 t + \frac{1}{2} a t^2$$

3. Substitute 0 for v_0 and rearrange the equation to solve for a. Substituting 0 for v_0 yields

Equation:

$$y = y_0 + \frac{1}{2} a t^2.$$

Solving for a gives

Equation:

$$a = \frac{2(y - y_0)}{t^2}.$$

4. Substitute known values yields

Equation:

$$a = \frac{2(-1.0000 \text{ m} - 0)}{(0.45173 \text{ s})^2} = -9.8010 \text{ m/s}^2,$$

so, because $a = -g$ with the directions we have chosen,

Equation:

$$g = 9.8010 \text{ m/s}^2.$$

Discussion

The negative value for a indicates that the gravitational acceleration is downward, as expected. We expect the value to be somewhere around the average value of 9.80 m/s^2, so 9.8010 m/s^2 makes sense. Since the data going into the calculation are relatively precise, this value for g is more precise than the average value of 9.80 m/s^2; it represents the local value for the acceleration due to gravity.

Exercise:

Check Your Understanding

Problem:

A chunk of ice breaks off a glacier and falls 30.0 meters before it hits the water. Assuming it falls freely (there is no air resistance), how long does it take to hit the water?

Solution:

We know that initial position $y_0 = 0$, final position $y = -30.0 \text{ m}$, and $a = -g = -9.80 \text{ m/s}^2$. We can then use the equation $y = y_0 + v_0 t + \frac{1}{2}at^2$ to solve for t. Inserting $a = -g$, we obtain

Equation:

$$\begin{aligned} y &= 0 + 0 - \tfrac{1}{2}gt^2 \\ t^2 &= \tfrac{2y}{-g} \\ t &= \pm\sqrt{\tfrac{2y}{-g}} = \pm\sqrt{\frac{2(-30.0 \text{ m})}{-9.80 \text{ m/s}^2}} = \pm\sqrt{6.12 \text{ s}^2} = 2.47 \text{ s} \approx 2.5 \text{ s} \end{aligned}$$

where we take the positive value as the physically relevant answer. Thus, it takes about 2.5 seconds for the piece of ice to hit the water.

Note:
PhET Explorations: Equation Grapher
Learn about graphing polynomials. The shape of the curve changes as the constants are adjusted. View the curves for the individual terms (e.g. $y = \text{bx}$) to see how they add to generate the polynomial curve.
https://phet.colorado.edu/sims/equation-grapher/equation-grapher_en.html

Section Summary

- An object in free-fall experiences constant acceleration if air resistance is negligible.
- On Earth, all free-falling objects have an acceleration due to gravity g, which averages
  Equation:
  $$g = 9.80 \text{ m/s}^2.$$
- Whether the acceleration a should be taken as $+g$ or $-g$ is determined by your choice of coordinate system. If you choose the upward direction as positive, $a = -g = -9.80 \text{ m/s}^2$ is negative. In the opposite case, $a = +\text{g} = 9.80 \text{ m/s}^2$ is positive. Since acceleration is constant, the kinematic equations above can be applied with the appropriate $+g$ or $-g$ substituted for a.
- For objects in free-fall, up is normally taken as positive for displacement, velocity, and acceleration.

Conceptual Questions

Exercise:

Problem:

What is the acceleration of a rock thrown straight upward on the way up? At the top of its flight? On the way down?

Exercise:

Problem:

An object that is thrown straight up falls back to Earth. This is one-dimensional motion. (a) When is its velocity zero? (b) Does its velocity change direction? (c) Does the acceleration due to gravity have the same sign on the way up as on the way down?

Exercise:

Problem:

Suppose you throw a rock nearly straight up at a coconut in a palm tree, and the rock misses on the way up but hits the coconut on the way down. Neglecting air resistance, how does the speed of the rock when it hits the coconut on the way down compare with what it would have been if it had hit the coconut on the way up? Is it more likely to dislodge the coconut on the way up or down? Explain.

Exercise:

Problem:

If an object is thrown straight up and air resistance is negligible, then its speed when it returns to the starting point is the same as when it was released. If air resistance were not negligible, how would its speed upon return compare with its initial speed? How would the maximum height to which it rises be affected?

Exercise:

Problem:

The severity of a fall depends on your speed when you strike the ground. All factors but the acceleration due to gravity being the same, how many times higher could a safe fall on the Moon be than on Earth (gravitational acceleration on the Moon is about 1/6 that of the Earth)?

Exercise:

Problem:

How many times higher could an astronaut jump on the Moon than on Earth if his takeoff speed is the same in both locations (gravitational acceleration on the Moon is about 1/6 of g on Earth)?

Problems & Exercises

Assume air resistance is negligible unless otherwise stated.

Exercise:

Problem:

Calculate the displacement and velocity at times of (a) 0.500, (b) 1.00, (c) 1.50, and (d) 2.00 s for a ball thrown straight up with an initial velocity of 15.0 m/s. Take the point of release to be $y_0 = 0$.

Solution:

(a) $y_1 = 6.28\text{ m}$; $v_1 = 10.1\text{ m/s}$

(b) $y_2 = 10.1\text{ m}$; $v_2 = 5.20\text{ m/s}$

(c) $y_3 = 11.5\text{ m}$; $v_3 = 0.300\text{ m/s}$

(d) $y_4 = 10.4\text{ m}$; $v_4 = -4.60\text{ m/s}$

Exercise:

Problem:

Calculate the displacement and velocity at times of (a) 0.500, (b) 1.00, (c) 1.50, (d) 2.00, and (e) 2.50 s for a rock thrown straight down with an initial velocity of 14.0 m/s from the Verrazano Narrows Bridge in New York City. The roadway of this bridge is 70.0 m above the water.

Exercise:

Problem:

A basketball referee tosses the ball straight up for the starting tip-off. At what velocity must a basketball player leave the ground to rise 1.25 m above the floor in an attempt to get the ball?

Solution:

$v_0 = 4.95 \text{ m/s}$

Exercise:

Problem:

A rescue helicopter is hovering over a person whose boat has sunk. One of the rescuers throws a life preserver straight down to the victim with an initial velocity of 1.40 m/s and observes that it takes 1.8 s to reach the water. (a) List the knowns in this problem. (b) How high above the water was the preserver released? Note that the downdraft of the helicopter reduces the effects of air resistance on the falling life preserver, so that an acceleration equal to that of gravity is reasonable.

Exercise:

Problem:

A dolphin in an aquatic show jumps straight up out of the water at a velocity of 13.0 m/s. (a) List the knowns in this problem. (b) How high does his body rise above the water? To solve this part, first note that the final velocity is now a known and identify its value. Then identify the unknown, and discuss how you chose the appropriate equation to solve for it. After choosing the equation, show your steps in solving for the unknown, checking units, and discuss whether the answer is reasonable. (c) How long is the dolphin in the air? Neglect any effects due to his size or orientation.

Solution:

(a) $a = -9.80 \text{ m/s}^2$; $v_0 = 13.0 \text{ m/s}$; $y_0 = 0 \text{ m}$

(b) $v = 0 \text{m/s}$. Unknown is distance y to top of trajectory, where velocity is zero. Use equation $v^2 = v_0^2 + 2a(y - y_0)$ because it contains all known values except for y, so we can solve for y. Solving for y gives

Equation:

$$\begin{array}{lll} v^2 - v_0^2 & = & 2a(y - y_0) \\ \frac{v^2 - v_0^2}{2a} & = & y - y_0 \\ y & = & y_0 + \frac{v^2 - v_0^2}{2a} = 0 \text{ m} + \frac{(0 \text{ m/s})^2 - (13.0 \text{ m/s})^2}{2\left(-9.80 \text{ m/s}^2\right)} = 8.62 \text{ m} \end{array}$$

Dolphins measure about 2 meters long and can jump several times their length out of the water, so this is a reasonable result.

(c) 2.65 s

Exercise:

Problem:

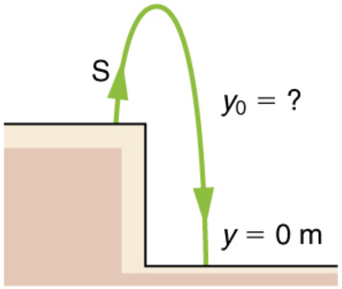
A swimmer bounces straight up from a diving board and falls feet first into a pool. She starts with a velocity of 4.00 m/s, and her takeoff point is 1.80 m above the pool. (a) How long are her feet in the air? (b) What is her highest point above the board? (c) What is her velocity when her feet hit the water?

Exercise:

Problem:

(a) Calculate the height of a cliff if it takes 2.35 s for a rock to hit the ground when it is thrown straight up from the cliff with an initial velocity of 8.00 m/s. (b) How long would it take to reach the ground if it is thrown straight down with the same speed?

Solution:



(a) 8.26 m

(b) 0.717 s

Exercise:

Problem:

A very strong, but inept, shot putter puts the shot straight up vertically with an initial velocity of 11.0 m/s. How long does he have to get out of the way if the shot was released at a height of 2.20 m, and he is 1.80 m tall?

Exercise:

Problem:

You throw a ball straight up with an initial velocity of 15.0 m/s. It passes a tree branch on the way up at a height of 7.00 m. How much additional time will pass before the ball passes the tree branch on the way back down?

Solution:

1.91 s

Exercise:

Problem:

A kangaroo can jump over an object 2.50 m high. (a) Calculate its vertical speed when it leaves the ground. (b) How long is it in the air?

Exercise:

Problem:

Standing at the base of one of the cliffs of Mt. Arapiles in Victoria, Australia, a hiker hears a rock break loose from a height of 105 m. He can't see the rock right away but then does, 1.50 s later. (a) How far above the hiker is the rock when he can see it? (b) How much time does he have to move before the rock hits his head?

Solution:

(a) 94.0 m

(b) 3.13 s

Exercise:

Problem:

An object is dropped from a height of 75.0 m above ground level. (a) Determine the distance traveled during the first second. (b) Determine the final velocity at which the object hits the ground. (c) Determine the distance traveled during the last second of motion before hitting the ground.

Exercise:

Problem:

There is a 250-m-high cliff at Half Dome in Yosemite National Park in California. Suppose a boulder breaks loose from the top of this cliff. (a) How fast will it be going when it strikes the ground? (b) Assuming a reaction time of 0.300 s, how long will a tourist at the bottom have to get out of the way after hearing the sound of the rock breaking loose (neglecting the height of the tourist, which would become negligible anyway if hit)? The speed of sound is 335 m/s on this day.

Solution:

(a) -70.0 m/s (downward)

(b) 6.10 s

Exercise:

Problem:

A ball is thrown straight up. It passes a 2.00-m-high window 7.50 m off the ground on its path up and takes 0.312 s to go past the window. What was the ball’s initial velocity? Hint: First consider only the distance along the window, and solve for the ball's velocity at the bottom of the window. Next, consider only the distance from the ground to the bottom of the window, and solve for the initial velocity using the velocity at the bottom of the window as the final velocity.

Exercise:

Problem:

Suppose you drop a rock into a dark well and, using precision equipment, you measure the time for the sound of a splash to return. (a) Neglecting the time required for sound to travel up the well, calculate the distance to the water if the sound returns in 2.0000 s. (b) Now calculate the distance taking into account the time for sound to travel up the well. The speed of sound is 332.00 m/s in this well.

Solution:

(a) 19.6 m

(b) 18.5 m

Exercise:

Problem:

A steel ball is dropped onto a hard floor from a height of 1.50 m and rebounds to a height of 1.45 m. (a) Calculate its velocity just before it strikes the floor. (b) Calculate its velocity just after it leaves the floor on its way back up. (c) Calculate its acceleration during contact with the floor if that contact lasts 0.0800 ms $(8.00\times 10^{-5}\text{ s})$. (d) How much did the ball compress during its collision with the floor, assuming the floor is absolutely rigid?

Exercise:

Problem:

A coin is dropped from a hot-air balloon that is 300 m above the ground and rising at 10.0 m/s upward. For the coin, find (a) the maximum height reached, (b) its position and velocity 4.00 s after being released, and (c) the time before it hits the ground.

Solution:

(a) 305 m

(b) 262 m, -29.2 m/s

(c) 8.91 s

Exercise:

Problem:

A soft tennis ball is dropped onto a hard floor from a height of 1.50 m and rebounds to a height of 1.10 m. (a) Calculate its velocity just before it strikes the floor. (b) Calculate its velocity just after it leaves the floor on its way back up. (c) Calculate its acceleration during contact with the floor if that contact lasts 3.50 ms $(3.50 \times 10^{-3}\text{ s})$. (d) How much did the ball compress during its collision with the floor, assuming the floor is absolutely rigid?

Glossary

free-fall
: the state of movement that results from gravitational force only

acceleration due to gravity
: acceleration of an object as a result of gravity

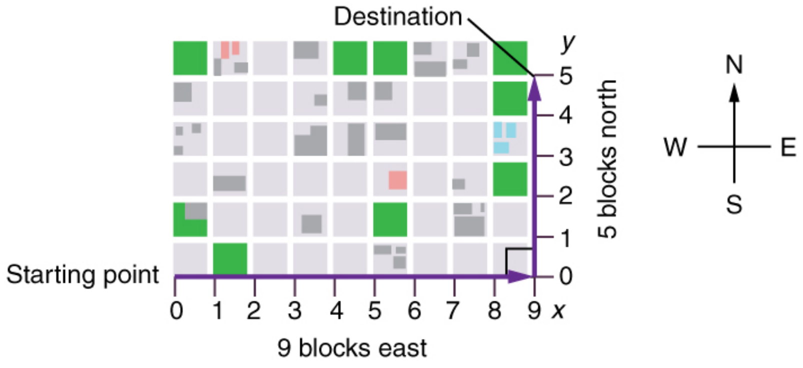
Kinematics in Two Dimensions: An Introduction

- Observe that motion in two dimensions consists of horizontal and vertical components.
- Understand the independence of horizontal and vertical vectors in two-dimensional motion.

Walkers and drivers in a city like New York are rarely able to travel in straight lines to reach their destinations. Instead, they must follow roads and sidewalks, making two-dimensional, zigzagged paths. (credit: Margaret W. Carruthers)

Two-Dimensional Motion: Walking in a City

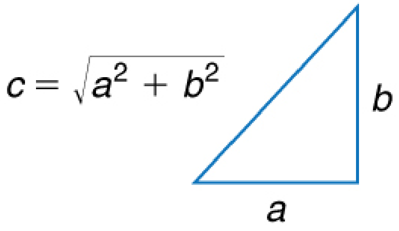
Suppose you want to walk from one point to another in a city with uniform square blocks, as pictured in [link].



A pedestrian walks a two-dimensional path between two points in a city. In this scene, all blocks are square and are the same size.

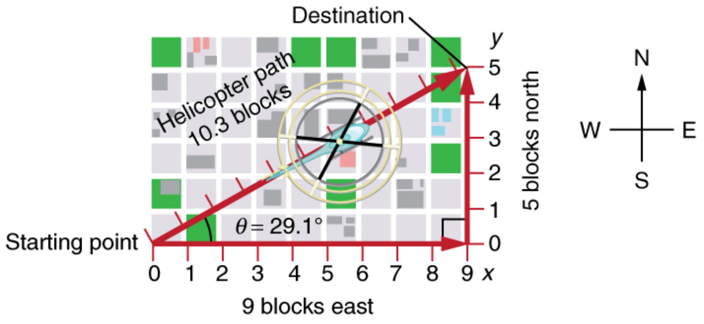
The straight-line path that a helicopter might fly is blocked to you as a pedestrian, and so you are forced to take a two-dimensional path, such as the one shown. You walk 14 blocks in all, 9 east followed by 5 north. What is the straight-line distance?

An old adage states that the shortest distance between two points is a straight line. The two legs of the trip and the straight-line path form a right triangle, and so the Pythagorean theorem, $a^2 + b^2 = c^2$, can be used to find the straight-line distance.



The Pythagorean theorem relates the length of the legs of a right triangle, labeled a and b, with the hypotenuse, labeled c. The relationship is given by: $a^2 + b^2 = c^2$. This can be rewritten, solving for c : $c = \sqrt{a^2 + b^2}$.

The hypotenuse of the triangle is the straight-line path, and so in this case its length in units of city blocks is $\sqrt{(9\text{ blocks})^2 + (5\text{ blocks})^2} = 10.3\text{ blocks}$, considerably shorter than the 14 blocks you walked. (Note that we are using three significant figures in the answer. Although it appears that “9” and “5” have only one significant digit, they are discrete numbers. In this case “9 blocks” is the same as “9.0 or 9.00 blocks.” We have decided to use three significant figures in the answer in order to show the result more precisely.)



The straight-line path followed by a helicopter between the two points is shorter than the 14 blocks walked by the pedestrian. All blocks are square and the same size.

The fact that the straight-line distance (10.3 blocks) in [link] is less than the total distance walked (14 blocks) is one example of a general characteristic of vectors. (Recall that **vectors** are quantities that have both magnitude and direction.)

As for one-dimensional kinematics, we use arrows to represent vectors. The length of the arrow is proportional to the vector's magnitude. The arrow's length is indicated by hash marks in [link] and [link]. The arrow points in the same direction as the vector. For two-dimensional motion, the path of an object can be represented with three vectors: one vector shows the straight-line path between the initial and final points of the motion, one vector shows the horizontal component of the motion, and one vector shows the vertical component of the motion. The horizontal and vertical components of the motion add together to give the straight-line path. For example, observe the three vectors in [link]. The first represents a 9-block displacement east. The second represents a 5-block displacement north. These vectors are added to give the third vector, with a 10.3-block total displacement. The third vector is the straight-line path between the two points. Note that in this example, the vectors that we are adding are perpendicular to each other and thus form a right triangle. This means that we can use the Pythagorean theorem to calculate the magnitude of the total displacement. (Note that we cannot use the Pythagorean theorem to add vectors that are not perpendicular. We will develop techniques for adding vectors having any direction, not just those perpendicular to one another, in Vector Addition and Subtraction: Graphical Methods and Vector Addition and Subtraction: Analytical Methods.)

The Independence of Perpendicular Motions

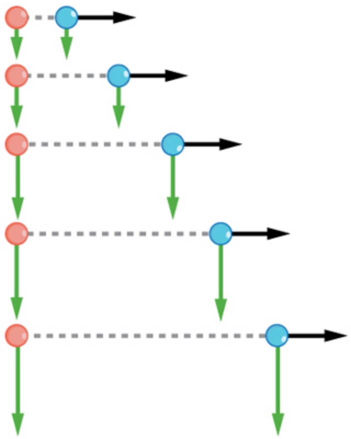
The person taking the path shown in [link] walks east and then north (two perpendicular directions). How far he or she walks east is only affected by his or her motion eastward. Similarly, how far he or she walks north is only affected by his or her motion northward.

Note:

Independence of Motion

The horizontal and vertical components of two-dimensional motion are independent of each other. Any motion in the horizontal direction does not affect motion in the vertical direction, and vice versa.

This is true in a simple scenario like that of walking in one direction first, followed by another. It is also true of more complicated motion involving movement in two directions at once. For example, let’s compare the motions of two baseballs. One baseball is dropped from rest. At the same instant, another is thrown horizontally from the same height and follows a curved path. A stroboscope has captured the positions of the balls at fixed time intervals as they fall.



This shows the motions of two identical balls—one falls from rest, the other has an initial horizontal velocity. Each subsequent position is an equal time interval. Arrows represent horizontal and vertical velocities at each position. The ball on the right has an initial horizontal velocity, while the ball on the left has no horizontal velocity. Despite the difference in horizontal velocities, the vertical velocities and positions are identical for both balls. This shows that the vertical and horizontal motions are independent.

It is remarkable that for each flash of the strobe, the vertical positions of the two balls are the same. This similarity implies that the vertical motion is independent of whether or not the ball is moving horizontally. (Assuming no air resistance, the vertical motion of a falling object is influenced by gravity only, and not by any horizontal forces.) Careful examination of the ball thrown horizontally shows that it travels the same horizontal distance between flashes. This is due to the fact that there are no additional forces on the ball in the horizontal direction after it is thrown. This result means that the horizontal velocity is constant, and affected neither by vertical motion nor by gravity (which is vertical). Note that this case is true only for ideal conditions. In the real world, air resistance will affect the speed of the balls in both directions.

The two-dimensional curved path of the horizontally thrown ball is composed of two independent one-dimensional motions (horizontal and vertical). The key to analyzing such motion, called *projectile motion*, is to *resolve* (break) it into motions along perpendicular directions. Resolving two-dimensional motion into perpendicular components is possible because the components are independent. We shall see how to resolve vectors in Vector Addition and Subtraction: Graphical Methods and Vector Addition and Subtraction: Analytical Methods. We will find such techniques to be useful in many areas of physics.

Note:
PhET Explorations: Ladybug Motion 2D
Learn about position, velocity and acceleration vectors. Move the ladybug by setting the position, velocity or acceleration, and see how the vectors change. Choose linear, circular or elliptical motion, and record and playback the motion to analyze the behavior.
https://archive.cnx.org/specials/317a2b1e-2fbd-11e5-99b5-e38ffb545fe6/ladybug-motion/#sim-ladybug-motion

Summary

- The shortest path between any two points is a straight line. In two dimensions, this path can be represented by a vector with horizontal and vertical components.
- The horizontal and vertical components of a vector are independent of one another. Motion in the horizontal direction does not affect motion

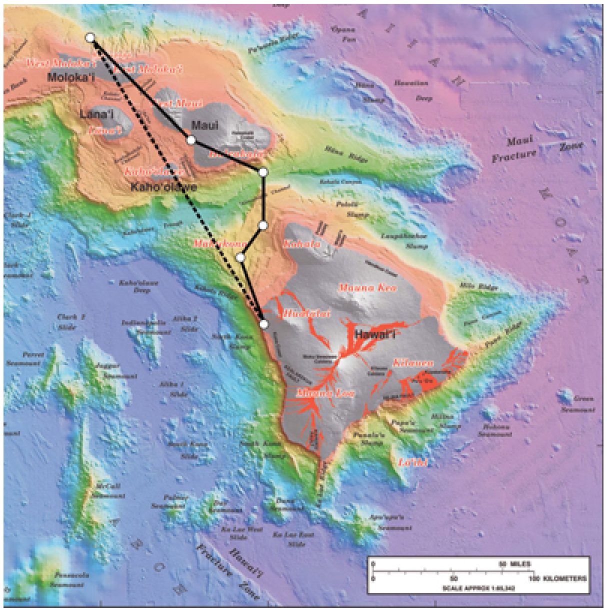
in the vertical direction, and vice versa.

Glossary

vector	a quantity that has both magnitude and direction; an arrow used to represent quantities with both magnitude and direction

Vector Addition and Subtraction: Graphical Methods

- Understand the rules of vector addition, subtraction, and multiplication.
- Apply graphical methods of vector addition and subtraction to determine the displacement of moving objects.



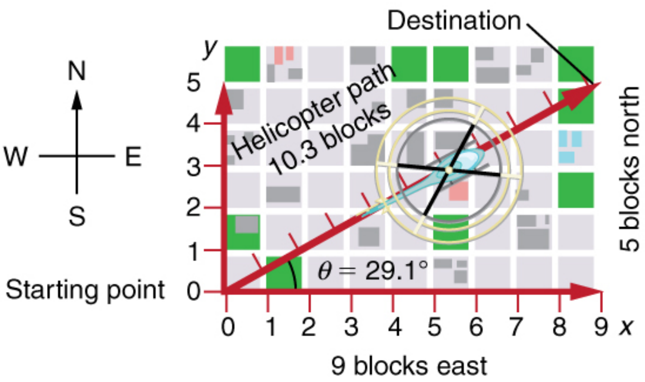
Displacement can be determined graphically using a scale map, such as this one of the Hawaiian Islands. A journey from Hawai'i to Moloka'i has a number of legs, or journey segments. These segments can be added graphically with a ruler to determine the total two-dimensional displacement of the journey. (credit: US Geological Survey)

Vectors in Two Dimensions

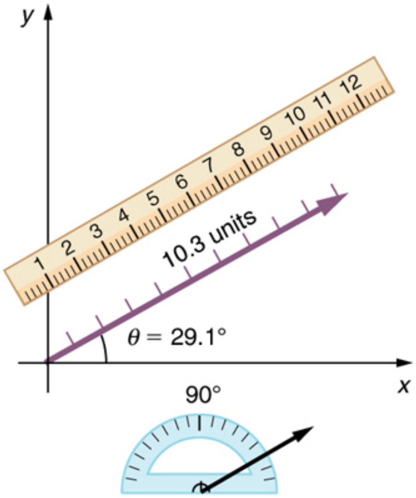
A **vector** is a quantity that has magnitude and direction. Displacement, velocity, acceleration, and force, for example, are all vectors. In one-dimensional, or straight-line, motion, the direction of a vector can be given simply by a plus or minus sign. In two dimensions (2-d), however, we specify the direction of a vector relative to some reference frame (i.e., coordinate system), using an arrow having length proportional to the vector's magnitude and pointing in the direction of the vector.

[link] shows such a *graphical representation of a vector*, using as an example the total displacement for the person walking in a city considered in Kinematics in Two Dimensions: An Introduction. We shall use the notation that a boldface symbol, such as $\mathbf{D}$, stands for a vector. Its magnitude is represented by the symbol in italics, D, and its direction by θ.

Note:
Vectors in this Text
In this text, we will represent a vector with a boldface variable. For example, we will represent the quantity force with the vector $\mathbf{F}$, which has both magnitude and direction. The magnitude of the vector will be represented by a variable in italics, such as F, and the direction of the variable will be given by an angle θ.



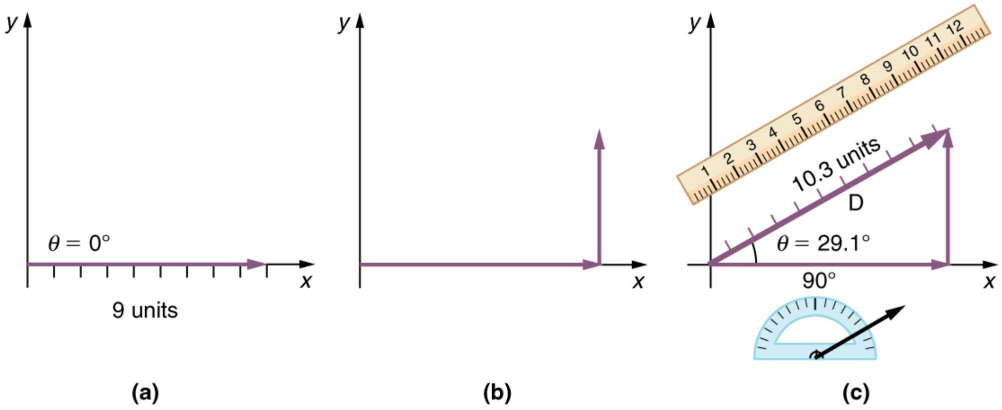
A person walks 9 blocks east and 5 blocks north. The displacement is 10.3 blocks at an angle 29.1° north of east.



To describe the resultant vector for the person walking in a city considered in [link] graphically, draw an arrow to represent the total displacement vector $\mathbf{D}$. Using a protractor, draw a line at an angle θ relative to the east-west axis. The length D of the arrow is proportional to the vector's magnitude and is measured along the line with a ruler. In this example, the magnitude D of the vector is 10.3 units, and the direction θ is 29.1° north of east.

Vector Addition: Head-to-Tail Method

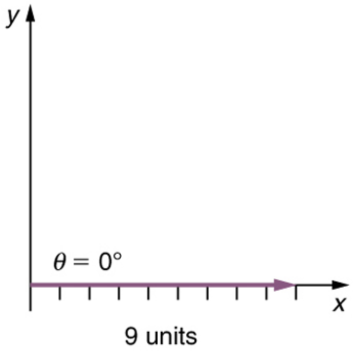
The **head-to-tail method** is a graphical way to add vectors, described in [link] below and in the steps following. The **tail** of the vector is the starting point of the vector, and the **head** (or tip) of a vector is the final, pointed end of the arrow.



Head-to-Tail Method: The head-to-tail method of graphically adding vectors is illustrated for the two displacements of the person walking in a city considered in [link]. (a) Draw a vector representing the displacement to the east. (b) Draw a vector representing the displacement to the north. The tail of this vector should originate from the head of the first, east-pointing vector.

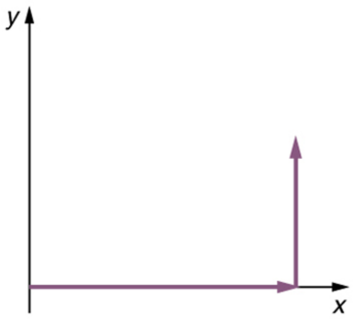
(c) Draw a line from the tail of the east-pointing vector to the head of the north-pointing vector to form the sum or **resultant vector** D. The length of the arrow D is proportional to the vector's magnitude and is measured to be 10.3 units . Its direction, described as the angle with respect to the east (or horizontal axis) θ is measured with a protractor to be 29.1°.

Step 1. *Draw an arrow to represent the first vector (9 blocks to the east) using a ruler and protractor*.



(a)

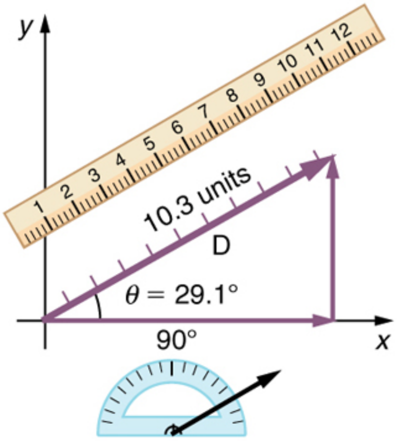
Step 2. Now draw an arrow to represent the second vector (5 blocks to the north). *Place the tail of the second vector at the head of the first vector*.



(b)

Step 3. *If there are more than two vectors, continue this process for each vector to be added. Note that in our example, we have only two vectors, so we have finished placing arrows tip to tail.*

Step 4. *Draw an arrow from the tail of the first vector to the head of the last vector*. This is the **resultant**, or the sum, of the other vectors.



(c)

Step 5. To get the **magnitude** of the resultant, *measure its length with a ruler. (Note that in most calculations, we will use the Pythagorean theorem to determine this length.)*

Step 6. To get the **direction** of the resultant, *measure the angle it makes with the reference frame using a protractor. (Note that in most calculations, we will use trigonometric relationships to determine this angle.)*

The graphical addition of vectors is limited in accuracy only by the precision with which the drawings can be made and the precision of the measuring tools. It is valid for any number of vectors.

Example:

Adding Vectors Graphically Using the Head-to-Tail Method: A Woman Takes a Walk

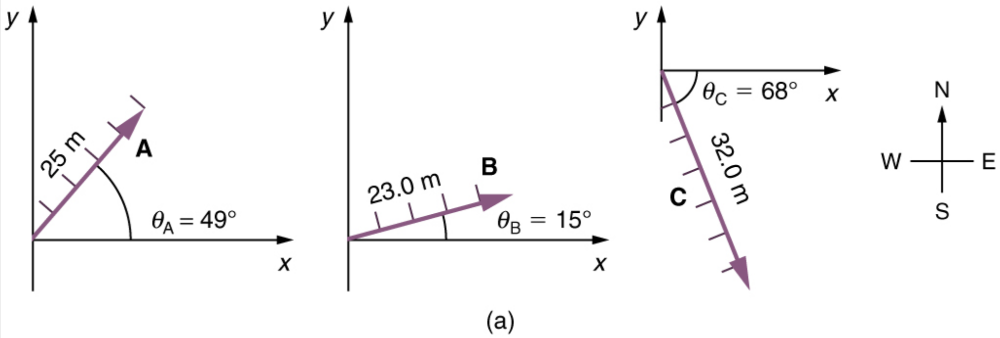
Use the graphical technique for adding vectors to find the total displacement of a person who walks the following three paths (displacements) on a flat field. First, she walks 25.0 m in a direction 49.0º north of east. Then, she walks 23.0 m heading 15.0º north of east. Finally, she turns and walks 32.0 m in a direction 68.0° south of east.

Strategy

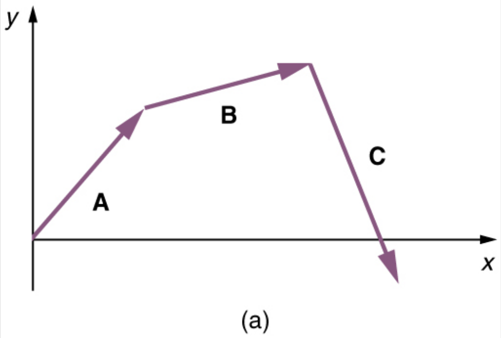
Represent each displacement vector graphically with an arrow, labeling the first A, the second B, and the third C, making the lengths proportional to the distance and the directions as specified relative to an east-west line. The head-to-tail method outlined above will give a way to determine the magnitude and direction of the resultant displacement, denoted **R**.

Solution

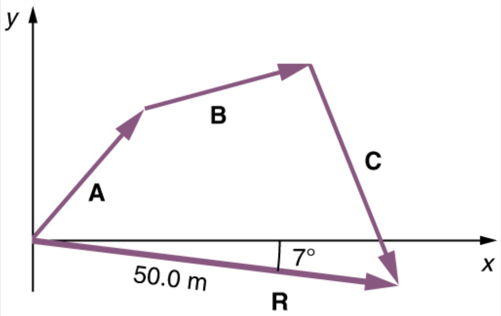
(1) Draw the three displacement vectors.



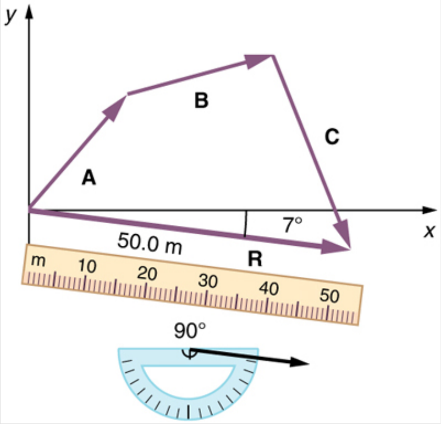
(2) Place the vectors head to tail retaining both their initial magnitude and direction.



(3) Draw the resultant vector, R.



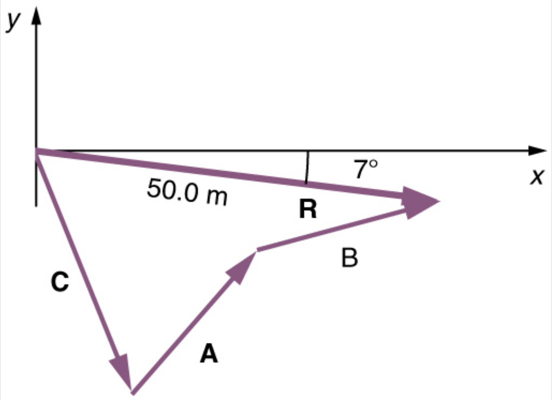
(4) Use a ruler to measure the magnitude of $\mathbf{R}$, and a protractor to measure the direction of R. While the direction of the vector can be specified in many ways, the easiest way is to measure the angle between the vector and the nearest horizontal or vertical axis. Since the resultant vector is south of the eastward pointing axis, we flip the protractor upside down and measure the angle between the eastward axis and the vector.



In this case, the total displacement $\mathbf{R}$ is seen to have a magnitude of 50.0 m and to lie in a direction 7.0° south of east. By using its magnitude and direction, this vector can be expressed as $R = 50.0\ \mathrm{m}$ and $\theta = 7.0^{\circ}$ south of east.

Discussion

The head-to-tail graphical method of vector addition works for any number of vectors. It is also important to note that the resultant is independent of the order in which the vectors are added. Therefore, we could add the vectors in any order as illustrated in [link] and we will still get the same solution.



Here, we see that when the same vectors are added in a different order, the result is the same. This characteristic is true in every case and is an important characteristic of vectors. Vector addition is **commutative**. Vectors can be added in any order.

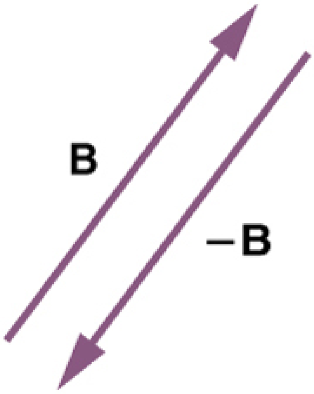
Equation:

$$\mathbf{A}+\mathbf{B}=\mathbf{B}+\mathbf{A}.$$

(This is true for the addition of ordinary numbers as well—you get the same result whether you add $\mathbf{2}+\mathbf{3}$ or $\mathbf{3}+\mathbf{2}$, for example).

Vector Subtraction

Vector subtraction is a straightforward extension of vector addition. To define subtraction (say we want to subtract $\mathbf{B}$ from $\mathbf{A}$, written $\mathbf{A}–\mathbf{B}$, we must first define what we mean by subtraction. The *negative* of a vector $\mathbf{B}$ is defined to be $–\mathbf{B}$; that is, graphically *the negative of any vector has the same magnitude but the opposite direction*, as shown in [link]. In other words, $\mathbf{B}$ has the same length as $–\mathbf{B}$, but points in the opposite direction. Essentially, we just flip the vector so it points in the opposite direction.



The negative of a vector is just another vector of the same magnitude but pointing in the opposite direction. So $\mathbf{B}$ is the negative of $-\mathbf{B}$; it has the same length but opposite direction.

The *subtraction* of vector $\mathbf{B}$ from vector $\mathbf{A}$ is then simply defined to be the addition of $-\mathbf{B}$ to $\mathbf{A}$. Note that vector subtraction is the addition of a negative vector. The order of subtraction does not affect the results.

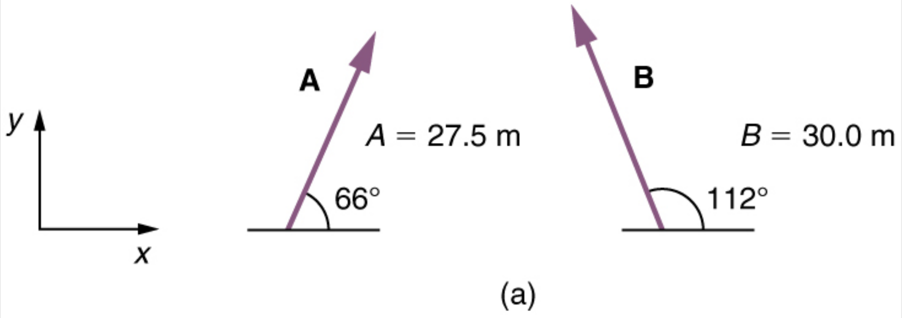
Equation:

$$\mathrm{A} - \mathrm{B} = \mathrm{A} + (-\mathbf{B}).$$

This is analogous to the subtraction of scalars (where, for example, $5 - 2 = 5 + (-2)$). Again, the result is independent of the order in which the subtraction is made. When vectors are subtracted graphically, the techniques outlined above are used, as the following example illustrates.

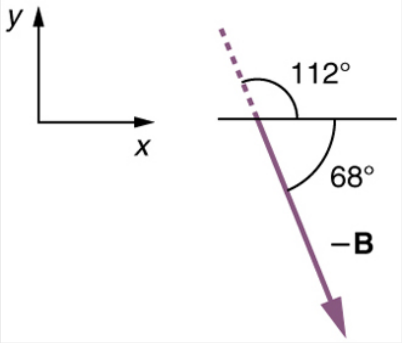
Example:
Subtracting Vectors Graphically: A Woman Sailing a Boat

A woman sailing a boat at night is following directions to a dock. The instructions read to first sail 27.5 m in a direction 66.0° north of east from her current location, and then travel 30.0 m in a direction 112° north of east (or 22.0° west of north). If the woman makes a mistake and travels in the *opposite* direction for the second leg of the trip, where will she end up? Compare this location with the location of the dock.



Strategy

We can represent the first leg of the trip with a vector $\mathbf{A}$, and the second leg of the trip with a vector $\mathbf{B}$. The dock is located at a location $\mathbf{A}+\mathbf{B}$. If the woman mistakenly travels in the *opposite* direction for the second leg of the journey, she will travel a distance B (30.0 m) in the direction $180^{\circ}-112^{\circ}=68^{\circ}$ south of east. We represent this as $-\mathbf{B}$, as shown below. The vector $-\mathbf{B}$ has the same magnitude as $\mathbf{B}$ but is in the opposite direction. Thus, she will end up at a location $\mathbf{A}+(-\mathbf{B})$, or $\mathbf{A}-\mathbf{B}$.



We will perform vector addition to compare the location of the dock, $\mathrm{A}+\mathbf{B}$, with the location at which the woman mistakenly arrives, $\mathrm{A}+(–\mathrm{B})$.

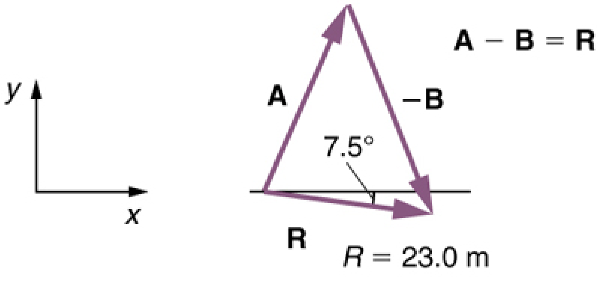
Solution

(1) To determine the location at which the woman arrives by accident, draw vectors $\mathbf{A}$ and $–\mathbf{B}$.

(2) Place the vectors head to tail.

(3) Draw the resultant vector $\mathbf{R}$.

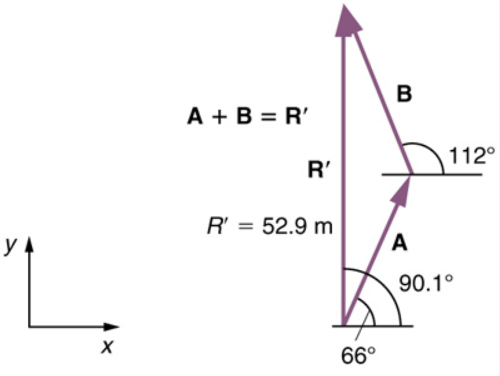
(4) Use a ruler and protractor to measure the magnitude and direction of $\mathbf{R}$.



(b)

In this case, $R=23.0\text{ m}$ and $\theta=7.5^{\circ}$ south of east.

(5) To determine the location of the dock, we repeat this method to add vectors $\mathbf{A}$ and $\mathbf{B}$. We obtain the resultant vector $\mathbf{R}'$:



(c)

In this case $R=52.9\text{ m}$ and $\theta=90.1^{\circ}$ north of east.

We can see that the woman will end up a significant distance from the dock if she travels in the opposite direction for the second leg of the trip.

Discussion

Because subtraction of a vector is the same as addition of a vector with the opposite direction, the graphical method of subtracting vectors works the same as for addition.

Multiplication of Vectors and Scalars

If we decided to walk three times as far on the first leg of the trip considered in the preceding example, then we would walk $3 \times 27.5 \text{ m}$, or 82.5 m, in a direction 66.0º north of east. This is an example of multiplying a vector by a positive **scalar**. Notice that the magnitude changes, but the direction stays the same.

If the scalar is negative, then multiplying a vector by it changes the vector's magnitude and gives the new vector the *opposite* direction. For example, if you multiply by –2, the magnitude doubles but the direction changes. We can summarize these rules in the following way: When vector $\mathbf{A}$ is multiplied by a scalar c,

- the magnitude of the vector becomes the absolute value of cA,
- if c is positive, the direction of the vector does not change,
- if c is negative, the direction is reversed.

In our case, $c = 3$ and $A = 27.5 \text{ m}$. Vectors are multiplied by scalars in many situations. Note that division is the inverse of multiplication. For example, dividing by 2 is the same as multiplying by the value (1/2). The rules for multiplication of vectors by scalars are the same for division; simply treat the divisor as a scalar between 0 and 1.

Resolving a Vector into Components

In the examples above, we have been adding vectors to determine the resultant vector. In many cases, however, we will need to do the opposite. We will need to take a single vector and find what other vectors added together produce it. In most cases, this involves determining the perpendicular **components** of a single vector, for example the *x- and y*-components, or the north-south and east-west components.

For example, we may know that the total displacement of a person walking in a city is 10.3 blocks in a direction 29.0º north of east and want to find out how many blocks east and north had to be walked. This method is called *finding the components (or parts)* of the displacement in the east and north directions, and it is the inverse of the process followed to find the total displacement. It is one example of finding the components of a vector. There are many applications in physics where this is a useful thing to do. We will see this soon in Projectile Motion, and much more when we cover **forces** in Dynamics: Newton’s Laws of Motion. Most of these involve finding components along perpendicular axes (such as north and east), so that right triangles are involved. The analytical techniques presented in Vector Addition and Subtraction: Analytical Methods are ideal for finding vector components.

Note:
PhET Explorations: Maze Game
Learn about position, velocity, and acceleration in the "Arena of Pain". Use the green arrow to move the ball. Add more walls to the arena to make the game more difficult. Try to make a goal as fast as you can.
https://archive.cnx.org/specials/30e37034-2fbd-11e5-83a2-03be60006ece/maze-game/#sim-maze-game

Summary

- The **graphical method of adding vectors** $\mathbf{A}$ and $\mathbf{B}$ involves drawing vectors on a graph and adding them using the head-to-tail method. The resultant vector $\mathbf{R}$ is defined such that $\mathbf{A}+\mathbf{B}=\mathbf{R}$. The magnitude and direction of $\mathbf{R}$ are then determined with a ruler and protractor, respectively.
- The **graphical method of subtracting vector** $\mathbf{B}$ from $\mathbf{A}$ involves adding the opposite of vector $\mathbf{B}$, which is defined as $-\mathbf{B}$. In this case, $\mathbf{A}-\mathbf{B}=\mathbf{A}+(-\mathbf{B})=\mathbf{R}$. Then, the head-to-tail method of addition is followed in the usual way to obtain the resultant vector $\mathbf{R}$.
- Addition of vectors is **commutative** such that $\mathbf{A}+\mathbf{B}=\mathbf{B}+\mathbf{A}$.
- The **head-to-tail method** of adding vectors involves drawing the first vector on a graph and then placing the tail of each subsequent vector at

the head of the previous vector. The resultant vector is then drawn from the tail of the first vector to the head of the final vector.

- If a vector $\mathbf{A}$ is multiplied by a scalar quantity c, the magnitude of the product is given by cA. If c is positive, the direction of the product points in the same direction as $\mathbf{A}$; if c is negative, the direction of the product points in the opposite direction as $\mathbf{A}$.

Conceptual Questions

Exercise:

Problem:

Which of the following is a vector: a person's height, the altitude on Mt. Everest, the age of the Earth, the boiling point of water, the cost of this book, the Earth's population, the acceleration of gravity?

Exercise:

Problem:

Give a specific example of a vector, stating its magnitude, units, and direction.

Exercise:

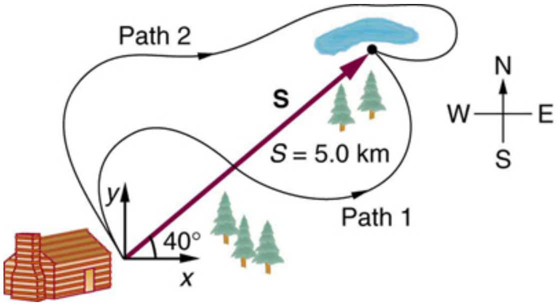
Problem:

What do vectors and scalars have in common? How do they differ?

Exercise:

Problem:

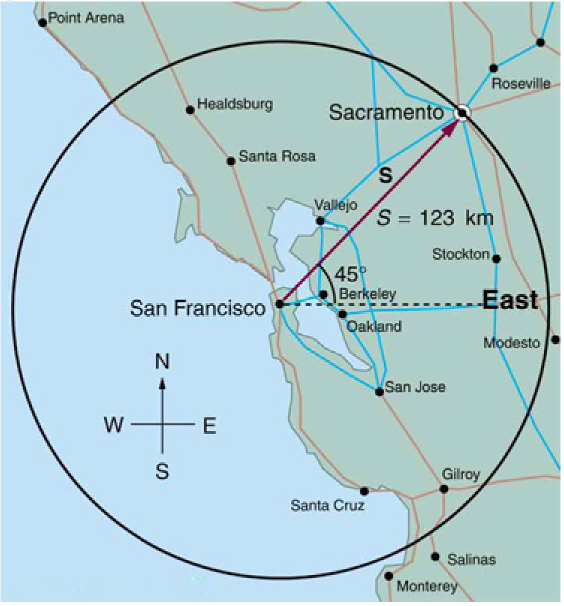
Two campers in a national park hike from their cabin to the same spot on a lake, each taking a different path, as illustrated below. The total distance traveled along Path 1 is 7.5 km, and that along Path 2 is 8.2 km. What is the final displacement of each camper?



Exercise:

Problem:

If an airplane pilot is told to fly 123 km in a straight line to get from San Francisco to Sacramento, explain why he could end up anywhere on the circle shown in [link]. What other information would he need to get to Sacramento?



Exercise:

Problem:

Suppose you take two steps $\mathbf{A}$ and $\mathbf{B}$ (that is, two nonzero displacements). Under what circumstances can you end up at your starting point? More generally, under what circumstances can two nonzero vectors add to give zero? Is the maximum distance you can end up from the starting point $\mathbf{A}+\mathbf{B}$ the sum of the lengths of the two steps?

Exercise:

Problem: Explain why it is not possible to add a scalar to a vector.

Exercise:

Problem:

If you take two steps of different sizes, can you end up at your starting point? More generally, can two vectors with different magnitudes ever add to zero? Can three or more?

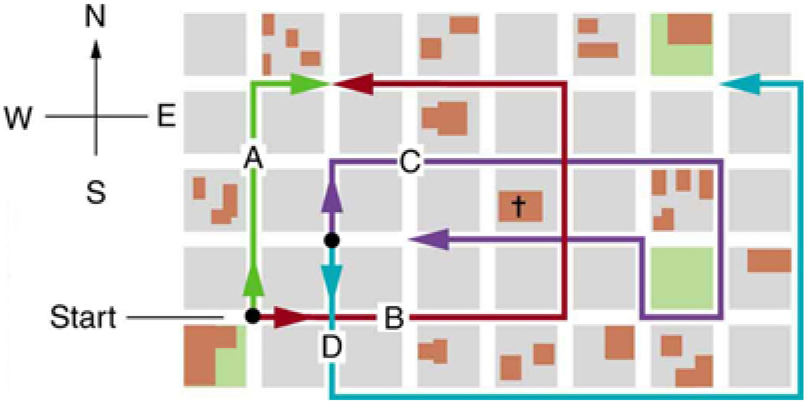
Problems & Exercises

Use graphical methods to solve these problems. You may assume data taken from graphs is accurate to three digits.

Exercise:

Problem:

Find the following for path A in [link]: (a) the total distance traveled, and (b) the magnitude and direction of the displacement from start to finish.



The various lines represent paths taken by different people walking in a city. All blocks are 120 m on a side.

Solution:

(a) 480 m

(b) 379 m, 18.4º east of north

Exercise:

Problem:

Find the following for path B in [link]: (a) the total distance traveled, and (b) the magnitude and direction of the displacement from start to finish.

Exercise:

Problem:

Find the north and east components of the displacement for the hikers shown in [link].

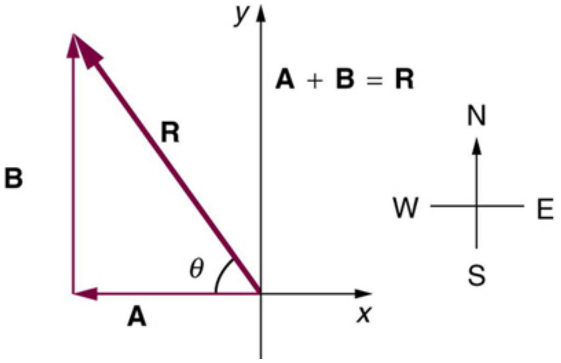
Solution:

north component 3.21 km, east component 3.83 km

Exercise:

Problem:

Suppose you walk 18.0 m straight west and then 25.0 m straight north. How far are you from your starting point, and what is the compass direction of a line connecting your starting point to your final position? (If you represent the two legs of the walk as vector displacements $\mathbf{A}$ and $\mathbf{B}$, as in [link], then this problem asks you to find their sum $\mathbf{R} = \mathbf{A} + \mathbf{B}$.)

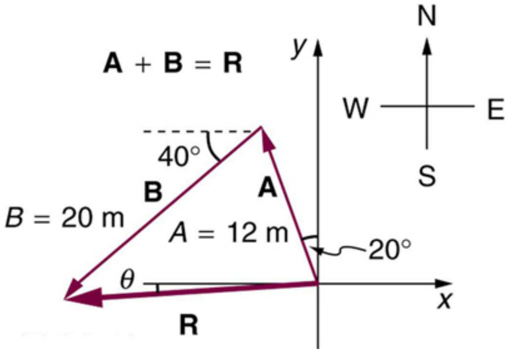


The two displacements $\mathbf{A}$ and $\mathbf{B}$ add to give a total displacement $\mathbf{R}$ having magnitude R and direction θ.

Exercise:

Problem:

Suppose you first walk 12.0 m in a direction 20º west of north and then 20.0 m in a direction 40.0º south of west. How far are you from your starting point, and what is the compass direction of a line connecting your starting point to your final position? (If you represent the two legs of the walk as vector displacements $\mathbf{A}$ and $\mathbf{B}$, as in [link], then this problem finds their sum $\text{R} = \text{A} + \text{B}$.)



Solution:

19.5 m, 4.65º south of west

Exercise:

Problem:

Repeat the problem above, but reverse the order of the two legs of the walk; show that you get the same final result. That is, you first walk leg $\mathbf{B}$, which is 20.0 m in a direction exactly 40° south of west, and then leg $\mathbf{A}$, which is 12.0 m in a direction exactly 20° west of north. (This problem shows that $\mathbf{A} + \mathbf{B} = \mathbf{B} + \mathbf{A}$.)

Exercise:

Problem:

(a) Repeat the problem two problems prior, but for the second leg you walk 20.0 m in a direction 40.0° north of east (which is equivalent to subtracting $\mathbf{B}$ from $\mathbf{A}$ —that is, to finding $\mathbf{R}' = \mathbf{A} - \mathbf{B}$). (b) Repeat the problem two problems prior, but now you first walk 20.0 m in a direction 40.0° south of west and then 12.0 m in a direction 20.0° east of south (which is equivalent to subtracting $\mathbf{A}$ from $\mathbf{B}$ —that is, to finding $\mathbf{R}'' = \mathbf{B} - \mathbf{A} = -\mathbf{R}'$). Show that this is the case.

Solution:

(a) $26.6\text{ m}, 65.1^\circ$ north of east

(b) $26.6\text{ m}, 65.1^\circ$ south of west

Exercise:

Problem:

Show that the *order* of addition of three vectors does not affect their sum. Show this property by choosing any three vectors $\mathbf{A}$, $\mathbf{B}$, and $\mathbf{C}$, all having different lengths and directions. Find the sum $A + B + C$ then find their sum when added in a different order and show the result is the same. (There are five other orders in which $\mathbf{A}$, $\mathbf{B}$, and $\mathbf{C}$ can be added; choose only one.)

Exercise:

Problem:

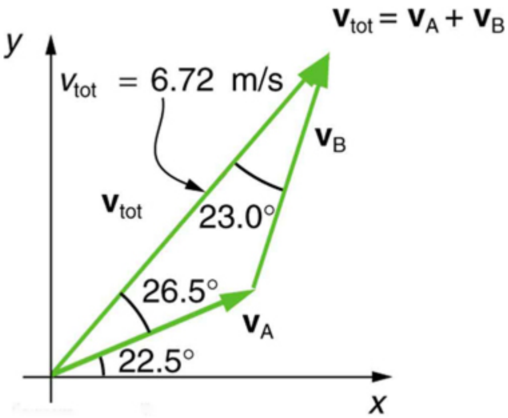
Show that the sum of the vectors discussed in [link] gives the result shown in [link].

Solution:

52.9 m, 90.1° with respect to the *x*-axis.

Exercise:

Problem: Find the magnitudes of velocities v_{A} and v_{B} in [link]



$\mathbf{v}_{\text{tot}} = \mathbf{v}_{\text{A}} + \mathbf{v}_{\text{B}}$
y
v_{tot} = 6.72 m/s
$\mathbf{v}_{\text{B}}$
$\mathbf{v}_{\text{tot}}$
23.0°
26.5°
$\mathbf{v}_{\text{A}}$
22.5°
x

The two velocities $\mathbf{v}_{\text{A}}$ and $\mathbf{v}_{\text{B}}$ add to give a total $\mathbf{v}_{\text{tot}}$.

Exercise:

Problem:

Find the components of v_{tot} along the *x*- and *y*-axes in [link].

Solution:

x-component 4.41 m/s

y-component 5.07 m/s

Exercise:

Problem:

Find the components of v_{tot} along a set of perpendicular axes rotated 30º counterclockwise relative to those in [link].

Glossary

component (of a 2-d vector)	a piece of a vector that points in either the vertical or the horizontal direction; every 2-d vector can be expressed as a sum of two vertical and horizontal vector components
commutative	refers to the interchangeability of order in a function; vector addition is commutative because the order in which vectors are added together does not affect the final sum
direction (of a vector)	the orientation of a vector in space
head (of a vector)	the end point of a vector; the location of the tip of the vector's arrowhead; also referred to as the "tip"
head-to-tail method	a method of adding vectors in which the tail of each vector is placed at the head of the previous vector
magnitude (of a vector)	the length or size of a vector; magnitude is a scalar quantity
resultant	the sum of two or more vectors

resultant vector
the vector sum of two or more vectors

scalar
a quantity with magnitude but no direction

tail
the start point of a vector; opposite to the head or tip of the arrow

Introduction to Dynamics: Newton’s Laws of Motion class="introduction"

Newton’s laws of motion describe the motion of the dolphin’s path. (credit: Jin Jang)

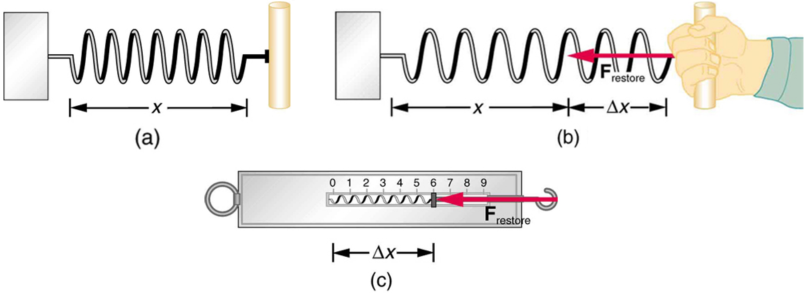
Motion draws our attention. Motion itself can be beautiful, causing us to marvel at the forces needed to achieve spectacular motion, such as that of a dolphin jumping out of the water, or a pole vaulter, or the flight of a bird, or the orbit of a satellite. The study of motion is kinematics, but kinematics only *describes* the way objects move—their velocity and their acceleration. **Dynamics** considers the forces that affect the motion of moving objects and systems. Newton’s laws of motion are the foundation of dynamics. These laws provide an example of the breadth and simplicity of principles under which nature functions. They are also universal laws in that they apply to similar situations on Earth as well as in space.

Development of Force Concept

- Understand the definition of force.

Dynamics is the study of the forces that cause objects and systems to move. To understand this, we need a working definition of force. Our intuitive definition of **force**—that is, a push or a pull—is a good place to start.

A more quantitative definition of force can be based on some standard force, just as distance is measured in units relative to a standard distance. One possibility is to stretch a spring a certain fixed distance, as illustrated in [link], and use the force it exerts to pull itself back to its relaxed shape—called a *restoring force*—as a standard.



The force exerted by a stretched spring can be used as a standard unit of force. (a) This spring has a length x when undistorted. (b) When stretched a distance Δx, the spring exerts a restoring force, $\mathbf{F}_{\text{restore}}$, which is reproducible. (c) A spring scale is one device that uses a spring to measure force. The force $\mathbf{F}_{\text{restore}}$ is exerted on whatever is attached to the hook. Here $\mathbf{F}_{\text{restore}}$ has a magnitude of 6 units in the force standard being employed.

Note:

Take-Home Experiment: Force Standards

To investigate force standards and cause and effect, get two identical rubber bands. Hang one rubber band vertically on a hook. Find a small household item that could be attached to the rubber band using a paper clip, and use this item as a weight to investigate the stretch of the rubber band. Measure the amount of stretch produced in the rubber band with one, two, and four of these (identical) items suspended from the rubber band. What is the relationship between the number of items and the amount of stretch? How large a stretch would you expect for the same number of items suspended from two rubber bands? What happens to the amount of stretch of the rubber band (with the weights attached) if the weights are also pushed to the side with a pencil?

Section Summary

- **Dynamics** is the study of how forces affect the motion of objects.
- **Force** is a push or pull that can be defined in terms of various standards, and it is a vector having both magnitude and direction.
- **External forces** are any outside forces that act on a body. A **free-body diagram** is a drawing of all external forces acting on a body.

Conceptual Questions

Exercise:

Problem:

Propose a force standard different from the example of a stretched spring discussed in the text. Your standard must be capable of producing the same force repeatedly.

Exercise:

Problem:

What properties do forces have that allow us to classify them as vectors?

Glossary

dynamics
the study of how forces affect the motion of objects and systems

external force
a force acting on an object or system that originates outside of the object or system

free-body diagram
a sketch showing all of the external forces acting on an object or system; the system is represented by a dot, and the forces are represented by vectors extending outward from the dot

force
a push or pull on an object with a specific magnitude and direction; can be represented by vectors; can be expressed as a multiple of a standard force

Newton’s First Law of Motion: Inertia

- Define mass and inertia.
- Understand Newton's first law of motion.

Experience suggests that an object at rest will remain at rest if left alone, and that an object in motion tends to slow down and stop unless some effort is made to keep it moving. What **Newton’s first law of motion** states, however, is the following:

Note:
Newton’s First Law of Motion
A body at rest remains at rest, or, if in motion, remains in motion at a constant velocity unless acted on by a net external force.

Note the repeated use of the verb “remains.” We can think of this law as preserving the status quo of motion.

Rather than contradicting our experience, **Newton’s first law of motion** states that there must be a *cause* (which is a net external force) *for there to be any change in velocity (either a change in magnitude or direction)*. We will define *net external force* in the next section. An object sliding across a table or floor slows down due to the net force of friction acting on the object. If friction disappeared, would the object still slow down?

The genius of Galileo, who first developed the idea for the first law, and Newton, who clarified it, was to ask the fundamental question, “What is the cause?”

Mass

The property of a body to remain at rest or to remain in motion with constant velocity is called **inertia**. Newton’s first law is often called the **law of inertia**. As we know from experience, some objects have more inertia than others. It is obviously more difficult to change the motion of a large boulder than that of a basketball, for example. The inertia of an object is measured by its **mass**. Roughly speaking, mass is a measure of the amount of “stuff” (or matter) in something. The quantity or amount of matter in an object is determined by the numbers of atoms and molecules of various types it contains. Unlike weight, mass does not vary with location. The mass of an object is the same on Earth, in orbit, or on the surface of the Moon. In practice, it is very difficult to count and identify all of the atoms and molecules in an object, so masses are not often determined in this manner. Operationally, the masses of objects are determined by comparison with the standard kilogram.

Exercise:
Check Your Understanding

Problem:

Which has more mass: a kilogram of cotton balls or a kilogram of gold?

Solution:
Answer

They are equal. A kilogram of one substance is equal in mass to a kilogram of another substance. The quantities that might differ between them are volume and density.

Section Summary

- **Newton’s first law of motion** states that a body at rest remains at rest, or, if in motion, remains in motion at a constant velocity unless acted on by a net external force. This is also known as the **law of inertia**.
- **Inertia** is the tendency of an object to remain at rest or remain in motion. Inertia is related to an object’s mass.
- **Mass** is the quantity of matter in a substance.

Conceptual Questions

Exercise:

Problem: How are inertia and mass related?

Exercise:

Problem:

What is the relationship between weight and mass? Which is an intrinsic, unchanging property of a body?

Glossary

inertia	the tendency of an object to remain at rest or remain in motion
law of inertia	see Newton’s first law of motion
mass	the quantity of matter in a substance; measured in kilograms
Newton’s first law of motion	a body at rest remains at rest, or, if in motion, remains in motion at a constant velocity unless acted on by a net external force; also known as the law of inertia

Newton’s Second Law of Motion

- Define net force, external force, and system.
- Understand Newton’s second law of motion.
- Apply Newton’s second law to determine the weight of an object.

Newton’s second law of motion is closely related to Newton’s first law of motion. It mathematically states the cause and effect relationship between force and changes in motion. Newton’s second law of motion is more quantitative and is used extensively to calculate what happens in situations involving a force. Before we can write down Newton’s second law as a simple equation giving the exact relationship of force, mass, and acceleration, we need to sharpen some ideas that have already been mentioned.

First, what do we mean by a change in motion? The answer is that a change in motion is equivalent to a change in velocity. A change in velocity means, by definition, that there is an **acceleration**. Newton’s first law says that a net external force causes a change in motion; thus, we see that a *net external force causes acceleration*.

Another question immediately arises. What do we mean by an external force? An intuitive notion of external is correct—an **external force** acts from outside the **system** of interest.

To obtain an equation for Newton’s second law, we first write the relationship of acceleration and net external force as the proportionality
Equation:

$$\mathbf{a} \propto \mathbf{F}_{\text{net}},$$

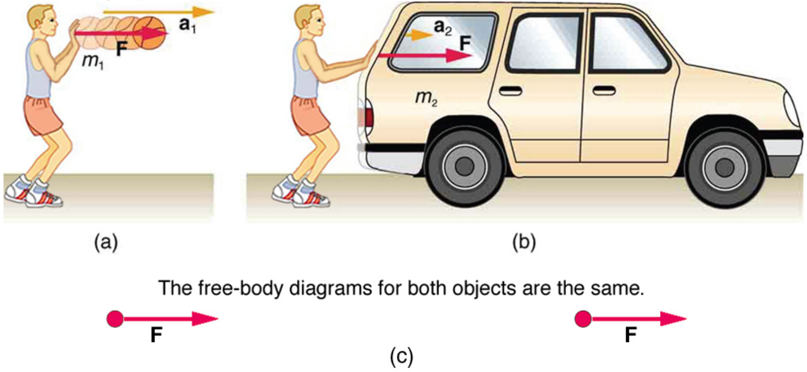
where the symbol $\propto$ means “proportional to,” and $\mathbf{F}_{\text{net}}$ is the **net external force**. (The net external force is the vector sum of all external forces and can be determined graphically, using the head-to-tail method. This proportionality states what we have said in words—*acceleration is directly proportional to the net external force*.

Now, it also seems reasonable that acceleration should be inversely proportional to the mass of the system. In other words, the larger the mass (the inertia), the smaller the acceleration produced by a given force. And indeed, as illustrated in [link], the same net external force applied to a car produces a much smaller acceleration than when applied to a basketball. The proportionality is written as

Equation:

$$\mathbf{a} \propto \frac{1}{m}$$

where m is the mass of the system. Experiments have shown that acceleration is exactly inversely proportional to mass, just as it is exactly linearly proportional to the net external force.


a₁
F
m₁
a₂
F
m₂
(a)
(b)
The free-body diagrams for both objects are the same.
F
F
(c)

The same force exerted on systems of different masses produces different accelerations. (a) A basketball player pushes on a basketball to make a pass. (The effect of gravity on the ball is ignored.) (b) The same player exerts an identical force on a stalled SUV and produces a far smaller acceleration (even if friction is negligible). (c) The free-body diagrams are identical, permitting direct comparison of the two situations. A series of patterns for the free-body diagram will emerge as you do more problems.

It has been found that the acceleration of an object depends *only* on the net external force and the mass of the object. Combining the two proportionalities just given yields Newton's second law of motion.

Note:
Newton’s Second Law of Motion
The acceleration of a system is directly proportional to and in the same direction as the net external force acting on the system, and inversely proportional to its mass.
In equation form, Newton’s second law of motion is
Equation:

$$\mathbf{a} = \frac{\mathbf{F}_{\text{net}}}{m}.$$

This is often written in the more familiar form
Equation:

$$\mathbf{F}_{\text{net}} = m\mathbf{a}.$$

When only the magnitude of force and acceleration are considered, this equation is simply
Equation:

$$F_{\text{net}} = \text{ma}.$$

Units of Force

$\mathbf{F}_{\text{net}} = m\mathbf{a}$ is used to define the units of force in terms of the three basic units for mass, length, and time. The SI unit of force is called the **newton**

(abbreviated N) and is the force needed to accelerate a 1-kg system at the rate of $1\mathrm{m/s}^2$. That is, since $\mathbf{F}_{\text{net}} = m\mathbf{a}$,
Equation:

$$1\text{ N} = 1\text{ kg}\cdot\mathrm{m/s}^2.$$

Weight and the Gravitational Force

When an object is dropped, it accelerates toward the center of Earth. Newton’s second law states that a net force on an object is responsible for its acceleration. If air resistance is negligible, the net force on a falling object is the gravitational force, commonly called its **weight $\mathbf{w}$**. Weight can be denoted as a vector $\mathbf{w}$ because it has a direction; *down* is, by definition, the direction of gravity, and hence weight is a downward force. The magnitude of weight is denoted as w. Galileo was instrumental in showing that, in the absence of air resistance, all objects fall with the same acceleration g. Using Galileo’s result and Newton’s second law, we can derive an equation for weight.

Consider an object with mass m falling downward toward Earth. It experiences only the downward force of gravity, which has magnitude w. Newton’s second law states that the magnitude of the net external force on an object is $F_{\text{net}} = \mathrm{ma}$.

Since the object experiences only the downward force of gravity, $F_{\text{net}} = w$. We know that the acceleration of an object due to gravity is g, or $a = g$. Substituting these into Newton’s second law gives

Note:
Weight
This is the equation for *weight*—the gravitational force on a mass m:
Equation:

$$w = \mathrm{mg}.$$

Since $g = 9.80\ \mathrm{m/s^2}$ on Earth, the weight of a 1.0 kg object on Earth is 9.8 N, as we see:

Equation:

$$w = \mathrm{mg} = (1.0\ \mathrm{kg})(9.80\ \mathrm{m/s^2}) = 9.8\ \mathrm{N}.$$

Recall that *g* can take a positive or negative value, depending on the positive direction in the coordinate system. Be sure to take this into consideration when solving problems with weight.

When the net external force on an object is its weight, we say that it is in **free-fall**. That is, the only force acting on the object is the force of gravity. In the real world, when objects fall downward toward Earth, they are never truly in free-fall because there is always some upward force from the air acting on the object.

The acceleration due to gravity *g* varies slightly over the surface of Earth, so that the weight of an object depends on location and is not an intrinsic property of the object. Weight varies dramatically if one leaves Earth's surface. On the Moon, for example, the acceleration due to gravity is only $1.67\ \mathrm{m/s^2}$. A 1.0-kg mass thus has a weight of 9.8 N on Earth and only about 1.7 N on the Moon.

It is important to be aware that weight and mass are very different physical quantities, although they are closely related. Mass is the quantity of matter (how much "stuff") and does not vary in classical physics, whereas weight is the gravitational force and does vary depending on gravity. It is tempting to equate the two, since most of our examples take place on Earth, where the weight of an object only varies a little with the location of the object. Furthermore, the terms *mass* and *weight* are used interchangeably in everyday language; for example, our medical records often show our "weight" in kilograms, but never in the correct units of newtons.

Note:

Common Misconceptions: Mass vs. Weight

Mass and weight are often used interchangeably in everyday language. However, in science, these terms are distinctly different from one another. Mass is a measure of how much matter is in an object. The typical measure of mass is the kilogram. Weight, on the other hand, is a measure of the force of gravity acting on an object. Weight is equal to the mass of an object (m) multiplied by the acceleration due to gravity (g). Like any other force, weight is measured in terms of newtons.

Note:

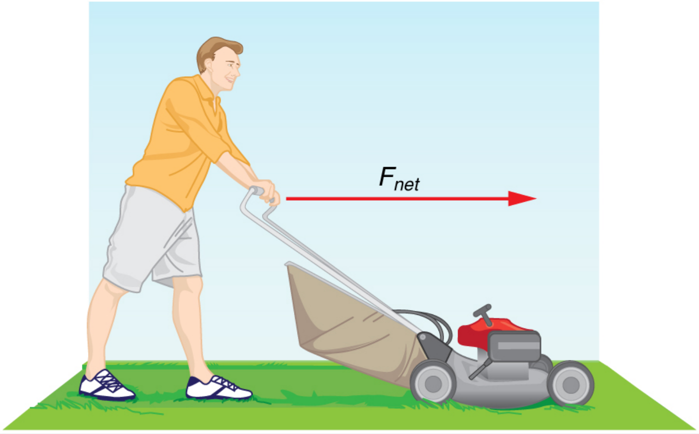
Take-Home Experiment: Mass and Weight

What do bathroom scales measure? When you stand on a bathroom scale, what happens to the scale? It depresses slightly. The scale contains springs that compress in proportion to your weight—similar to rubber bands expanding when pulled. The springs provide a measure of your weight (for an object which is not accelerating). This is a force in newtons (or pounds). In most countries, the measurement is divided by 9.80 to give a reading in mass units of kilograms. The scale measures weight but is calibrated to provide information about mass. While standing on a bathroom scale, push down on a table next to you. What happens to the reading? Why? Would your scale measure the same “mass” on Earth as on the Moon?

Example:

What Acceleration Can a Person Produce when Pushing a Lawn Mower?

Suppose that the net external force (push minus friction) exerted on a lawn mower is 51 N (about 11 lb) parallel to the ground. The mass of the mower is 24 kg. What is its acceleration?



The net force on a lawn mower is 51 N to the right. At what rate does the lawn mower accelerate to the right?

Strategy

Since $\mathbf{F}_{\text{net}}$ and m are given, the acceleration can be calculated directly from Newton's second law as stated in $\mathbf{F}_{\text{net}} = m\mathbf{a}$.

Solution

The magnitude of the acceleration a is $a = \frac{F_{\text{net}}}{m}$. Entering known values gives

Equation:

$$a = \frac{51\ \text{N}}{24\ \text{kg}}$$

Substituting the units $\text{kg} \cdot \text{m/s}^2$ for N yields

Equation:

$$a = \frac{51\ \text{kg} \cdot \text{m/s}^2}{24\ \text{kg}} = 2.1\ \text{m/s}^2.$$

Section Summary

- Acceleration, $\mathbf{a}$, is defined as a change in velocity, meaning a change in its magnitude or direction, or both.
- An external force is one acting on a system from outside the system, as opposed to internal forces, which act between components within the system.
- Newton's second law of motion states that the acceleration of a system is directly proportional to and in the same direction as the net external force acting on the system, and inversely proportional to its mass.
- In equation form, Newton's second law of motion is $\mathbf{a} = \frac{\mathbf{F}_{\text{net}}}{m}$.
- This is often written in the more familiar form: $\mathbf{F}_{\text{net}} = m\mathbf{a}$.
- The weight $\mathbf{w}$ of an object is defined as the force of gravity acting on an object of mass m. The object experiences an acceleration due to gravity $\mathbf{g}$:
  Equation:

$$\mathbf{w} = m\mathbf{g}.$$

- If the only force acting on an object is due to gravity, the object is in free fall.
- Friction is a force that opposes the motion past each other of objects that are touching.

Conceptual Questions

Exercise:

Problem:

Which statement is correct? (a) Net force causes motion. (b) Net force causes change in motion. Explain your answer and give an example.

Exercise:

Problem:

Why can we neglect forces such as those holding a body together when we apply Newton's second law of motion?

Exercise:

Problem:

Explain how the choice of the “system of interest” affects which forces must be considered when applying Newton’s second law of motion.

Exercise:

Problem:

Describe a situation in which the net external force on a system is not zero, yet its speed remains constant.

Exercise:

Problem:

A system can have a nonzero velocity while the net external force on it *is* zero. Describe such a situation.

Exercise:

Problem:

A rock is thrown straight up. What is the net external force acting on the rock when it is at the top of its trajectory?

Exercise:

Problem:

(a) Give an example of different net external forces acting on the same system to produce different accelerations. (b) Give an example of the same net external force acting on systems of different masses, producing different accelerations. (c) What law accurately describes both effects? State it in words and as an equation.

Exercise:

Problem:

If the acceleration of a system is zero, are no external forces acting on it? What about internal forces? Explain your answers.

Exercise:

Problem:

If a constant, nonzero force is applied to an object, what can you say about the velocity and acceleration of the object?

Exercise:

Problem:

The gravitational force on the basketball in [link] is ignored. When gravity *is* taken into account, what is the direction of the net external force on the basketball—above horizontal, below horizontal, or still horizontal?

Problem Exercises

You may assume data taken from illustrations is accurate to three digits.

Exercise:

Problem:

A 63.0-kg sprinter starts a race with an acceleration of 4.20 m/s^2. What is the net external force on him?

Solution:

265 N

Exercise:

Problem:

If the sprinter from the previous problem accelerates at that rate for 20 m, and then maintains that velocity for the remainder of the 100-m dash, what will be his time for the race?

Exercise:

Problem:

A cleaner pushes a 4.50-kg laundry cart in such a way that the net external force on it is 60.0 N. Calculate the magnitude of its acceleration.

Solution:

$13.3\ \text{m/s}^2$

Exercise:

Problem:

Since astronauts in orbit are apparently weightless, a clever method of measuring their masses is needed to monitor their mass gains or losses to adjust diets. One way to do this is to exert a known force on an astronaut and measure the acceleration produced. Suppose a net external force of 50.0 N is exerted and the astronaut’s acceleration is measured to be $0.893\ \text{m/s}^2$. (a) Calculate her mass. (b) By exerting a force on the astronaut, the vehicle in which they orbit experiences an equal and opposite force. Discuss how this would affect the measurement of the astronaut’s acceleration. Propose a method in which recoil of the vehicle is avoided.

Exercise:

Problem:

In [link], the net external force on the 24-kg mower is stated to be 51 N. If the force of friction opposing the motion is 24 N, what force F (in newtons) is the person exerting on the mower? Suppose the mower is moving at 1.5 m/s when the force F is removed. How far will the mower go before stopping?

Exercise:

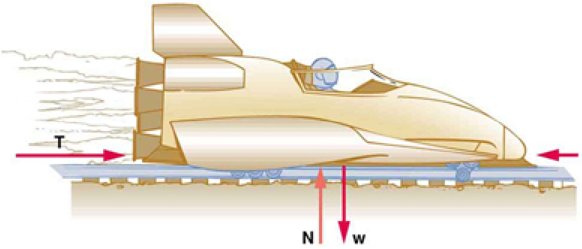
Problem:

(a) If the rocket sled shown in [link] starts with only one rocket burning, what is the magnitude of its acceleration? Assume that the mass of the system is 2100 kg, the thrust T is 2.4×10^4 N, and the force of friction opposing the motion is known to be 650 N. (b) Why is the acceleration not one-fourth of what it is with all rockets burning?

Solution:

(a) $12\ \text{m/s}^2$.

(b) The acceleration is not one-fourth of what it was with all rockets burning because the frictional force is still as large as it was with all rockets burning.



Exercise:

Problem:

What is the deceleration of the rocket sled if it comes to rest in 1.1 s from a speed of 1000 km/h? (Such deceleration caused one test subject to black out and have temporary blindness.)

Exercise:

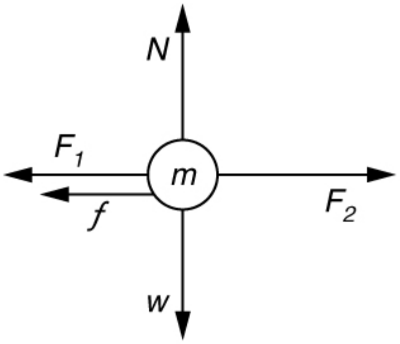
Problem:

Suppose two children push horizontally, but in exactly opposite directions, on a third child in a wagon. The first child exerts a force of 75.0 N, the second a force of 90.0 N, friction is 12.0 N, and the mass of the third child plus wagon is 23.0 kg. (a) What is the system of interest if the acceleration of the child in the wagon is to be calculated? (b) Draw a free-body diagram, including all forces acting on the system. (c) Calculate the acceleration. (d) What would the acceleration be if friction were 15.0 N?

Solution:

(a) The system is the child in the wagon plus the wagon.

(b



(c) $a = 0.130\ \text{m/s}^2$ in the direction of the second child’s push.

(d) $a = 0.00\ \text{m/s}^2$

Exercise:

Problem:

A powerful motorcycle can produce an acceleration of $3.50\ \text{m/s}^2$ while traveling at 90.0 km/h. At that speed the forces resisting motion, including friction and air resistance, total 400 N. (Air resistance is analogous to air friction. It always opposes the motion of an object.) What is the magnitude of the force the motorcycle exerts backward on the ground to produce its acceleration if the mass of the motorcycle with rider is 245 kg?

Exercise:

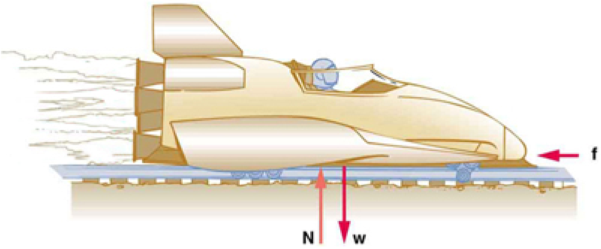
Problem:

The rocket sled shown in [link] accelerates at a rate of $49.0\ \text{m/s}^2$. Its passenger has a mass of 75.0 kg. (a) Calculate the horizontal component of the force the seat exerts against his body. Compare this with his weight by using a ratio. (b) Calculate the direction and magnitude of the total force the seat exerts against his body.

Solution:

(a) $3.68 \times 10^3\ \text{N}$. This force is 5.00 times greater than his weight.

(b) $3750\ \text{N}$; 11.3^{o} above horizontal



Exercise:

Problem:

Repeat the previous problem for the situation in which the rocket sled decelerates at a rate of $201\ \text{m/s}^2$. In this problem, the forces are exerted by the seat and restraining belts.

Exercise:

Problem:

The weight of an astronaut plus his space suit on the Moon is only 250 N. How much do they weigh on Earth? What is the mass on the Moon? On Earth?

Solution:

$1.5 \times 10^3\ \text{N}, 150\ \text{kg}, 150\ \text{kg}$

Exercise:

Problem:

Suppose the mass of a fully loaded module in which astronauts take off from the Moon is 10,000 kg. The thrust of its engines is 30,000 N. (a) Calculate its the magnitude of acceleration in a vertical takeoff from the Moon. (b) Could it lift off from Earth? If not, why not? If it could, calculate the magnitude of its acceleration.

Glossary

acceleration	the rate at which an object’s velocity changes over a period of time
free-fall	a situation in which the only force acting on an object is the force due to gravity
friction	

a force past each other of objects that are touching; examples include rough surfaces and air resistance

net external force

the vector sum of all external forces acting on an object or system; causes a mass to accelerate

Newton’s second law of motion

the net external force $\mathbf{F}_{\text{net}}$ on an object with mass m is proportional to and in the same direction as the acceleration of the object, $\mathbf{a}$, and inversely proportional to the mass; defined mathematically as $\mathbf{a}=\frac{\mathbf{F}_{\text{net}}}{m}$

system

defined by the boundaries of an object or collection of objects being observed; all forces originating from outside of the system are considered external forces

weight

the force $\mathbf{w}$due to gravity acting on an object of mass m; defined mathematically as: $\mathbf{w}=m\mathbf{g}$, where $\mathbf{g}$ is the magnitude and direction of the acceleration due to gravity

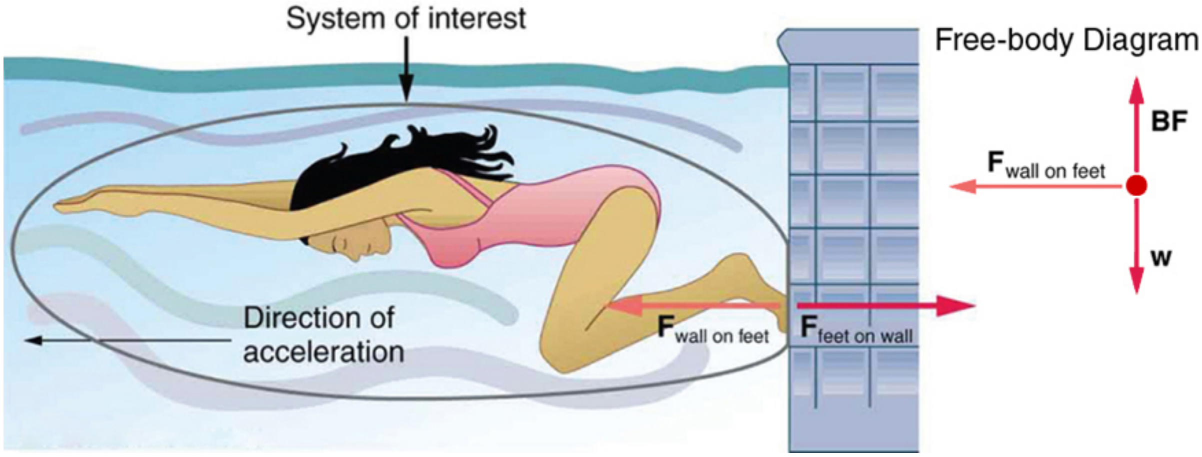
Newton’s Third Law of Motion

- Understand Newton's third law of motion.
- Apply Newton's third law to define systems and solve problems of motion.

Note:
Newton’s Third Law of Motion
Whenever one body exerts a force on a second body, the first body experiences a force that is equal in magnitude and opposite in direction to the force that it exerts.

This law represents a certain *symmetry in nature*: Forces always occur in pairs, and one body cannot exert a force on another without experiencing a force itself. We sometimes refer to this law loosely as “action-reaction,” where the force exerted is the action and the force experienced as a consequence is the reaction. Newton’s third law has practical uses in analyzing the origin of forces and understanding which forces are external to a system.

We can readily see Newton’s third law at work by taking a look at how people move about. Consider a swimmer pushing off from the side of a pool, as illustrated in [link]. She pushes against the pool wall with her feet and accelerates in the direction *opposite* to that of her push. The wall has exerted an equal and opposite force back on the swimmer. You might think that two equal and opposite forces would cancel, but they do not *because they act on different systems*. In this case, there are two systems that we could investigate: the swimmer or the wall. If we select the swimmer to be the system of interest, as in the figure, then $\mathbf{F}_{\text{wall on feet}}$ is an external force on this system and affects its motion. The swimmer moves in the direction of $\mathbf{F}_{\text{wall on feet}}$. In contrast, the force $\mathbf{F}_{\text{feet on wall}}$ acts on the wall and not on our system of interest. Thus $\mathbf{F}_{\text{feet on wall}}$ does not directly affect the motion of the system and does not cancel $\mathbf{F}_{\text{wall on feet}}$. Note that the swimmer pushes in the direction opposite to that in which she wishes to move. The reaction to her push is thus in the desired direction.

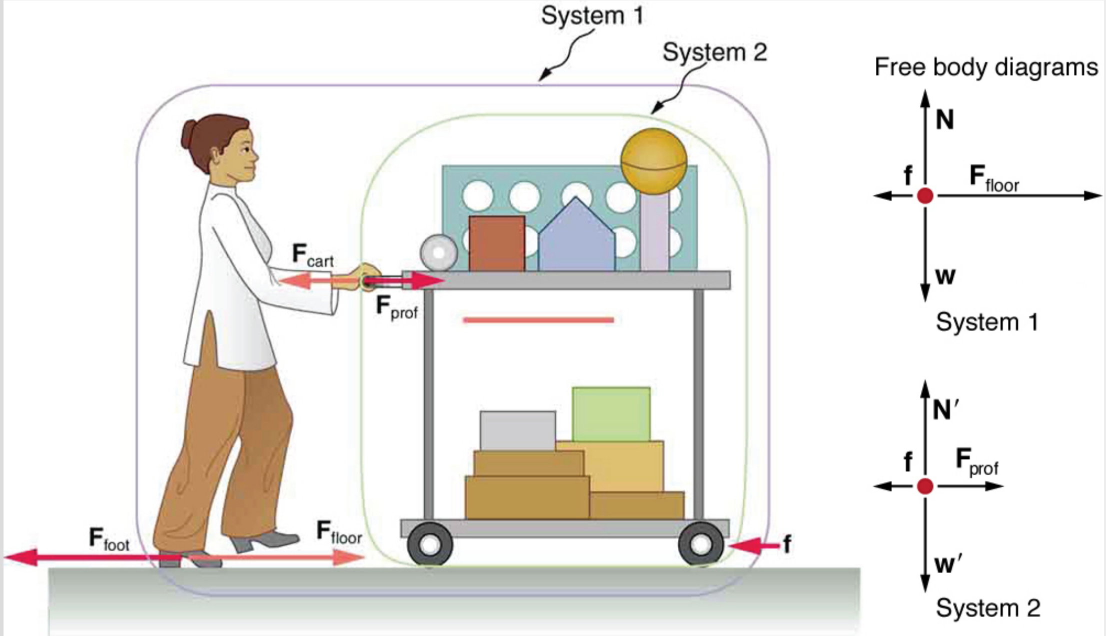

System of interest
Free-body Diagram
BF
$\mathbf{F}_{\text{wall on feet}}$
w
Direction of acceleration
$\mathbf{F}_{\text{wall on feet}}$
$\mathbf{F}_{\text{feet on wall}}$

When the swimmer exerts a force $\mathbf{F}_{\text{feet on wall}}$ on the wall, she accelerates in the direction opposite to that of her push. This means the net external force on her is in the direction opposite to $\mathbf{F}_{\text{feet on wall}}$. This opposition occurs because, in accordance with Newton's third law of motion, the wall exerts a force $\mathbf{F}_{\text{wall on feet}}$ on her, equal in magnitude but in the direction opposite to the one she exerts on it. The line around the swimmer indicates the system of interest. Note that $\mathbf{F}_{\text{feet on wall}}$ does not act on this system (the swimmer) and, thus, does not cancel $\mathbf{F}_{\text{wall on feet}}$. Thus the free-body diagram shows only $\mathbf{F}_{\text{wall on feet}}$, $\mathbf{w}$, the gravitational force, and $\mathbf{BF}$, the buoyant force of the water supporting the swimmer's weight. The vertical forces $\mathbf{w}$ and $\mathbf{BF}$ cancel since there is no vertical motion.

Example:

Getting Up To Speed: Choosing the Correct System

A physics professor pushes a cart of demonstration equipment to a lecture hall, as seen in [link]. Her mass is 65.0 kg, the cart's is 12.0 kg, and the equipment's is 7.0 kg. Calculate the acceleration produced when the professor exerts a backward force of 150 N on the floor. All forces opposing the motion, such as friction on the cart's wheels and air resistance, total 24.0 N.



A professor pushes a cart of demonstration equipment. The lengths of the arrows are proportional to the magnitudes of the forces (except for $\mathbf{f}$, since it is too small to draw to scale). Different questions are asked in each example; thus, the system of interest must be defined differently for each. System 1 is appropriate for this example, since it asks for the acceleration of the entire group of objects. Only $\mathbf{F}_{\text{floor}}$ and $\mathbf{f}$ are external forces acting on System 1 along the line of motion. All other forces either cancel or act on the outside world. System 2 is chosen for [link] so that $\mathbf{F}_{\text{prof}}$ will be an external force and enter into Newton's second law. Note that the free-body diagrams, which allow us to apply Newton's second law, vary with the system chosen.

Strategy

Since they accelerate as a unit, we define the system to be the professor, cart, and equipment. This is System 1 in [link]. The professor pushes backward with a force $\mathbf{F}_{\text{foot}}$ of 150 N. According to Newton's third law, the floor exerts a forward reaction force $\mathbf{F}_{\text{floor}}$ of 150 N on System 1.

Because all motion is horizontal, we can assume there is no net force in the vertical direction. The problem is therefore one-dimensional along the horizontal direction. As noted, $\mathbf{f}$ opposes the motion and is thus in the opposite direction of $\mathbf{F}_{\text{floor}}$. Note that we do not include the forces $\mathbf{F}_{\text{prof}}$ or $\mathbf{F}_{\text{cart}}$ because these are internal forces, and we do not include $\mathbf{F}_{\text{foot}}$ because it acts on the floor, not on the system. There are no other significant forces acting on System 1. If the net external force can be found from all this information, we can use Newton's second law to find the acceleration as requested. See the free-body diagram in the figure.

Solution

Newton's second law is given by

Equation:

$$a=\frac{F_{\text{net}}}{m}.$$

The net external force on System 1 is deduced from [link] and the discussion above to be

Equation:

$$F_{\text{net}}=F_{\text{floor}}-f=150\text{ N}-24.0\text{ N}=126\text{ N}.$$

The mass of System 1 is

Equation:

$$m=(65.0+12.0+7.0)\text{ kg}=84\text{ kg}.$$

These values of F_{net} and m produce an acceleration of

Equation:

$$\begin{aligned}a&=\tfrac{F_{\text{net}}}{m},\\ a&=\tfrac{126\text{ N}}{84\text{ kg}}=1.5\text{ m/s}^2.\end{aligned}$$

Note:

PhET Explorations: Gravity Force Lab

Visualize the gravitational force that two objects exert on each other. Change properties of the objects in order to see how it changes the gravity force.
https://phet.colorado.edu/sims/html/gravity-force-lab/latest/gravity-force-lab_en.html

Section Summary

- **Newton's third law of motion** represents a basic symmetry in nature. It states: Whenever one body exerts a force on a second body, the first body experiences a force that is equal in magnitude and opposite in direction to the force that the first body exerts.
- A **thrust** is a reaction force that pushes a body forward in response to a backward force. Rockets, airplanes, and cars are pushed forward by a thrust reaction force.

Conceptual Questions

Exercise:

Problem:

When you take off in a jet aircraft, there is a sensation of being pushed back into the seat. Explain why you move backward in the seat—is there really a force backward on you? (The same reasoning explains whiplash injuries, in which the head is apparently thrown backward.)

Exercise:

Problem:

A device used since the 1940s to measure the kick or recoil of the body due to heart beats is the “ballistocardiograph.” What physics principle(s) are involved here to measure the force of cardiac contraction? How might we construct such a device?

Exercise:

Problem:

Describe a situation in which one system exerts a force on another and, as a consequence, experiences a force that is equal in magnitude and opposite in direction. Which of Newton's laws of motion apply?

Exercise:

Problem:

Why does an ordinary rifle recoil (kick backward) when fired? The barrel of a recoilless rifle is open at both ends. Describe how Newton's third law applies when one is fired. Can you safely stand close behind one when it is fired?

Exercise:

Problem:

An American football lineman reasons that it is senseless to try to out-push the opposing player, since no matter how hard he pushes he will experience an equal and opposite force from the other player. Use Newton's laws and draw a free-body diagram of an appropriate system to explain how he can still out-push the opposition if he is strong enough.

Exercise:

Problem:

Newton's third law of motion tells us that forces always occur in pairs of equal and opposite magnitude. Explain how the choice of the "system of interest" affects whether one such pair of forces cancels.

Problem Exercises

Exercise:

Problem:

What net external force is exerted on a 1100-kg artillery shell fired from a battleship if the shell is accelerated at $2.40\times10^{4}\ \mathrm{m/s}^{2}$? What is the magnitude of the force exerted on the ship by the artillery shell?

Solution:

Force on shell: $2.64 \times 10^{7}\ \mathrm{N}$

Force exerted on ship = $-2.64 \times 10^{7}\ \mathrm{N}$, by Newton's third law

Exercise:

Problem:

A brave but inadequate rugby player is being pushed backward by an opposing player who is exerting a force of 800 N on him. The mass of the losing player plus equipment is 90.0 kg, and he is accelerating at $1.20\ \mathrm{m/s}^{2}$ backward. (a) What is the force of friction between the losing player's feet and the grass? (b) What force does the winning player exert on the ground to move forward if his mass plus equipment is 110 kg? (c) Draw a sketch of the situation showing the system of interest used to solve each part. For this situation, draw a free-body diagram and write the net force equation.

Glossary

Newton's third law of motion
: whenever one body exerts a force on a second body, the first body experiences a force that is equal in magnitude and opposite in direction to the force that the first body exerts

thrust
: a reaction force that pushes a body forward in response to a backward force; rockets, airplanes, and cars are pushed forward by a thrust reaction force

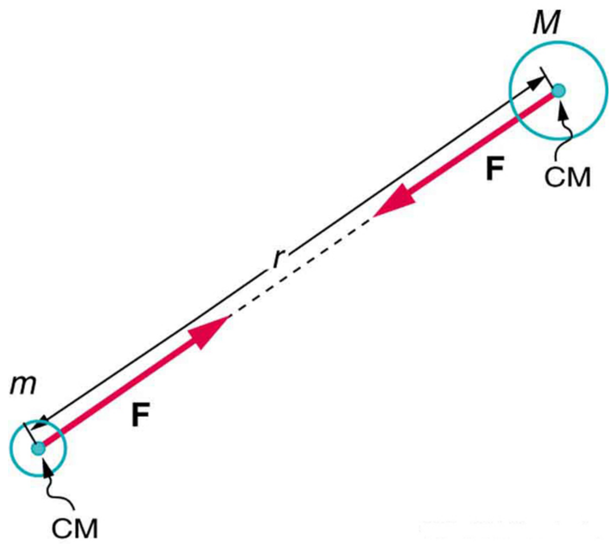
Newton’s Universal Law of Gravitation

- Explain Earth’s gravitational force.
- Describe the gravitational effect of the Moon on Earth.
- Discuss weightlessness in space.
- Examine the Cavendish experiment

Sir Isaac Newton was the first scientist to precisely define the gravitational force, and to show that it could explain both falling bodies and astronomical motions. See [link].

According to early accounts, Newton was inspired to make the connection between falling bodies and astronomical motions when he saw an apple fall from a tree and realized that if the gravitational force could extend above the ground to a tree, it might also reach the Sun. The inspiration of Newton’s apple is a part of worldwide folklore and may even be based in fact. Great importance is attached to it because Newton’s universal law of gravitation and his laws of motion answered very old questions about nature and gave tremendous support to the notion of underlying simplicity and unity in nature. Scientists still expect underlying simplicity to emerge from their ongoing inquiries into nature.

The gravitational force is relatively simple. It is always attractive, and it depends only on the masses involved and the distance between them. Stated in modern language, **Newton’s universal law of gravitation** states that every particle in the universe attracts every other particle with a force along a line joining them. The force is directly proportional to the product of their masses and inversely proportional to the square of the distance between them.



Gravitational attraction is along a line joining the centers of mass of these two bodies. The magnitude of the force is the same on each, consistent with Newton’s third law.

Note:

Misconception Alert

The magnitude of the force on each object (one has larger mass than the other) is the same, consistent with Newton’s third law.

Equation:

$$F=G\frac{\text{mM}}{r^2},$$

where F is the magnitude of the gravitational force and G is a proportionality factor called the **gravitational constant**. G is a universal gravitational constant—that is, it is thought to be the same everywhere in the universe. It has been measured experimentally to be

Equation:

$$G = 6.674 \times 10^{-11} \frac{\mathrm{N} \cdot \mathrm{m}^2}{\mathrm{kg}^2}$$

Recall that the acceleration due to gravity g is about $9.80\ \mathrm{m/s}^2$ on Earth. We can now determine why this is so. The weight of an object mg is the gravitational force between it and Earth. Substituting mg for F in Newton's universal law of gravitation gives

Equation:

$$\mathrm{mg} = G\frac{\mathrm{mM}}{r^2},$$

where m is the mass of the object, M is the mass of Earth, and r is the distance to the center of Earth (the distance between the centers of mass of the object and Earth). See [link]. The mass m of the object cancels, leaving an equation for g:

Equation:

$$g = G\frac{M}{r^2}.$$

Substituting known values for Earth's mass and radius (to three significant figures),

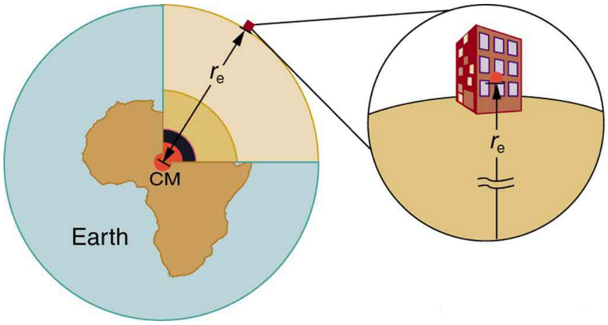
Equation:

$$g = \left(6.67 \times 10^{-11} \frac{\mathrm{N} \cdot \mathrm{m}^2}{\mathrm{kg}^2}\right) \times \frac{5.98 \times 10^{24}\ \mathrm{kg}}{(6.38 \times 10^6\ \mathrm{m})^2},$$

and we obtain a value for the acceleration of a falling body:

Equation:

$$g = 9.80\ \text{m/s}^2.$$



The distance between the centers of mass of Earth and an object on its surface is very nearly the same as the radius of Earth, because Earth is so much larger than the object.

This is the expected value *and is independent of the body's mass*. Newton's law of gravitation takes Galileo's observation that all masses fall with the same acceleration a step further, explaining the observation in terms of a force that causes objects to fall—in fact, in terms of a universally existing force of attraction between masses.

Note:
Take-Home Experiment
Take a marble, a ball, and a spoon and drop them from the same height. Do they hit the floor at the same time? If you drop a piece of paper as well, does it behave like the other objects? Explain your observations.

Section Summary

- Newton's universal law of gravitation: Every particle in the universe attracts every other particle with a force along a line joining them. The force is directly proportional to the product of their masses and inversely proportional to the square of the distance between them. In equation form, this is
  Equation:

  $$F = G\frac{\mathrm{mM}}{r^2},$$

  where F is the magnitude of the gravitational force. G is the gravitational constant, given by $G = 6.674 \times 10^{-11}\ \mathrm{N \cdot m^2/kg^2}$.
- Newton's law of gravitation applies universally.

Conceptual Questions

Exercise:

Problem:

Action at a distance, such as is the case for gravity, was once thought to be illogical and therefore untrue. What is the ultimate determinant of the truth in physics, and why was this action ultimately accepted?

Exercise:

Problem:

Two friends are having a conversation. Anna says a satellite in orbit is in freefall because the satellite keeps falling toward Earth. Tom says a satellite in orbit is not in freefall because the acceleration due to gravity is not $9.80\ \mathrm{m/s}^2$. Who do you agree with and why?

Exercise:

Problem:

Draw a free body diagram for a satellite in an elliptical orbit showing why its speed increases as it approaches its parent body and decreases as it moves away.

Exercise:

Problem:

Newton’s laws of motion and gravity were among the first to convincingly demonstrate the underlying simplicity and unity in nature. Many other examples have since been discovered, and we now expect to find such underlying order in complex situations. Is there proof that such order will always be found in new explorations?

Problem Exercises

Exercise:

Problem:

(a) Calculate Earth’s mass given the acceleration due to gravity at the North Pole is $9.830\ \text{m/s}^2$ and the radius of the Earth is 6371 km from center to pole.

(b) Compare this with the accepted value of $5.979 \times 10^{24}\ \text{kg}$.

Solution:

a) $5.979 \times 10^{24}\ \text{kg}$

b) This is identical to the best value to three significant figures.

Exercise:

Problem:

(a) Calculate the magnitude of the acceleration due to gravity on the surface of Earth due to the Moon.

(b) Calculate the magnitude of the acceleration due to gravity at Earth due to the Sun.

(c) Take the ratio of the Moon’s acceleration to the Sun’s and comment on why the tides are predominantly due to the Moon in spite of this number.

Exercise:

Problem:

(a) What is the acceleration due to gravity on the surface of the Moon?

(b) On the surface of Mars? The mass of Mars is $6.418 \times 10^{23}\ \text{kg}$ and its radius is $3.38 \times 10^{6}\ \text{m}$.

Solution:

a) $1.62\ \ \text{m/s}^2$

b) $3.75\ \ \text{m/s}^2$

Exercise:

Problem:

(a) Calculate the acceleration due to gravity on the surface of the Sun.

(b) By what factor would your weight increase if you could stand on the Sun? (Never mind that you cannot.)

Exercise:

Problem:

The Moon and Earth rotate about their common center of mass, which is located about 4700 km from the center of Earth. (This is 1690 km below the surface.)

(a) Calculate the magnitude of the acceleration due to the Moon's gravity at that point.

(b) Calculate the magnitude of the centripetal acceleration of the center of Earth as it rotates about that point once each lunar month (about 27.3 d) and compare it with the acceleration found in part (a). Comment on whether or not they are equal and why they should or should not be.

Solution:

a) $3.42 \times 10^{-5}\ \text{m/s}^2$

b) $3.34 \times 10^{-5}\ \text{m/s}^2$

The values are nearly identical. One would expect the gravitational force to be the same as the centripetal force at the core of the system.

Exercise:

Problem: Solve part (b) of [link] using $a_c = v^2/r$.

Exercise:

Problem:

Astrology, that unlikely and vague pseudoscience, makes much of the position of the planets at the moment of one's birth. The only known force a planet exerts on Earth is gravitational.

(a) Calculate the magnitude of the gravitational force exerted on a 4.20 kg baby by a 100 kg father 0.200 m away at birth (he is assisting, so he is close to the child).

(b) Calculate the magnitude of the force on the baby due to Jupiter if it is at its closest distance to Earth, some 6.29×10^{11} m away. How does the force of Jupiter on the baby compare to the force of the father on the baby? Other objects in the room and the hospital building also exert similar gravitational forces. (Of course, there could be an unknown force acting, but scientists first need to be convinced that there is even an effect, much less that an unknown force causes it.)

Solution:

a) 7.01×10^{-7} N

b) 1.35×10^{-6} N, 0.521

Exercise:

Problem:

The existence of the dwarf planet Pluto was proposed based on irregularities in Neptune’s orbit. Pluto was subsequently discovered near its predicted position. But it now appears that the discovery was fortuitous, because Pluto is small and the irregularities in Neptune’s orbit were not well known. To illustrate that Pluto has a minor effect on the orbit of Neptune compared with the closest planet to Neptune:

(a) Calculate the acceleration due to gravity at Neptune due to Pluto when they are 4.50×10^{12} m apart, as they are at present. The mass of Pluto is 1.4×10^{22} kg.

(b) Calculate the acceleration due to gravity at Neptune due to Uranus, presently about 2.50×10^{12} m apart, and compare it with that due to Pluto. The mass of Uranus is 8.62×10^{25} kg.

Exercise:

Problem:

(a) The Sun orbits the Milky Way galaxy once each $2.60 \text{ x } 10^8$ y, with a roughly circular orbit averaging $3.00 \text{ x } 10^4$ light years in radius. (A light year is the distance traveled by light in 1 y.) Calculate the centripetal acceleration of the Sun in its galactic orbit. Does your result support the contention that a nearly inertial frame of reference can be located at the Sun?

(b) Calculate the average speed of the Sun in its galactic orbit. Does the answer surprise you?

Solution:

a) $1.66 \times 10^{-10}\ \text{m/s}^2$

b) $2.17 \times 10^5\ \text{m/s}$

Exercise:

Problem: Unreasonable Result

A mountain 10.0 km from a person exerts a gravitational force on him equal to 2.00% of his weight.

(a) Calculate the mass of the mountain.

(b) Compare the mountain's mass with that of Earth.

(c) What is unreasonable about these results?

(d) Which premises are unreasonable or inconsistent? (Note that accurate gravitational measurements can easily detect the effect of nearby mountains and variations in local geology.)

Solution:

a) $2.937 \times 10^{17}\ \text{kg}$

b) 4.91×10^{-8}

of the Earth’s mass.

c) The mass of the mountain and its fraction of the Earth’s mass are too great.

d) The gravitational force assumed to be exerted by the mountain is too great.

Glossary

gravitational constant, *G*

a proportionality factor used in the equation for Newton’s universal law of gravitation; it is a universal constant—that is, it is thought to be the same everywhere in the universe

center of mass

the point where the entire mass of an object can be thought to be concentrated

microgravity

an environment in which the apparent net acceleration of a body is small compared with that produced by Earth at its surface

Newton’s universal law of gravitation

every particle in the universe attracts every other particle with a force along a line joining them; the force is directly proportional to the product of their masses and inversely proportional to the square of the distance between them

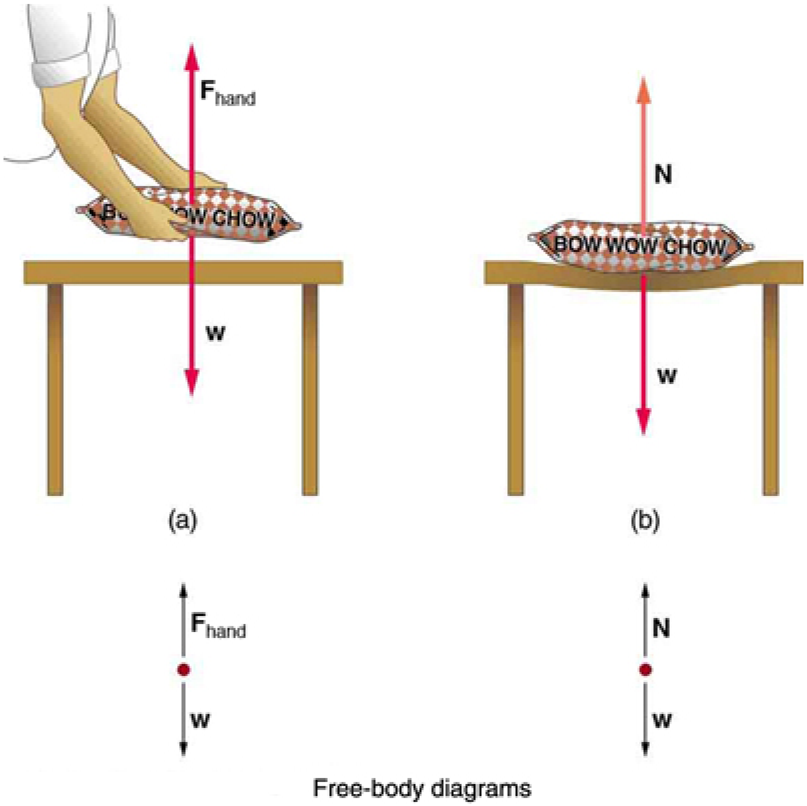
Normal, Tension, and Other Examples of Forces

- Define normal and tension forces.
- Apply Newton's laws of motion to solve problems involving a variety of forces.
- Use trigonometric identities to resolve weight into components.

Forces are given many names, such as push, pull, thrust, lift, weight, friction, and tension. Traditionally, forces have been grouped into several categories and given names relating to their source, how they are transmitted, or their effects. The most important of these categories are discussed in this section, together with some interesting applications. Further examples of forces are discussed later in this text.

Normal Force

Weight (also called force of gravity) is a pervasive force that acts at all times and must be counteracted to keep an object from falling. You definitely notice that you must support the weight of a heavy object by pushing up on it when you hold it stationary, as illustrated in [link](a). But how do inanimate objects like a table support the weight of a mass placed on them, such as shown in [link](b)? When the bag of dog food is placed on the table, the table actually sags slightly under the load. This would be noticeable if the load were placed on a card table, but even rigid objects deform when a force is applied to them. Unless the object is deformed beyond its limit, it will exert a restoring force much like a deformed spring (or trampoline or diving board). The greater the deformation, the greater the restoring force. So when the load is placed on the table, the table sags until the restoring force becomes as large as the weight of the load. At this point the net external force on the load is zero. That is the situation when the load is stationary on the table. The table sags quickly, and the sag is slight so we do not notice it. But it is similar to the sagging of a trampoline when you climb onto it.



(a) The person holding the bag of dog food must supply an upward force $\mathbf{F}_{\text{hand}}$ equal in magnitude and opposite in direction to the weight of the food $\mathbf{w}$. (b) The card table sags when the dog food is placed on it, much like a stiff trampoline. Elastic restoring forces in the table grow as it sags until they supply a force $\mathbf{N}$ equal in magnitude and opposite in direction to the weight of the load.

We must conclude that whatever supports a load, be it animate or not, must supply an upward force equal to the weight of the load, as we assumed in a few of the previous examples. If the force supporting a load is perpendicular to the surface of contact between the load and its support, this force is defined to be a **normal force** and here is given the symbol $\mathbf{N}$. (This is not the unit for force N.) The word *normal* means perpendicular to a surface. The normal force can be less than the object's weight if the object is on an incline, as you will see in the next example.

Note:

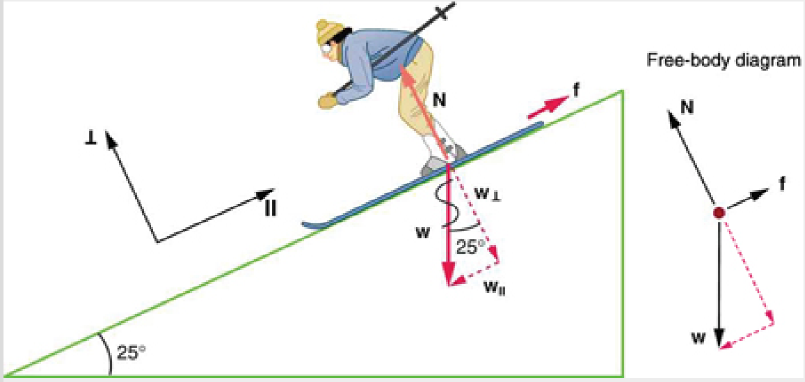
Common Misconception: Normal Force (N) vs. Newton (N)

In this section we have introduced the quantity normal force, which is represented by the variable $\mathbf{N}$. This should not be confused with the symbol for the newton, which is also represented by the letter N. These symbols are particularly important to distinguish because the units of a normal force ($\mathbf{N}$) happen to be newtons (N). For example, the normal force $\mathbf{N}$ that the floor exerts on a chair might be $\mathbf{N} = 100\text{ N}$. One important difference is that normal force is a vector, while the newton is simply a unit. Be careful not to confuse these letters in your calculations! You will encounter more similarities among variables and units as you proceed in physics. Another example of this is the quantity work (W) and the unit watts (W).

Example:

Weight on an Incline, a Two-Dimensional Problem

Consider the skier on a slope shown in [link]. Her mass including equipment is 60.0 kg. (a) What is her acceleration if friction is negligible? (b) What is her acceleration if friction is known to be 45.0 N?



Since motion and friction are parallel to the slope, it is most convenient to project all forces onto a coordinate system where one axis is parallel to the slope and the other is perpendicular (axes shown to left of skier). **N** is perpendicular to the slope and **f** is parallel to the slope, but **w** has components along both axes, namely $\mathbf{w}_\perp$ and $\mathbf{w}_\parallel$. **N** is equal in magnitude to $\mathbf{w}_\perp$, so that there is no motion perpendicular to the slope, but f is less than $w_\parallel$, so that there is a downslope acceleration (along the parallel axis).

Strategy

This is a two-dimensional problem, since the forces on the skier (the system of interest) are not parallel. The approach we have used in two-dimensional kinematics also works very well here. Choose a convenient coordinate system and project the vectors onto its axes, creating *two* connected *one*-dimensional problems to solve. The most convenient coordinate system for motion on an incline is one that has one coordinate parallel to the slope and one perpendicular to the slope. (Remember that motions along mutually perpendicular axes are independent.) We use the symbols $\perp$ and $\parallel$ to represent perpendicular and parallel, respectively. This choice of axes simplifies this type of problem, because there is no motion perpendicular to the slope and because friction is always parallel to the surface between two objects. The only external forces acting on the system are the skier’s weight, friction, and the support of the slope, respectively labeled **w**, **f**, and **N** in [link]. **N** is always perpendicular to the slope, and **f** is parallel to it. But **w** is not in the direction of either axis, and so the first step we take is to project it into components along the chosen axes, defining $w_\parallel$ to be the component of weight parallel to the slope and $w_\perp$ the component of weight perpendicular to the slope. Once this is done, we can consider the two separate problems of forces parallel to the slope and forces perpendicular to the slope.

Solution

The magnitude of the weight is mg = (60.0 kg)(9.80) = 588 N. Use the graphical method to find the parallel component = 249 N and the perpendicular component = 533 N

(a) Neglecting friction. Since the acceleration is parallel to the slope, we need only consider forces parallel to the slope. (Forces perpendicular to the slope add to zero, since there is no acceleration in that direction.) The forces parallel to the slope are the amount of the skier's weight parallel to the slope $w_{\parallel}$ and friction f. Using Newton's second law, with subscripts to denote quantities parallel to the slope,

Equation:

$$a_{\parallel} = \frac{F_{\text{net}\parallel}}{m}$$

Equation:

$$= 4.14\ \text{m/s}^2$$

is the acceleration.

(b) Including friction. We now have a given value for friction, and we know its direction is parallel to the slope and it opposes motion between surfaces in contact. So the net external force is now

Equation:

$$F_{\text{net}\parallel} = w_{\parallel} - f,$$

and substituting this into Newton's second law, $a_{\parallel} = \frac{F_{\text{net}\parallel}}{m}$, gives

We substitute known values to obtain

Equation:

$$a_{\parallel} = 3.39\ \text{m/s}^2,$$

which is the acceleration parallel to the incline when there is 45.0 N of opposing friction.

Discussion

Note:

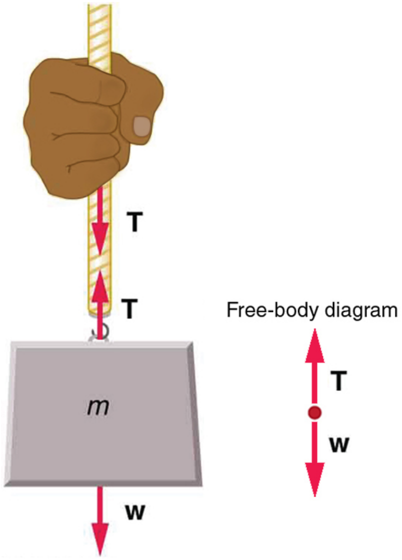
Take-Home Experiment: Force Parallel

To investigate how a force parallel to an inclined plane changes, find a rubber band, some objects to hang from the end of the rubber band, and a board you can position at different angles. How much does the rubber band stretch when you hang the object from the end of the board? Now place the board at an angle so that the object slides off when placed on the board. How much does the rubber band extend if it is lined up parallel to the board and used to hold the object stationary on the board? Try two more angles. What does this show?

Tension

A **tension** is a force along the length of a medium, especially a force carried by a flexible medium, such as a rope or cable. The word “tension” comes from a Latin word meaning “to stretch.” Not coincidentally, the flexible cords that carry muscle forces to other parts of the body are called *tendons*. Any flexible connector, such as a string, rope, chain, wire, or cable, can exert pulls only parallel to its length; thus, a force carried by a flexible connector is a tension with direction parallel to the connector. It is important to understand that tension is a pull in a connector. In contrast, consider the phrase: “You can’t push a rope.” The tension force pulls outward along the two ends of a rope.

Consider a person holding a mass on a rope as shown in [link].



When a perfectly flexible connector (one requiring no force to bend it) such as this rope transmits a force $\mathbf{T}$, that force must be parallel to the length of the rope, as shown. The pull such a flexible connector exerts is a tension. Note that the rope pulls with equal force but in opposite directions on the hand and the supported mass (neglecting the weight of the rope). This is an example of Newton’s third law. The rope is the medium that carries the equal and opposite forces between the two objects. The tension anywhere in the rope between the hand and the mass is equal. Once you have determined the tension in one location, you have determined the tension at all locations along the rope.

Tension in the rope must equal the weight of the supported mass, as we can prove using Newton’s second law. If the 5.00-kg mass in the figure is stationary, then its acceleration is zero, and thus $\mathbf{F}_{\text{net}} = 0$. The only external forces acting on the mass are its weight $\mathbf{w}$ and the tension $\mathbf{T}$ supplied by the rope. Thus,

Equation:

$$F_{\text{net}} = T - w = 0,$$

where T and w are the magnitudes of the tension and weight and their signs indicate direction, with up being positive here. Thus, just as you would expect, the tension equals the weight of the supported mass:

Equation:

$$T = w = \text{mg}.$$

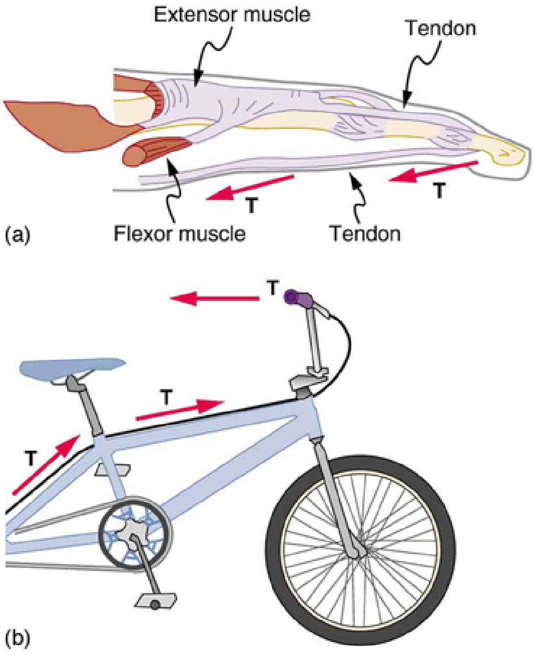
For a 5.00-kg mass, then (neglecting the mass of the rope) we see that

Equation:

$$T = \mathrm{mg} = (5.00\ \mathrm{kg})(9.80\ \mathrm{m/s^2}) = 49.0\ \mathrm{N}.$$

If we cut the rope and insert a spring, the spring would extend a length corresponding to a force of 49.0 N, providing a direct observation and measure of the tension force in the rope.

Flexible connectors are often used to transmit forces around corners, such as in a hospital traction system, a finger joint, or a bicycle brake cable. If there is no friction, the tension is transmitted undiminished. Only its direction changes, and it is always parallel to the flexible connector. This is illustrated in [link] (a) and (b).



(a) Tendons in the finger carry force $\mathbf{T}$ from the muscles to other parts of the finger, usually changing the force’s direction, but not its magnitude (the tendons are relatively friction free). (b) The brake cable on a bicycle carries the tension $\mathbf{T}$ from the handlebars to the brake mechanism. Again, the direction but not the magnitude of $\mathbf{T}$ is changed.

Note:

PhET Explorations: Forces in 1 Dimension

Explore the forces at work when you try to push a filing cabinet. Create an applied force and see the resulting friction force and total force acting on the cabinet. Charts show the forces, position, velocity, and acceleration vs. time. View a free-body diagram of all the forces (including gravitational and normal forces).

Forces in 1 Dimension

Section Summary

- When objects rest on a surface, the surface applies a force to the object that supports the weight of the object. This supporting force acts perpendicular to and away from the surface. It is called a normal force, $\mathbf{N}$.

- When objects rest on a non-accelerating horizontal surface, the magnitude of the normal force is equal to the weight of the object:
  Equation:

$$N = \text{mg}.$$

- When objects rest on an inclined plane that makes an angle θ with the horizontal surface, the weight of the object can be resolved into components that act perpendicular $(\mathbf{w}_{\perp})$ and parallel $(\mathbf{w}_{\|})$ to the surface of the plane. These components can be calculated using:
- The pulling force that acts along a stretched flexible connector, such as a rope or cable, is called tension, $\mathbf{T}$. When a rope supports the weight of an object that is at rest, the tension in the rope is equal to the weight of the object:
  Equation:

$$T = \text{mg}.$$

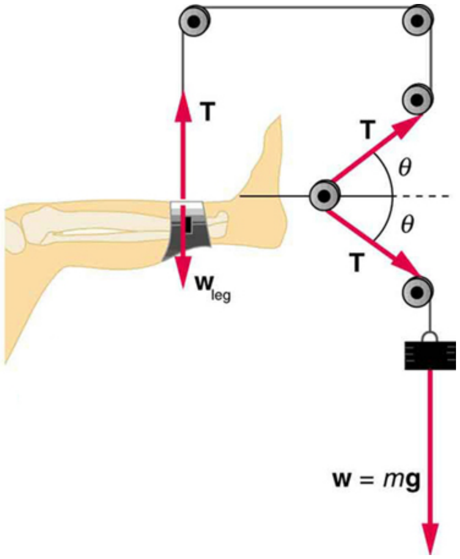
- In any inertial frame of reference (one that is not accelerated or rotated), Newton’s laws have the simple forms given in this chapter and all forces are real forces having a physical origin.

Conceptual Questions

Exercise:

Problem:

If a leg is suspended by a traction setup as shown in [link], what is the tension in the rope?



A leg is suspended by a traction system in which wires are used to transmit forces. Frictionless pulleys change the direction of the force *T* without changing its magnitude.

Exercise:

Problem:

In a traction setup for a broken bone, with pulleys and rope available, how might we be able to increase the force along the tibia using the same weight? (See [link].) (Note that the tibia is the shin bone shown in this image.)

Problem Exercises

Exercise:

Problem:

Two teams of nine members each engage in a tug of war. Each of the first team’s members has an average mass of 68 kg and exerts an average force of 1350 N horizontally. Each of the second team’s members has an average mass of 73 kg and exerts an average force of 1365 N horizontally. (a) What is magnitude of the acceleration of the two teams? (b) What is the tension in the section of rope between the teams?

Solution:

a. 0.11 m/s^2
b. $1.2 \times 10^4\text{ N}$

Exercise:

Problem:

What force does a trampoline have to apply to a 45.0-kg gymnast to accelerate her straight up at 7.50 m/s^2? Note that the answer is independent of the velocity of the gymnast—she can be moving either up or down, or be stationary.

Exercise:

Problem:

(a) Calculate the tension in a vertical strand of spider web if a spider of mass $8.00 \times 10^{-5}\text{ kg}$ hangs motionless on it. (b) Calculate the tension in a horizontal strand of spider web if the same spider sits motionless in the middle of it much like the tightrope walker in [link]. The strand sags at an angle of 12º below the horizontal. Compare this with the tension in the vertical strand (find their ratio).

Solution:

(a) $7.84 \times 10^{-4}\text{ N}$

(b) $1.89 \times 10^{-3}\ \text{N}$. This is 2.41 times the tension in the vertical strand.

Exercise:

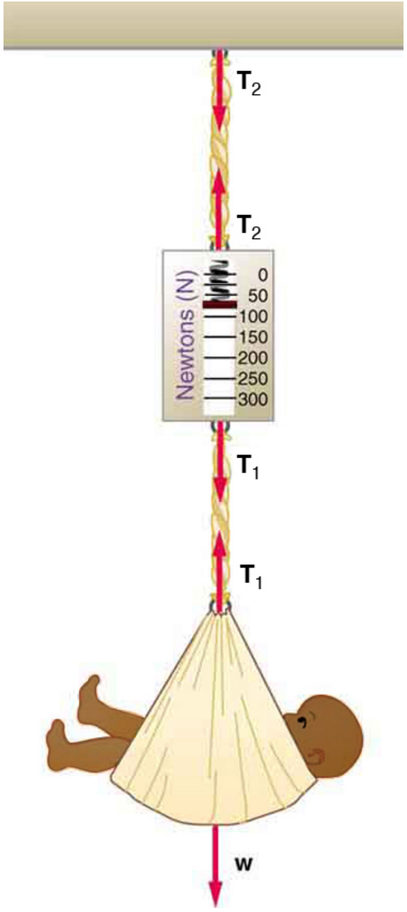
Problem:

Suppose a 60.0-kg gymnast climbs a rope. (a) What is the tension in the rope if he climbs at a constant speed? (b) What is the tension in the rope if he accelerates upward at a rate of $1.50\ \text{m/s}^2$?

Exercise:

Problem:

Consider the baby being weighed in [link]. (a) What is the mass of the child and basket if a scale reading of 55 N is observed? (b) What is the tension T_1 in the cord attaching the baby to the scale? (c) What is the tension T_2 in the cord attaching the scale to the ceiling, if the scale has a mass of 0.500 kg? (d) Draw a sketch of the situation indicating the system of interest used to solve each part. The masses of the cords are negligible.



A baby is weighed using a spring scale.

Glossary

inertial frame of reference
: a coordinate system that is not accelerating; all forces acting in an inertial frame of reference are real forces, as opposed to fictitious forces that are observed due to an accelerating frame of reference

normal force
: the force that a surface applies to an object to support the weight of the object; acts perpendicular to the surface on which the object rests

tension

the pulling force that acts along a medium, especially a stretched flexible connector, such as a rope or cable; when a rope supports the weight of an object, the force on the object due to the rope is called a tension force

Friction

- Discuss the general characteristics of friction.
- Describe the various types of friction.
- Calculate the magnitude of static and kinetic friction.

Friction is a force that is around us all the time that opposes relative motion between surfaces in contact but also allows us to move (which you have discovered if you have ever tried to walk on ice). While a common force, the behavior of friction is actually very complicated and is still not completely understood. We have to rely heavily on observations for whatever understandings we can gain. However, we can still deal with its more elementary general characteristics and understand the circumstances in which it behaves.

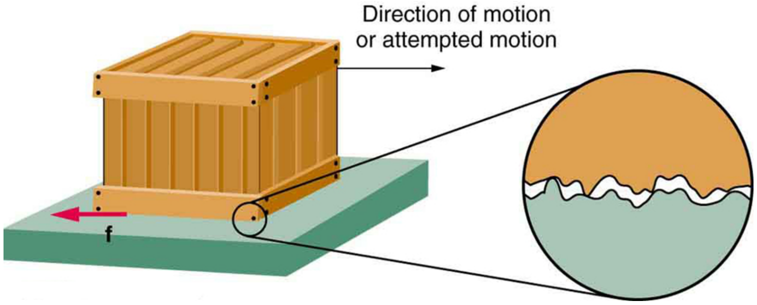
Note:
Friction
Friction is a force that opposes relative motion between surfaces in contact.

One of the simpler characteristics of friction is that it is parallel to the contact surface between surfaces and always in a direction that opposes motion or attempted motion of the systems relative to each other. If two surfaces are in contact and moving relative to one another, then the friction between them is called **kinetic friction**. For example, friction slows a hockey puck sliding on ice. But when objects are stationary, **static friction** can act between them; the static friction is usually greater than the kinetic friction between the surfaces.

Note:
Kinetic Friction
If two surfaces are in contact and moving relative to one another, then the friction between them is called kinetic friction.

Imagine, for example, trying to slide a heavy crate across a concrete floor—you may push harder and harder on the crate and not move it at all. This means that the static friction responds to what you do—it increases to be equal to and in the opposite direction of your push. But if you finally push hard enough, the crate seems to slip suddenly and starts to move. Once in motion it is easier to keep it in motion than it was to get it started, indicating that the kinetic friction force is less than the static friction force. If you add mass to the crate, say by placing a box on top of it, you need to push even harder to get it started and also to keep it moving. Furthermore, if you oiled the concrete you would find it to be easier to get the crate started and keep it going (as you might expect).

[link] is a crude pictorial representation of how friction occurs at the interface between two objects. Close-up inspection of these surfaces shows them to be rough. So when you push to get an object moving (in this case, a crate), you must raise the object until it can skip along with just the tips of the surface hitting, break off the points, or do both. A considerable force can be resisted by friction with no apparent motion. The harder the surfaces are pushed together (such as if another box is placed on the crate), the more force is needed to move them. Part of the friction is due to adhesive forces between the surface molecules of the two objects, which explain the dependence of friction on the nature of the substances. Adhesion varies with substances in contact and is a complicated aspect of surface physics. Once an object is moving, there are fewer points of contact (fewer molecules adhering), so less force is required to keep the object moving. At small but nonzero speeds, friction is nearly independent of speed.



Frictional forces, such as f, always oppose motion or attempted motion between surfaces in contact. Friction arises in part because of the roughness of the surfaces in contact, as seen in the expanded view. In order for the object to move, it must rise to where the peaks can skip along the bottom surface. Thus a force is required just to set the object in motion. Some of the peaks will be broken off, also requiring a force to maintain motion. Much of the friction is actually due to attractive forces between molecules making up the two objects, so that even perfectly smooth surfaces are not friction-free. Such adhesive forces also depend on the substances the surfaces are made of, explaining, for example, why rubber-soled shoes slip less than those with leather soles.

The magnitude of the frictional force has two forms: one for static situations (static friction), the other for when there is motion (kinetic friction).

When there is no motion between the objects, the **magnitude of static friction $\mathbf{f}_{\mathrm{s}}$** is
Equation:

$$f_{\mathrm{s}} \leq \mu_{\mathrm{s}} N,$$

where μ_{s} is the coefficient of static friction and N is the magnitude of the normal force (the force perpendicular to the surface).

Note:
Magnitude of Static Friction
Magnitude of static friction f_{s} is

Equation:

$$f_{\mathrm{s}} \leq \mu_{\mathrm{s}} N,$$

where μ_{s} is the coefficient of static friction and N is the magnitude of the normal force.

The symbol $\leq$ means *less than or equal to*, implying that static friction can have a minimum and a maximum value of $\mu_{\mathrm{s}} N$. Static friction is a responsive force that increases to be equal and opposite to whatever force is exerted, up to its maximum limit. Once the applied force exceeds $f_{\mathrm{s(max)}}$, the object will move. Thus

Equation:

$$f_{\mathrm{s(max)}} = \mu_{\mathrm{s}} N.$$

Once an object is moving, the **magnitude of kinetic friction $\mathbf{f}_{\mathrm{k}}$** is given by

Equation:

$$f_{\mathrm{k}} = \mu_{\mathrm{k}} N,$$

where μ_{k} is the coefficient of kinetic friction. A system in which $f_{\mathrm{k}} = \mu_{\mathrm{k}} N$ is described as a system in which *friction behaves simply*.

Note:

Magnitude of Kinetic Friction

The magnitude of kinetic friction f_{k} is given by

Equation:

$$f_{\mathrm{k}} = \mu_{\mathrm{k}} N,$$

where μ_{k} is the coefficient of kinetic friction.

As seen in [link], the coefficients of kinetic friction are less than their static counterparts. That values of μ in [link] are stated to only one or, at most, two digits is an indication of the approximate description of friction given by the above two equations.

System	**Static friction** μ_{s}	**Kinetic friction** μ_{k}
Rubber on dry concrete	1.0	0.7
Rubber on wet concrete	0.7	0.5
Wood on wood	0.5	0.3
Waxed wood on wet snow	0.14	0.1
Metal on wood	0.5	0.3
Steel on steel (dry)	0.6	0.3
Steel on steel (oiled)	0.05	0.03
Teflon on steel	0.04	0.04
Bone lubricated by synovial fluid	0.016	0.015
Shoes on wood	0.9	0.7

System	**Static friction** μ_s	**Kinetic friction** μ_k
Shoes on ice	0.1	0.05
Ice on ice	0.1	0.03
Steel on ice	0.04	0.02

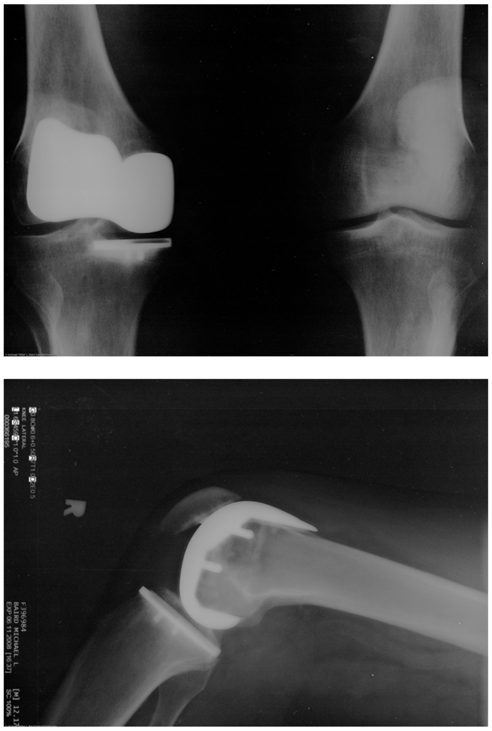
Coefficients of Static and Kinetic Friction

The equations given earlier include the dependence of friction on materials and the normal force. The direction of friction is always opposite that of motion, parallel to the surface between objects, and perpendicular to the normal force. For example, if the crate you try to push (with a force parallel to the floor) has a mass of 100 kg, then the normal force would be equal to its weight, $W = mg = (100\text{ kg})(9.80\text{ m/s}^2) = 980\text{ N}$, perpendicular to the floor. If the coefficient of static friction is 0.45, you would have to exert a force parallel to the floor greater than $f_{\text{s(max)}} = \mu_s N = (0.45)(980\text{ N}) = 440\text{ N}$ to move the crate. Once there is motion, friction is less and the coefficient of kinetic friction might be 0.30, so that a force of only 290 N ($f_k = \mu_k N = (0.30)(980\text{ N}) = 290\text{ N}$) would keep it moving at a constant speed. If the floor is lubricated, both coefficients are considerably less than they would be without lubrication. Coefficient of friction is a unit less quantity with a magnitude usually between 0 and 1.0. The coefficient of the friction depends on the two surfaces that are in contact.

Note:
Take-Home Experiment
Find a small plastic object (such as a food container) and slide it on a kitchen table by giving it a gentle tap. Now spray water on the table,

simulating a light shower of rain. What happens now when you give the object the same-sized tap? Now add a few drops of (vegetable or olive) oil on the surface of the water and give the same tap. What happens now? This latter situation is particularly important for drivers to note, especially after a light rain shower. Why?

Many people have experienced the slipperiness of walking on ice. However, many parts of the body, especially the joints, have much smaller coefficients of friction—often three or four times less than ice. A joint is formed by the ends of two bones, which are connected by thick tissues. The knee joint is formed by the lower leg bone (the tibia) and the thighbone (the femur). The hip is a ball (at the end of the femur) and socket (part of the pelvis) joint. The ends of the bones in the joint are covered by cartilage, which provides a smooth, almost glassy surface. The joints also produce a fluid (synovial fluid) that reduces friction and wear. A damaged or arthritic joint can be replaced by an artificial joint ([link]). These replacements can be made of metals (stainless steel or titanium) or plastic (polyethylene), also with very small coefficients of friction.



Artificial knee replacement is a procedure that has been performed for more than 20 years. In this figure, we see the post-op X-rays of the right knee joint replacement. (credit: Mike Baird, Flickr)

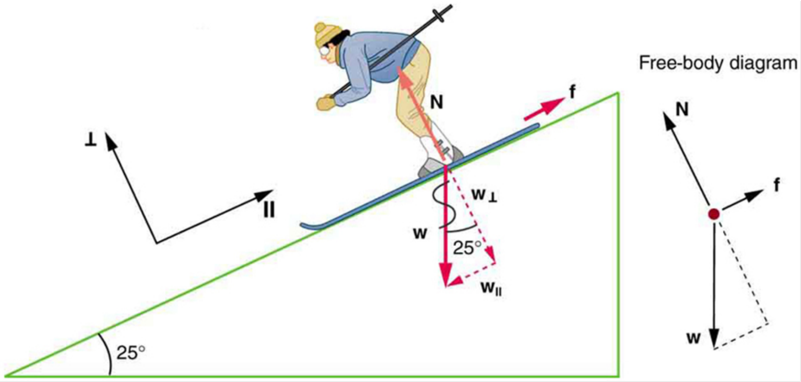
Other natural lubricants include saliva produced in our mouths to aid in the swallowing process, and the slippery mucus found between organs in the body, allowing them to move freely past each other during heartbeats, during breathing, and when a person moves. Artificial lubricants are also common in hospitals and doctor’s clinics. For example, when ultrasonic imaging is carried out, the gel that couples the transducer to the skin also serves to to lubricate the surface between the transducer and the skin—thereby reducing the coefficient of friction between the two surfaces. This allows the transducer to move freely over the skin.

Example:
Skiing Exercise

A skier with a mass of 62 kg is sliding down a snowy slope. Find the coefficient of kinetic friction for the skier if friction is known to be 45.0 N.

Strategy

The magnitude of kinetic friction was given in to be 45.0 N. Kinetic friction is related to the normal force N as $f_{\mathrm{k}} = \mu_{\mathrm{k}} N$; thus, the coefficient of kinetic friction can be found if we can find the normal force of the skier on a slope. The normal force is always perpendicular to the surface, and since there is no motion perpendicular to the surface, the normal force should equal the component of the skier’s weight perpendicular to the slope. (See the skier and free-body diagram in [link].)



The motion of the skier and friction are parallel to the slope and so it is most convenient to project all forces onto a coordinate system where one axis is parallel to the slope and the other is perpendicular (axes shown to left of skier). **N** (the normal force) is perpendicular to the slope, and **f** (the friction) is parallel to the slope, but **w** (the skier's weight) has components along both axes, namely $\mathbf{w}_{\perp}$ and $\mathbf{W}_{//}$. **N** is equal in magnitude to $\mathbf{w}_{\perp}$, so there is no motion perpendicular to the slope. However, **f** is less than $\mathbf{W}_{//}$ in magnitude, so there is acceleration down the slope (along the *x*-axis).

Note:

PhET Explorations: Forces and Motion

Explore the forces at work when you try to push a filing cabinet. Create an applied force and see the resulting friction force and total force acting on the cabinet. Charts show the forces, position, velocity, and acceleration vs. time. Draw a free-body diagram of all the forces (including gravitational and normal forces).

Forces and Motion

Section Summary

- Friction is a contact force between systems that opposes the motion or attempted motion between them. Simple friction is proportional to the normal force N pushing the systems together. (A normal force is always perpendicular to the contact surface between systems.) Friction depends on both of the materials involved. The magnitude of static friction f_{s} between systems stationary relative to one another is given by
  Equation:
  $$f_{\mathrm{s}} \leq \mu_{\mathrm{s}} N,$$
  where μ_{s} is the coefficient of static friction, which depends on both of the materials.
- The kinetic friction force f_{k} between systems moving relative to one another is given by
  Equation:
  $$f_{\mathrm{k}} = \mu_{\mathrm{k}} N,$$
  where μ_{k} is the coefficient of kinetic friction, which also depends on both materials.

Conceptual Questions

Exercise:

Problem:

Define normal force. What is its relationship to friction when friction behaves simply?

Exercise:

Problem:

The glue on a piece of tape can exert forces. Can these forces be a type of simple friction? Explain, considering especially that tape can stick to vertical walls and even to ceilings.

Exercise:

Problem:

When you learn to drive, you discover that you need to let up slightly on the brake pedal as you come to a stop or the car will stop with a jerk. Explain this in terms of the relationship between static and kinetic friction.

Exercise:

Problem:

When you push a piece of chalk across a chalkboard, it sometimes screeches because it rapidly alternates between slipping and sticking to the board. Describe this process in more detail, in particular explaining how it is related to the fact that kinetic friction is less than static friction. (The same slip-grab process occurs when tires screech on pavement.)

Problems & Exercises

Exercise:

Problem:

A physics major is cooking breakfast when he notices that the frictional force between his steel spatula and his Teflon frying pan is only 0.200 N. Knowing the coefficient of kinetic friction between the two materials, he quickly calculates the normal force. What is it?

Solution:

5.00 N

Exercise:

Problem:

(a) When rebuilding her car’s engine, a physics major must exert 300 N of force to insert a dry steel piston into a steel cylinder. What is the magnitude of the normal force between the piston and cylinder? (b) What is the magnitude of the force would she have to exert if the steel parts were oiled?

Exercise:

Problem:

(a) What is the maximum frictional force in the knee joint of a person who supports 66.0 kg of her mass on that knee? (b) During strenuous exercise it is possible to exert forces to the joints that are easily ten times greater than the weight being supported. What is the maximum force of friction under such conditions? The frictional forces in joints are relatively small in all circumstances except when the joints deteriorate, such as from injury or arthritis. Increased frictional forces can cause further damage and pain.

Exercise:

Problem:

Suppose you have a 120-kg wooden crate resting on a wood floor. (a) What maximum force can you exert horizontally on the crate without moving it? (b) If you continue to exert this force once the crate starts to slip, what will the magnitude of its acceleration then be?

Solution:

(a) 588 N

(b) $1.96\ \mathrm{m/s}^2$

Exercise:

Problem:

(a) If half of the weight of a small 1.00×10^3 kg utility truck is supported by its two drive wheels, what is the magnitude of the maximum acceleration it can achieve on dry concrete? (b) Will a metal cabinet lying on the wooden bed of the truck slip if it accelerates at this rate? (c) Solve both problems assuming the truck has four-wheel drive.

Exercise:

Problem:

A team of eight dogs pulls a sled with waxed wood runners on wet snow (mush!). The dogs have average masses of 19.0 kg, and the loaded sled with its rider has a mass of 210 kg. (a) Calculate the magnitude of the acceleration starting from rest if each dog exerts an average force of 185 N backward on the snow. (b) What is the magnitude of the acceleration once the sled starts to move? (c) For both situations, calculate the magnitude of the force in the coupling between the dogs and the sled.

Solution:

(a) 3.29 m/s^2

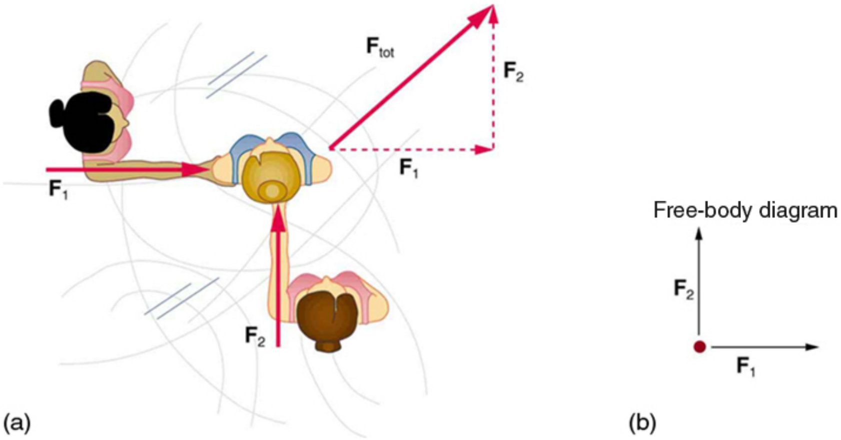
(b) 3.52 m/s^2

(c) 980 N; 945 N

Exercise:

Problem:

Consider the 65.0-kg ice skater being pushed by two others shown in [link]. (a) Find the direction and magnitude of $\mathbf{F}_{\text{tot}}$, the total force exerted on her by the others, given that the magnitudes F_1 and F_2 are 26.4 N and 18.6 N, respectively. (b) What is her initial acceleration if she is initially stationary and wearing steel-bladed skates that point in the direction of $\mathbf{F}_{\text{tot}}$? (c) What is her acceleration assuming she is already moving in the direction of $\mathbf{F}_{\text{tot}}$? (Remember that friction always acts in the direction opposite that of motion or attempted motion between surfaces in contact.)



Exercise:

Problem:

Show that the acceleration of any object down a frictionless incline that makes an angle θ with the horizontal is $a = g \sin \theta$. (Note that this acceleration is independent of mass.)

Exercise:

Problem:

Calculate the maximum deceleration of a car that is heading down a 6° slope (one that makes an angle of 6° with the horizontal) under the following road conditions. You may assume that the weight of the car is evenly distributed on all four tires and that the coefficient of static friction is involved—that is, the tires are not allowed to slip during the deceleration. (Ignore rolling.) Calculate for a car: (a) On dry concrete. (b) On wet concrete. (c) On ice, assuming that $\mu_{\mathrm{s}} = 0.100$, the same as for shoes on ice.

Exercise:

Problem:

Calculate the maximum acceleration of a car that is heading up a 4° slope (one that makes an angle of 4° with the horizontal) under the following road conditions. Assume that only half the weight of the car is supported by the two drive wheels and that the coefficient of static friction is involved—that is, the tires are not allowed to slip during the acceleration. (Ignore rolling.) (a) On dry concrete. (b) On wet concrete. (c) On ice, assuming that $\mu_s = 0.100$, the same as for shoes on ice.

Solution:

(a) $4.20\ \mathrm{m/s}^2$

(b) $2.74\ \mathrm{m/s}^2$

(c) $-0.195\ \mathrm{m/s}^2$

Exercise:

Problem: Repeat [link] for a car with four-wheel drive.

Exercise:

Problem:

A freight train consists of two 8.00×10^5-kg engines and 45 cars with average masses of 5.50×10^5 kg. (a) What force must each engine exert backward on the track to accelerate the train at a rate of $5.00 \times 10^{-2}\ \mathrm{m/s^2}$ if the force of friction is 7.50×10^5 N, assuming the engines exert identical forces? This is not a large frictional force for such a massive system. Rolling friction for trains is small, and consequently trains are very energy-efficient transportation systems. (b) What is the magnitude of the force in the coupling between the 37th and 38th cars (this is the force each exerts on the other), assuming all cars have the same mass and that friction is evenly distributed among all of the cars and engines?

Solution:

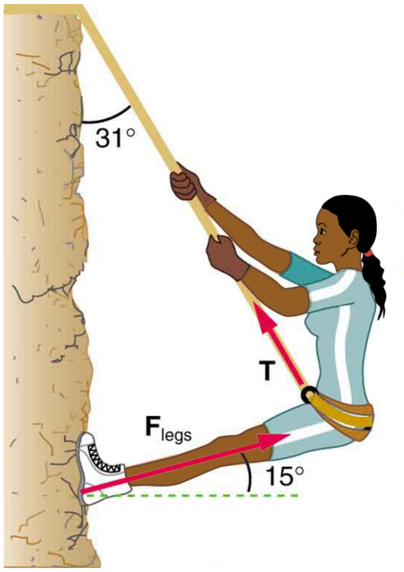
(a) 1.03×10^6 N

(b) 3.48×10^5 N

Exercise:

Problem:

Consider the 52.0-kg mountain climber in [link]. (a) Find the tension in the rope and the force that the mountain climber must exert with her feet on the vertical rock face to remain stationary. Assume that the force is exerted parallel to her legs. Also, assume negligible force exerted by her arms. (b) What is the minimum coefficient of friction between her shoes and the cliff?



Part of the climber’s weight is supported by her rope and part by friction between her feet and the rock face.

Exercise:

Problem:

A contestant in a winter sporting event pushes a 45.0-kg block of ice across a frozen lake as shown in [link](a). (a) Calculate the minimum force F he must exert to get the block moving. (b) What is the magnitude of its acceleration once it starts to move, if that force is maintained?

Solution:

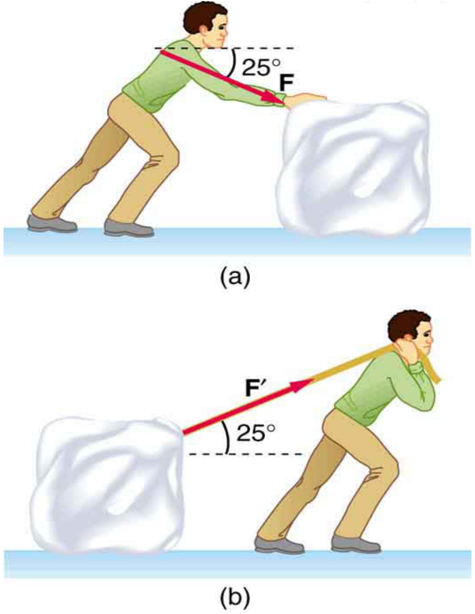
(a) 51.0 N

(b) 0.720 m/s^2

Exercise:

Problem:

Repeat [link] with the contestant pulling the block of ice with a rope over his shoulder at the same angle above the horizontal as shown in [link](b).



Which method of sliding a block of ice requires less force—(a) pushing or (b) pulling at the same angle above the horizontal?

Glossary

friction
: a force that opposes relative motion or attempts at motion between systems in contact

kinetic friction

a force that opposes the motion of two systems that are in contact and moving relative to one another

static friction

a force that opposes the motion of two systems that are in contact and are not moving relative to one another

magnitude of static friction

$f_\mathrm{s} \le \mu_\mathrm{s}\, N$, where μ_s is the coefficient of static friction and N is the magnitude of the normal force

magnitude of kinetic friction

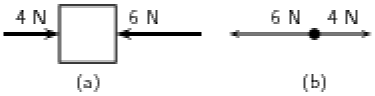
$f_\mathrm{k} = \mu_\mathrm{k} N$, where μ_k is the coefficient of kinetic friction

Forces: Introduction and key concepts

adding Force

The easiest way to determine a resultant force is to draw a free body diagram. Remember from Vectors that we use the length of the arrow to indicate the vector's magnitude and the direction of the arrow to show which direction it acts in.

After we have done this, we have a diagram of vectors and we simply find the sum of the vectors to get the resultant force.



(a) Force diagram of 2 forces acting on a box. (b) Free body diagram of the box.

For example, two people push on a box from opposite sides with forces of 4 N and 6 N respectively as shown in [link](a). The free body diagram in [link](b) shows the object represented by a dot and the two forces are represented by arrows with their tails on the dot.

As you can see, the arrows point in opposite directions and have different lengths. The resultant force is 2 N to the left. This result can be obtained algebraically too, since the two forces act along the same line. First, as in motion in one direction, choose a frame of reference. Secondly, add the two vectors taking their directions into account.

For the example, assume that the positive direction is to the right, then:

Equation:

$$\begin{aligned} F_R &= (+4\,\mathrm{N}) + (-6\,\mathrm{N}) \\ &= -2\,\mathrm{N} \\ &= 2\,\mathrm{N}\ \ \text{to the left} \end{aligned}$$

Remember that a negative answer means that the force acts in the *opposite* direction to the one that you chose to be positive. You can *choose* the positive direction to be any way you want, but once you have chosen it you *must* keep it.

As you work with more force diagrams in which the forces exactly balance, you may notice that you get a zero answer (e.g. 0 N). This simply means that the forces are balanced and that the object will not accelerate.

Once a force diagram has been drawn the techniques of vector addition introduced in Vectors can be used. Depending on the situation you might choose to use a graphical technique such as the tail-to-head method or the parallelogram method, or else an algebraic approach to determine the resultant. Since force is a vector quantity all of these methods apply.

Exercise:

Finding the resultant force

Problem:

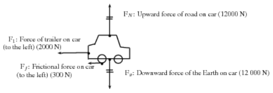
A car (mass 1200 kg) applies a force of 2000 N on a trailer (mass 250 kg). A constant frictional force of 200 N is acting on the trailer, and a constant frictional force of 300 N is acting on the car.

1. Draw a force diagram of all the forces acting on the car.
2. Draw a free body diagram of all the horizontal forces acting on the trailer.
3. Use the force diagram to determine the resultant force on the trailer.

Solution:

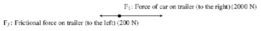
Draw the force diagram for the car.

The question asks us to draw all the forces on the car. This means that we must include horizontal and vertical forces.


F_N: Upward force of road on car (12000 N)
F_1: Force of trailer on car (to the left) (2000 N)
F_f: Frictional force on car (to the left) (300 N)
F_g: Downward force of the Earth on car (12 000 N)

Draw the free body diagram for the trailer.

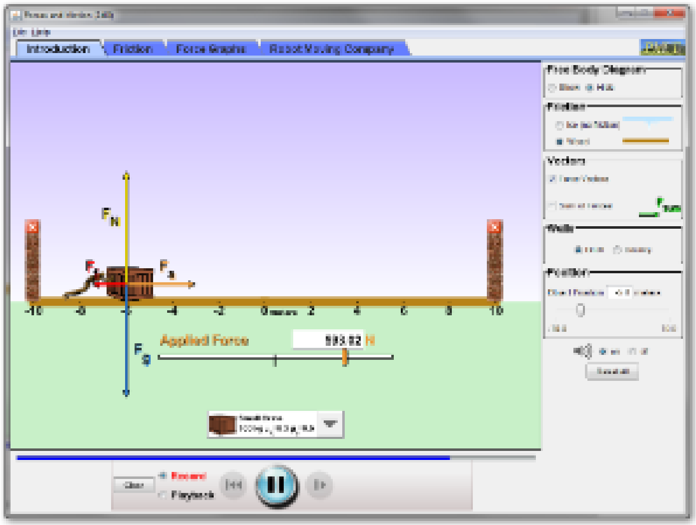
The question only asks for horizontal forces. We will therefore not include the force of the Earth on the trailer, or the force of the road on the trailer as these forces are in a vertical direction.


F_1: Force of car on trailer (to the right) (2000 N)
F_f: Frictional force on trailer (to the left) (200 N)

Determine the resultant force on the trailer.

To find the resultant force we need to add all the horizontal forces together. We do not add vertical forces as the movement of the car and trailer will be in a horizontal direction, and not up or down. $F_R = 2000 + (-200) = 1800$ N to the right.

run demo



Exercise

1. A force acts on an object. Name three effects that the force can have on the object.
2. Identify each of the following forces as contact or non-contact forces.

    a. The force between the north pole of a magnet and a paper clip.
    b. The force required to open the door of a taxi.
    c. The force required to stop a soccer ball.
    d. The force causing a ball, dropped from a height of 2 m, to fall to the floor.

3. A book of mass 2 kg is lying on a table. Draw a labeled force diagram indicating all the forces on the book.
4. A boy pushes a shopping trolley (mass 15 kg) with a constant force of 75 N. A constant frictional force of 20 N is present.

    a. Draw a labeled force diagram to identify all the forces acting on the shopping trolley.
    b. Draw a free body diagram of all the horizontal forces acting on the trolley.
    c. Determine the resultant force on the trolley.

5. A donkey (mass 250 kg) is trying to pull a cart (mass 80 kg) with a force of 400 N. The rope between the donkey and the cart makes an angle of 30° with the cart. The cart does not move.

    a. Draw a free body diagram of all the forces acting on the donkey.
    b. Draw a force diagram of all the forces acting on the cart.
    c. Find the magnitude and direction of the frictional force preventing the cart from moving.

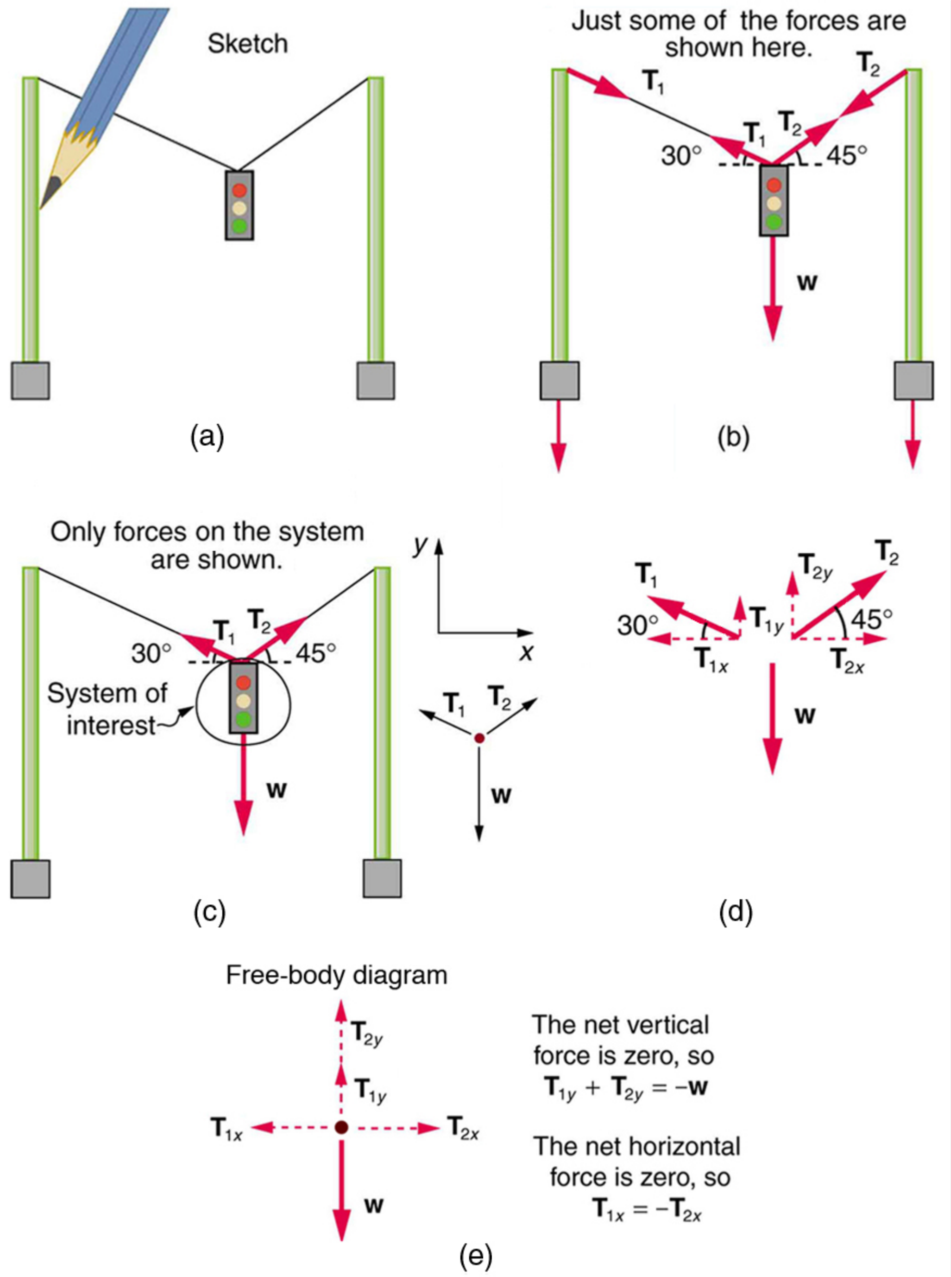
Further Applications of Newton’s Laws of Motion

- Apply problem-solving techniques to solve for quantities in more complex systems of forces.
- Integrate concepts from kinematics to solve problems using Newton's laws of motion.

Example:

Different Tensions at Different Angles

Consider the traffic light (mass 15.0 kg) suspended from two wires as shown in [link]. Find the tension in each wire, neglecting the masses of the wires.


Sketch
(a)
Just some of the forces are shown here.
$\mathbf{T}_1$
$\mathbf{T}_2$
30°
45°
$\mathbf{w}$
(b)
Only forces on the system are shown.
30°
45°
System of interest
y
x
(c)
$\mathbf{T}_{1x}$
$\mathbf{T}_{1y}$
$\mathbf{T}_{2x}$
$\mathbf{T}_{2y}$
(d)
Free-body diagram
The net vertical force is zero, so
$\mathbf{T}_{1y} + \mathbf{T}_{2y} = -\mathbf{w}$
The net horizontal force is zero, so
$\mathbf{T}_{1x} = -\mathbf{T}_{2x}$
(e)

A traffic light is suspended from two wires. (b) Some of the forces involved. (c) Only forces acting on the system are shown here. The free-body diagram for the traffic light is also shown. (d) The forces projected onto vertical (*y*) and horizontal (*x*) axes. The horizontal components of the tensions must cancel, and the sum of the vertical components of the tensions must equal the weight of the traffic light. (e)

The free-body diagram shows the vertical and horizontal forces acting on the traffic light.

Strategy

The system of interest is the traffic light, and its free-body diagram is shown in [link](c). The three forces involved are not parallel, and so they must be projected onto a coordinate system. The most convenient coordinate system has one axis vertical and one horizontal, and the vector projections on it are shown in part (d) of the figure. There are two unknowns in this problem (T_1 and T_2), so two equations are needed to find them. These two equations come from applying Newton's second law along the vertical and horizontal axes, noting that the net external force is zero along each axis because acceleration is zero.

Solution

First consider the horizontal or *x*-axis:

Equation:

$$F_{\text{net}x} = T_{2x} - T_{1x} = 0.$$

Thus, as you might expect,

Equation:

$$T_{1x} = T_{2x}.$$

Note that T_1 and T_2 are not equal in this case, because the angles on either side are not equal. It is reasonable that T_2 ends up being greater than T_1, because it is exerted more vertically than T_1.

Now consider the force components along the vertical or *y*-axis:

Equation:

$$F_{\text{net }y} = T_{1y} + T_{2y} - w = 0.$$

This implies

Equation:

$$T_{1y} + T_{2y} = w.$$

Substituting the expressions for the vertical components gives
There are two unknowns in this equation, but substituting the expression for T_2 in terms of T_1 reduces this to one equation with one unknown:
Equation:

$$T_1(0.500) + (1.225T_1)(0.707) = w = \text{mg},$$

Solving this last equation gives the magnitude of T_1 to be
Equation:

$$T_1 = 108\text{ N}.$$

Equation:

$$T_2 = 132\text{ N}.$$

Discussion
Both tensions would be larger if both wires were more horizontal, and they will be equal if and only if the angles on either side are the same (as they were in the earlier example of a tightrope walker).

Summary

- Newton's laws of motion can be applied in numerous situations to solve problems of motion.
- Some problems will contain multiple force vectors acting in different directions on an object. Be sure to draw diagrams, resolve all force vectors into horizontal and vertical components, and draw a free-body diagram. Always analyze the direction in which an object accelerates so that you can determine whether $F_{\text{net}} = \text{ma}$ or $F_{\text{net}} = 0$.
- The normal force on an object is not always equal in magnitude to the weight of the object. If an object is accelerating, the normal force will be less than or greater than the weight of the object. Also, if the object is on an inclined plane, the normal force will always be less than the full weight of the object.

- Some problems will contain various physical quantities, such as forces, acceleration, velocity, or position. You can apply concepts from kinematics and dynamics in order to solve these problems of motion.

Conceptual Questions

Exercise:

Problem:

To simulate the apparent weightlessness of space orbit, astronauts are trained in the hold of a cargo aircraft that is accelerating downward at g. Why will they appear to be weightless, as measured by standing on a bathroom scale, in this accelerated frame of reference? Is there any difference between their apparent weightlessness in orbit and in the aircraft?

Exercise:

Problem:

A cartoon shows the toupee coming off the head of an elevator passenger when the elevator rapidly stops during an upward ride. Can this really happen without the person being tied to the floor of the elevator? Explain your answer.

Problem Exercises

Exercise:

Problem:

A flea jumps by exerting a force of 1.20×10^{-5} N straight down on the ground. A breeze blowing on the flea parallel to the ground exerts a force of 0.500×10^{-6} N on the flea. Find the direction and magnitude of the acceleration of the flea if its mass is 6.00×10^{-7} kg. Do not neglect the gravitational force.

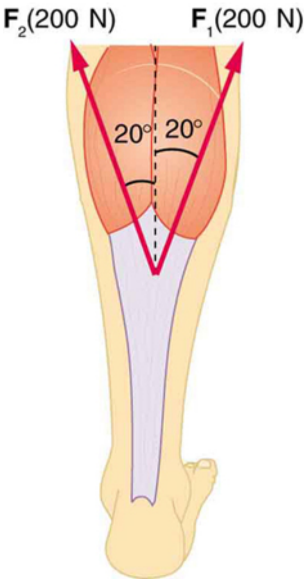
Solution:

$10.2\ \text{m/s}^2$, 4.67° from vertical

Exercise:

Problem:

Two muscles in the back of the leg pull upward on the Achilles tendon, as shown in [link]. (These muscles are called the medial and lateral heads of the gastrocnemius muscle.) Find the magnitude and direction of the total force on the Achilles tendon. What type of movement could be caused by this force?



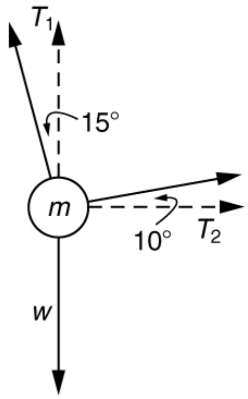
Achilles tendon

Exercise:

Problem:

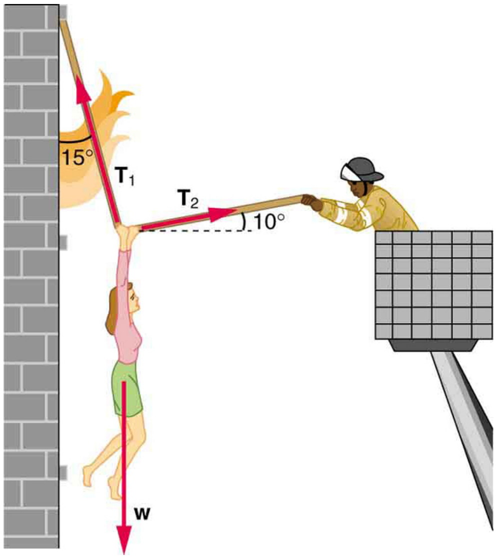
A 76.0-kg person is being pulled away from a burning building as shown in [link]. Calculate the tension in the two ropes if the person is momentarily motionless. Include a free-body diagram in your solution.

Solution:



$$T_1 = 736\text{ N}$$

$$T_2 = 194\text{ N}$$



The force $\mathbf{T}_2$ needed to

hold steady the person being rescued from the fire is less than her weight and less than the force $\mathbf{T}_1$ in the other rope, since the more vertical rope supports a greater part of her weight (a vertical force).

Exercise:

Problem:

Integrated Concepts A 35.0-kg dolphin decelerates from 12.0 to 7.50 m/s in 2.30 s to join another dolphin in play. What average force was exerted to slow him if he was moving horizontally? (The gravitational force is balanced by the buoyant force of the water.)

Exercise:

Problem:

Integrated Concepts When starting a foot race, a 70.0-kg sprinter exerts an average force of 650 N backward on the ground for 0.800 s. (a) What is his final speed? (b) How far does he travel?

Solution:

(a) $7.43\ \text{m/s}$

(b) 2.97 m

Exercise:

Problem:

Integrated Concepts A large rocket has a mass of $2.00 \times 10^6\ \text{kg}$ at takeoff, and its engines produce a thrust of $3.50 \times 10^7\ \text{N}$. (a) Find its initial acceleration if it takes off vertically. (b) How long does it take to reach a velocity of 120 km/h straight up, assuming constant mass and thrust? (c) In reality, the mass of a rocket decreases significantly as its fuel is consumed. Describe qualitatively how this affects the acceleration and time for this motion.

Exercise:

Problem:

Integrated Concepts A basketball player jumps straight up for a ball. To do this, he lowers his body 0.300 m and then accelerates through this distance by forcefully straightening his legs. This player leaves the floor with a vertical velocity sufficient to carry him 0.900 m above the floor. (a) Calculate his velocity when he leaves the floor. (b) Calculate his acceleration while he is straightening his legs. He goes from zero to the velocity found in part (a) in a distance of 0.300 m. (c) Calculate the force he exerts on the floor to do this, given that his mass is 110 kg.

Solution:

(a) $4.20\ \text{m/s}$

(b) $29.4\ \text{m/s}^2$

(c) $4.31 \times 10^3\ \text{N}$

Exercise:

Problem:

Integrated Concepts A 2.50-kg fireworks shell is fired straight up from a mortar and reaches a height of 110 m. (a) Neglecting air resistance (a poor assumption, but we will make it for this example), calculate the shell's velocity when it leaves the mortar. (b) The mortar itself is a tube 0.450 m long. Calculate the average acceleration of the shell in the tube as it goes from zero to the velocity found in (a). (c) What is the average force on the shell in the mortar? Express your answer in newtons and as a ratio to the weight of the shell.

Exercise:

Problem:

Integrated Concepts Repeat [link] for a shell fired at an angle 10.0º from the vertical.

Solution:

(a) 47.1 m/s

(b) $2.47 \times 10^3 \text{ m/s}^2$

(c) $6.18 \times 10^3 \text{ N}$. The average force is 252 times the shell's weight.

Exercise:

Problem:

Integrated Concepts An elevator filled with passengers has a mass of 1700 kg. (a) The elevator accelerates upward from rest at a rate of 1.20 m/s^2 for 1.50 s. Calculate the tension in the cable supporting the elevator. (b) The elevator continues upward at constant velocity for 8.50 s. What is the tension in the cable during this time? (c) The elevator decelerates at a rate of 0.600 m/s^2 for 3.00 s. What is the tension in the cable during deceleration? (d) How high has the elevator moved above its original starting point, and what is its final velocity?

Exercise:

Problem:

Unreasonable Results (a) What is the final velocity of a car originally traveling at 50.0 km/h that decelerates at a rate of $0.400\ \text{m/s}^2$ for 50.0 s? (b) What is unreasonable about the result? (c) Which premise is unreasonable, or which premises are inconsistent?

Exercise:

Problem:

Unreasonable Results A 75.0-kg man stands on a bathroom scale in an elevator that accelerates from rest to 30.0 m/s in 2.00 s. (a) Calculate the scale reading in newtons and compare it with his weight. (The scale exerts an upward force on him equal to its reading.) (b) What is unreasonable about the result? (c) Which premise is unreasonable, or which premises are inconsistent?

The First Condition for Equilibrium

- State the first condition of equilibrium.
- Explain static equilibrium.
- Explain dynamic equilibrium.

The first condition necessary to achieve equilibrium is the one already mentioned: the net external force on the system must be zero. Expressed as an equation, this is simply

Equation:

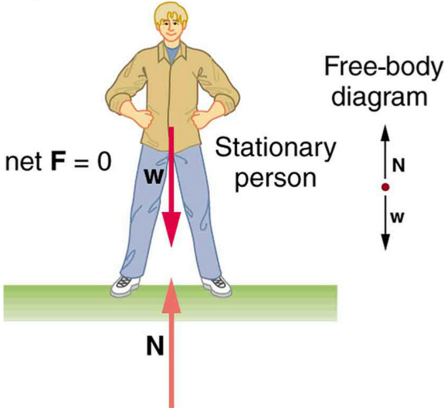
$$\text{net }\mathbf{F}=0$$

Note that if net **F** is zero, then the net external force in *any* direction is zero. For example, the net external forces along the typical *x*- and *y*-axes are zero. This is written as

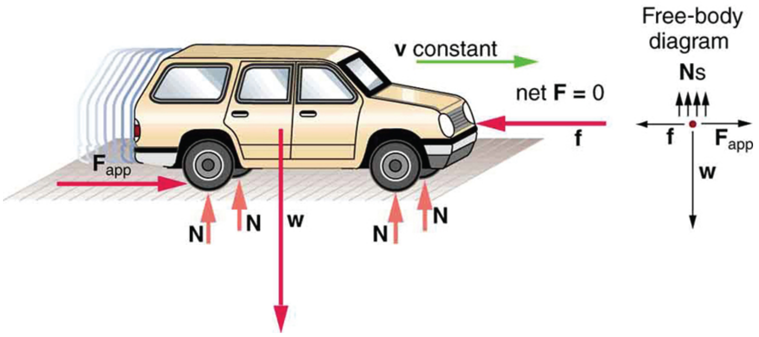
Equation:

$$\text{net }F_x=0\;\text{ and net }F_y=0$$

[link] and [link] illustrate situations where net $\mathbf{F}=0$ for both **static equilibrium** (motionless), and **dynamic equilibrium** (constant velocity).

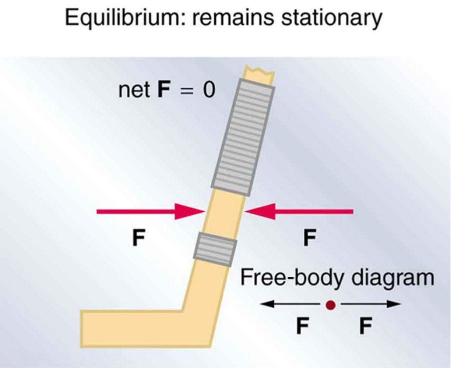


This motionless person is in static equilibrium. The forces acting on him add up to zero. Both forces are vertical in this case.

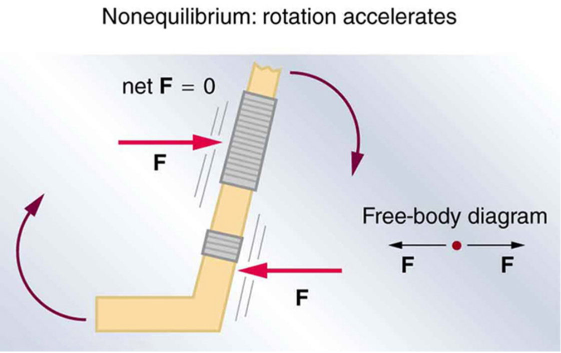


This car is in dynamic equilibrium because it is moving at constant velocity. There are horizontal and vertical forces, but the net external force in any direction is zero. The applied force $\mathbf{F}_{\text{app}}$ between the tires and the road is balanced by air friction, and the weight of the car is supported by the normal forces, here shown to be equal for all four tires.

However, it is not sufficient for the net external force of a system to be zero for a system to be in equilibrium. Consider the two situations illustrated in [link] and [link] where forces are applied to an ice hockey stick lying flat on ice. The net external force is zero in both situations shown in the figure; but in one case, equilibrium is achieved, whereas in the other, it is not. In [link], the ice hockey stick remains motionless. But in [link], with the same forces applied in different places, the stick experiences accelerated rotation. Therefore, we know that the point at which a force is applied is another factor in determining whether or not equilibrium is achieved. This will be explored further in the next section.



An ice hockey stick lying flat on ice with two equal and opposite horizontal forces applied to it. Friction is negligible, and the gravitational force is balanced by the support of the ice (a normal force). Thus, $\text{net }\mathbf{F} = 0$. Equilibrium is achieved, which is static equilibrium in this case.



The same forces are applied at other points and the stick rotates—in fact, it experiences an accelerated rotation. Here net $\mathbf{F} = 0$ but the system is *not* at equilibrium. Hence, the net $\mathbf{F} = 0$ is a necessary—but not sufficient—condition for achieving equilibrium.

Note:
PhET Explorations: Torque
Investigate how torque causes an object to rotate. Discover the relationships between angular acceleration, moment of inertia, angular momentum and torque. Click to open media in new browser.

Section Summary

- Statics is the study of forces in equilibrium.
- Two conditions must be met to achieve equilibrium, which is defined to be motion without linear or rotational acceleration.
- The first condition necessary to achieve equilibrium is that the net external force on the system must be zero, so that net $\mathbf{F} = 0$.

Conceptual Questions

Exercise:

Problem:

What can you say about the velocity of a moving body that is in dynamic equilibrium? Draw a sketch of such a body using clearly labeled arrows to represent all external forces on the body.

Exercise:

Problem:

Under what conditions can a rotating body be in equilibrium? Give an example.

Glossary

static equilibrium

a state of equilibrium in which the net external force and torque acting on a system is zero

dynamic equilibrium

a state of equilibrium in which the net external force and torque on a system moving with constant velocity are zero

Introduction to Statics and Torque class="introduction"

On a short time scale, rocks like these in Australia’s Kings Canyon are static, or motionless relative to the Earth. (credit: freeaussiestock.com)

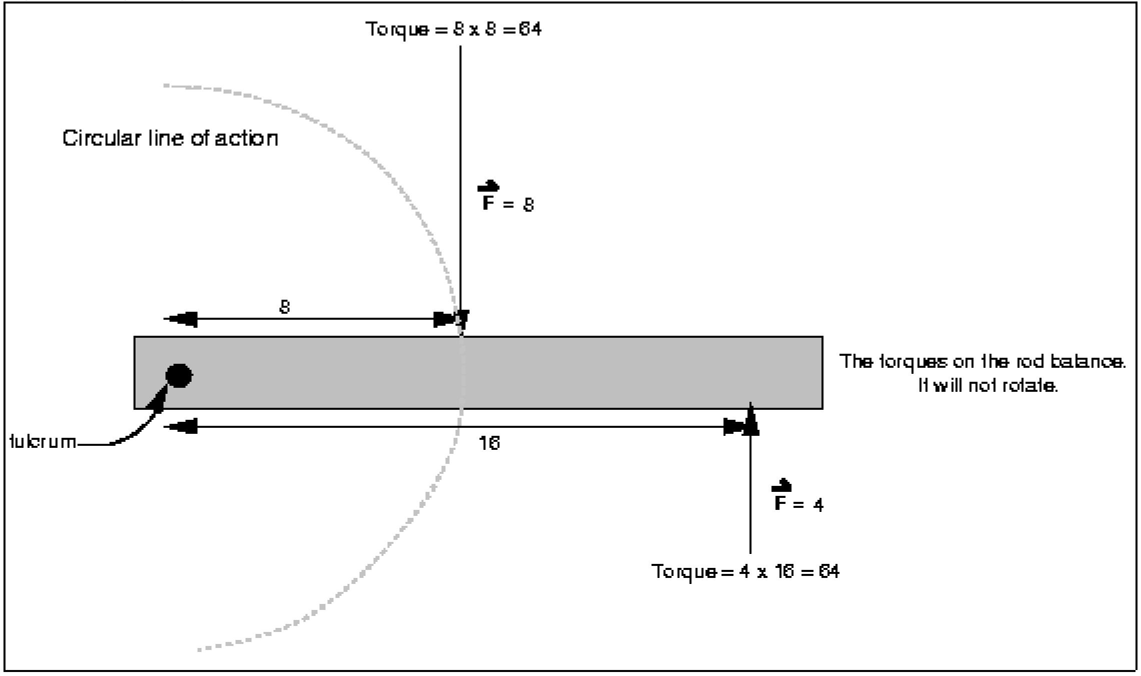
What might desks, bridges, buildings, trees, and mountains have in common—at least in the eyes of a physicist? The answer is that they are ordinarily motionless relative to the Earth. Furthermore, their acceleration is zero because they remain motionless. That means they also have something in common with a car moving at a constant velocity, because anything with a constant velocity also has an acceleration of zero. Now, the important part—Newton’s second law states that net , and so the net external force is zero for all stationary objects and for all objects moving at constant velocity. There are forces acting, but they are balanced. That is, they are in *equilibrium*.

Note:
Statics
Statics is the study of forces in equilibrium, a large group of situations that makes up a special case of Newton’s second law. We have already considered a few such situations; in this chapter, we cover the topic more thoroughly, including consideration of such possible effects as the rotation and deformation of an object by the forces acting on it.

How can we guarantee that a body is in equilibrium and what can we learn from systems that are in equilibrium? There are actually two conditions that must be satisfied to achieve equilibrium. These conditions are the topics of the first two sections of this chapter.
https://www.youtube.com/embed/gfZBqviOd-M

Torque

A **torque** is a force applied at a distance from a pivot point, such as an axle or shaft. When describing torques, one must include the magnitude and direction of the force, plus the perpendicular distance from the pivot where the force is applied. The magnitude of the torque is the **product** of the force and the distance from the pivot. Since torque is a product of force and distance, one may be "traded" for the other. The same torque can be created by applying a different force at a different distance from the pivot.



Illustrating Torque

Figure 1 shows a beam, such as a LEGO beam, with an axle pivot at one end, the fulcrum. The beam is free to rotate about the fulcrum, so a downward force will cause the beam to rotate downward, or clockwise, in Figure 1. The force of 8 units shown applied at a distance of 8 units from the fulcurm produces a downward, or clockwise, torque of 64 units.

If we want the beam to remain stationary, we must apply an equal torque in the upward direction. We can apply force at many points along the beam, but the force needed depends on where we apply it because of the force-distance trade-off of torque. We could apply an upward force at the same place as the downward force, having the same magnitude. Or, as shown, we could apply a smaller force, 4 units, at a larger distance, 16 units. The smaller force at a larger distance produces the same torque, 64 units. By the same reasoning, we would have to apply a larger force closer to the pivot to produce the same upward torque.

Figure 1 illustrates how a simple lever works and why it is useful. Applying a small force at a large distance from the pivot can produce or oppose a very large force close to the pivot. "Give me a long enough lever, and I can move the world."

In summary, **torque** is a force applied at a distance from a pivot. When describing torques, one must include magnitude, direction, and perpendicular distance from the pivot. Since torque is a **product** of force and distance, one may be "traded" for the other. The concept of trading distance traveled/applied for force experienced/applied is key to many simple machines.

Torque and levers

Torque and Levers

Torque

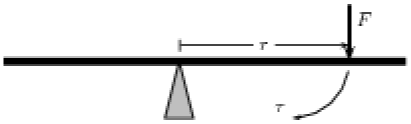
This chapter has dealt with forces and how they lead to motion in a straight line. In this section, we examine how forces lead to rotational motion.

When an object is fixed or supported at one point and a force acts on it a distance away from the support, it tends to make the object turn. The moment of force or *torque* (symbol, τ read *tau*) is defined as the product of the distance from the support or pivot (r) and the component of force perpendicular to the object, $F_{\perp}$.

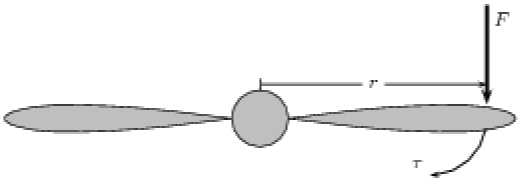
Equation:

$$\tau = F_{\perp} \cdot r$$

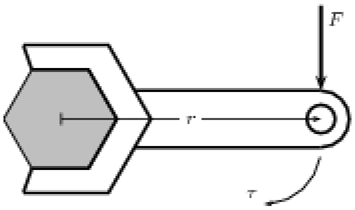
Torque can be seen as a rotational force. The unit of torque is N·m and torque is a vector quantity. Some examples of where torque arises are shown in Figures [link], [link] and [link].



The force exerted on one side of a see-saw causes it to swing.



The force exerted on the edge of a propellor causes the propellor to spin.



The force exerted on a spanner helps to loosen the bolt.

For example in [link], if a force F of 10 N is applied perpendicularly to the spanner at a distance r of 0,3 m from the center of the bolt, then the torque applied to the bolt is:

Equation:

$$\begin{aligned}\tau &= F_{\perp}\cdot r\\ &= (10\ \mathrm{N})(0,3\,\mathrm{m})\\ &= 3\,\mathrm{N}\cdot\mathrm{m}\end{aligned}$$

If the force of 10 N is now applied at a distance of 0,15 m from the centre of the bolt, then the torque is:

Equation:

$$\begin{aligned}\tau &= F_{\perp} \cdot r \\ &= (10\ \text{N})(0{,}15\,\text{m}) \\ &= 1{,}5\,\text{N} \cdot \text{m}\end{aligned}$$

This shows that there is less torque when the force is applied closer to the bolt than further away.

Note: Loosening a bolt

If you are trying to loosen (or tighten) a bolt, apply the force on the spanner further away from the bolt, as this results in a greater torque to the bolt making it easier to loosen.

Note: Any component of a force exerted parallel to an object will not cause the object to turn. Only perpendicular components cause turning.

Note: Torques

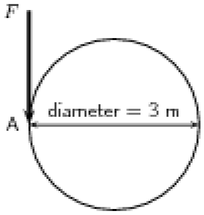
The direction of a torque is either clockwise or anticlockwise. When torques are added, choose one direction as positive and the opposite direction as negative. If equal clockwise and anticlockwise torques are applied to an object, they will cancel out and there will be no net turning effect.

Exercise:

Merry-go-round

Problem:

Several children are playing in the park. One child pushes the merry-go-round with a force of 50 N. The diameter of the merry-go-round is 3,0 m. What torque does the child apply if the force is applied perpendicularly at point A?



Solution:

Identify what has been given

The following has been given:

- the force applied, F = 50 N
- the diameter of the merry-go-round, $2r$ = 3 m, therefore r = 1,5 m.

The quantities are in SI units.

Decide how to approach the problem

We are required to determine the torque applied to the merry-go-round. We can do this by using:

Equation:

$$\tau = F_{\perp} \cdot r$$

We are given $F_{\perp}$ and we are given the diameter of the merry-go-round. Therefore, r = 1,5 m.

Solve the problem

Equation:

$$\begin{aligned}\tau &= F_{\perp} \cdot r \\ &= (50\,\mathrm{N})(1,5\,\mathrm{m}) \\ &= 75\,\mathrm{N}\cdot\mathrm{m}\end{aligned}$$

Write the final answer

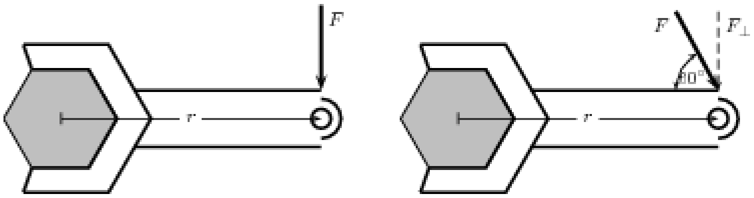
75 N · m of torque is applied to the merry-go-round.

Exercise:

Flat tyre

Problem:

Kevin is helping his dad replace the flat tyre on the car. Kevin has been asked to undo all the wheel nuts. Kevin holds the spanner at the same distance for all nuts, but applies the force at two angles (90° and 60°). If Kevin applies a force of 60 N, at a distance of 0,3 m away from the nut, which angle is the best to use? Prove your answer by means of calculations.



Solution:

Identify what has been given	The following has been given:	○ the force applied, F = 60 N ○ the angles at which the force is applied, $\theta = 90^\circ$ and $\theta = 60^\circ$ ○ the distance from the centre of the nut at which the force is applied, r = 0,3 m	The quantities are in SI units.
Decide how to approach the problem	We are required to determine which angle is more better to use. This means that we must find which angle gives the higher torque. We can use	**Equation:** $\tau = F_\perp \cdot r$	to determine the torque. We are given F for each situation. $F_\perp = F$ sin θ and we are given θ. We are also given the distance away from the nut, at which the force is applied.
Solve the problem for	$F_\perp = F$	**Equation:**	

$$\begin{aligned}\tau &= F_{\perp}\cdot r\\ &= (60\,\mathrm{N})(0,3\,\mathrm{m})\\ &= 18\,\mathrm{N}\cdot\mathrm{m}\end{aligned}$$

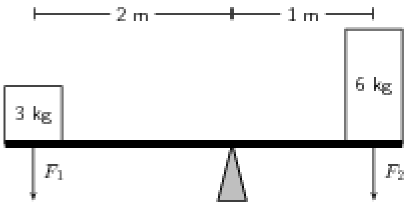
Solve the problem for **Equation:**

$$\begin{aligned}\tau &= F_{\perp}\cdot r\\ &= F\ \text{sin}\ \theta\cdot r\\ &= (60\,\mathrm{N})\ \text{sin}\ (\theta)(0,3\,\mathrm{m})\\ &= 15,6\,\mathrm{N}\cdot\mathrm{m}\end{aligned}$$

Write the final answer The torque from the perpendicular force is greater than the torque from the force applied at 60°. Therefore, the best angle is 90°.

Mechanical Advantage and Levers

We can use our knowlegde about the moments of forces (torque) to determine whether situations are balanced. For example two mass pieces are placed on a seesaw as shown in [link]. The one mass is 3 kg and the other is 6 kg. The masses are placed at distances of 2 m and 1 m (respectively) from the pivot. By looking at the clockwise and anti-clockwise moments, we can determine whether the seesaw will pivot (move) or not. If the sum of the clockwise and anti-clockwise moments is zero, the seesaw is in equilibrium (i.e. balanced).



The moments of
force are balanced.

The clockwise moment can be calculated as follows:

Equation:

$$\begin{aligned}\tau_1 &= F_\perp \cdot r \\ \tau_1 &= (6\ \text{kg})\left(9,8\,\text{m}\cdot\text{s}^{-2}\right)(1m) \\ \tau_1 &= 58,8N\cdot m\ \ \text{clockwise}\end{aligned}$$

The anti-clockwise moment can be calculated as follows:

Equation:

$$\begin{aligned}\tau_2 &= F_\perp \cdot r \\ \tau_2 &= (3\ \text{kg})\left(9,8\,\text{m}\cdot\text{s}^{-2}\right)(2m) \\ \tau_2 &= 58,8N\cdot m\ \text{anti}-\text{clockwise}\end{aligned}$$

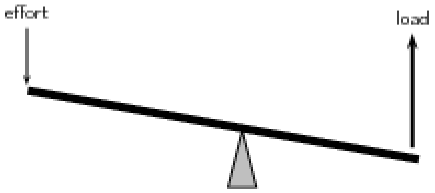
The sum of the moments of force will be zero:

The resultant moment is zero as the clockwise and anti-clockwise moments of force are in opposite directions and therefore cancel each other.

As we see in [link], we can use different distances away from a pivot to balance two different forces. This principle is applied to a lever to make lifting a heavy object much easier.

Lever

> A lever is a rigid object that is used with an appropriate fulcrum or pivot point to multiply the mechanical force that can be applied to another object.



A lever is used to put
in a small effort to
get out a large load.

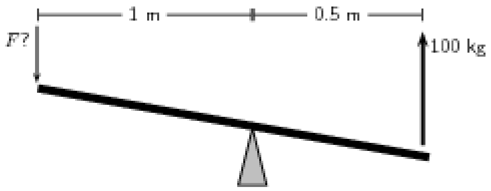
Note:
Interesting Fact
Archimedes reputedly said: *Give me a lever long enough and a fulcrum on which to place it, and I shall move the world.*

The concept of getting out more than the effort is termed mechanical advantage, and is one example of the principle of moments. The lever allows one to apply a smaller force over a greater distance. For instance to lift a certain unit of weight with a lever with an effort of half a unit we need a distance from the fulcrum in the effort's side to be twice the distance of the weight's side. It also means that to lift the weight 1 meter we need to push the lever for 2 meters. The amount of work done is always the same and independent of the dimensions of the lever (in an ideal lever). The lever only allows to trade force for distance.

Ideally, this means that the mechanical advantage of a system is the ratio of the force that performs the work (output or load) to the applied force (input or effort), assuming there is no friction in the system. In reality, the mechanical advantage will be less than the ideal value by an amount determined by the amount of friction.
Equation:

$$\text{mechanical advantage} = \frac{\text{load}}{\text{effort}}$$

For example, you want to raise an object of mass 100 kg. If the pivot is placed as shown in [link], what is the mechanical advantage of the lever?



A lever is used to put in a small effort to get out a large load.

In order to calculate mechanical advantage, we need to determine the load and effort.

Note: Effort is the input force and load is the output force.

The load is easy to calculate, it is simply the weight of the 100 kg object.
Equation:

$$F_{load} = m \cdot g = 100\,\text{kg} \cdot 9,8\,\text{m} \cdot \text{s}^{-2} = 980\,\text{N}$$

The effort is found by balancing torques.
Equation:

$$\begin{aligned} F_{load} \cdot r_{load} &= F_{effort} \cdot r_{effort} \\ 980\,\text{N} \cdot 0.5\,\text{m} &= F_{effort} \cdot 1\,\text{m} \\ F_{effort} &= \frac{980\,\text{N} \cdot 0.5\,\text{m}}{1\,\text{m}} \\ &= 490\,\text{N} \end{aligned}$$

The mechanical advantage is:
Equation:

$$\begin{aligned} \text{mechanical advantage} &= \frac{\text{load}}{\text{effort}} \\ &= \frac{980\,\text{N}}{490\,\text{N}} \\ &= 2 \end{aligned}$$

Since mechanical advantage is a ratio, it does not have any units.

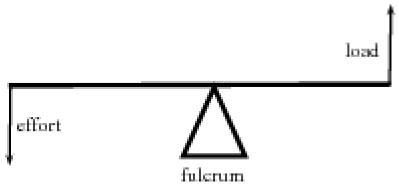
Pulleys

Pulleys change the direction of a tension force on a flexible material, e.g. a rope or cable. In addition, pulleys can be “added together” to create mechanical advantage, by having the flexible material looped over several pulleys in turn. More loops and pulleys increases the mechanical advantage.

Classes of levers

Class 1 levers

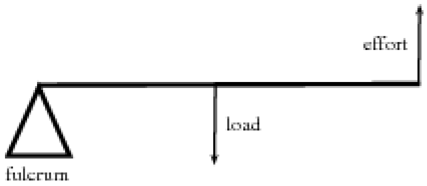
In a class 1 lever the fulcrum is between the effort and the load. Examples of class 1 levers are the seesaw, crowbar and equal-arm balance. The mechanical advantage of a class 1 lever can be increased by moving the fulcrum closer to the load.



Class 1 levers

Class 2 levers

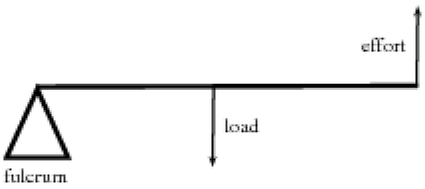
In class 2 levers the fulcrum is at the one end of the bar, with the load closer to the fulcrum and the effort on the other end of bar. The mechanical advantage of this type of lever can be increased by increasing the length of the bar. A bottle opener or wheel barrow are examples of class 2 levers.



Class 2 levers

Class 3 levers

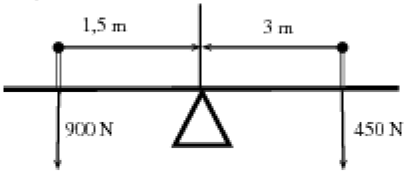
In class 3 levers the fulcrum is also at the end of the bar, but the effort is between the fulcrum and the load. An example of this type of lever is the human arm.



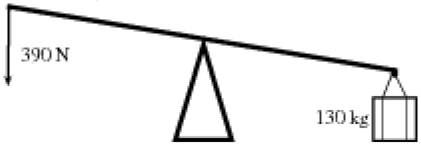
Class 3 levers

Exercise

1. Riyaad applies a force of 120 N on a spanner to undo a nut.
    1. Calculate the moment of the force if he applies the force 0,15 m from the bolt.
    2. The nut does not turn, so Riyaad moves his hand to the end of the spanner and applies the same force 0,2 m away from the bolt. Now the nut begins to move. Calculate the force. Is it bigger or smaller than before?
    3. Once the nuts starts to turn, the moment needed to turn it is less than it was to start it turning. It is now 20 N·m. Calculate the new moment of force that Riyaad now needs to apply 0,2 m away from the nut.
2. Calculate the clockwise and anticlockwise moments of force in the figure below to see if the see-saw is balanced.



3. Jeffrey uses a force of 390 N to lift a load of 130 kg.



    1. Calculate the mechanical advantage of the lever that he is using.

    2. What type of lever is he using? Give a reason for your answer.
    3. If the force is applied 1 m from the pivot, calculate the maximum distance between the pivot and the load.

4. A crowbar is used to lift a box of weight 400 N. The box is placed 75 cm from the pivot. A crow bar is a class 1 lever.

    1. Why is a crowbar a class 1 lever? Draw a diagram to explain your answer.
    2. What force F needs to be applied at a distance of 1,25 m from the pivot to balance the crowbar?
    3. If force F was applied at a distance of 2 m, what would the magnitude of F be?

5. A wheelbarrow is used to carry a load of 200 N. The load is 40 cm from the pivot and the force F is applied at a distance of 1,2 m from the pivot.

    1. What type of lever is a wheelbarrow?
    2. Calculate the force F that needs to be applied to lift the load.

6. The bolts holding a car wheel in place is tightened to a torque of 90 N · m. The mechanic has two spanners to undo the bolts, one with a length of 20 cm and one with a length of 30 cm. Which spanner should he use? Give a reason for your answer by showing calculations and explaining them.

The Second Condition for Equilibrium

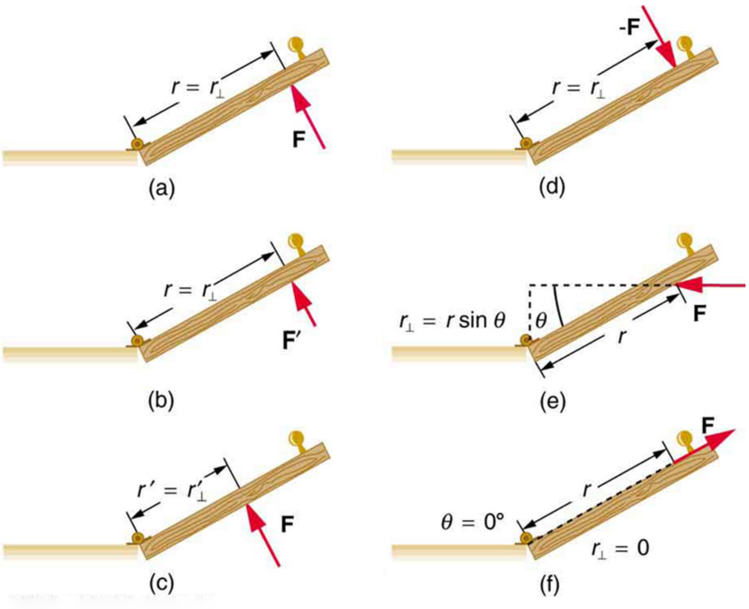
- State the second condition that is necessary to achieve equilibrium.
- Explain torque and the factors on which it depends.
- Describe the role of torque in rotational mechanics.

Note:

Torque

The second condition necessary to achieve equilibrium involves avoiding accelerated rotation (maintaining a constant angular velocity). A rotating body or system can be in equilibrium if its rate of rotation is constant and remains unchanged by the forces acting on it. To understand what factors affect rotation, let us think about what happens when you open an ordinary door by rotating it on its hinges.

Several familiar factors determine how effective you are in opening the door. See [link]. First of all, the larger the force, the more effective it is in opening the door—obviously, the harder you push, the more rapidly the door opens. Also, the point at which you push is crucial. If you apply your force too close to the hinges, the door will open slowly, if at all. Most people have been embarrassed by making this mistake and bumping up against a door when it did not open as quickly as expected. Finally, the direction in which you push is also important. The most effective direction is perpendicular to the door—we push in this direction almost instinctively.



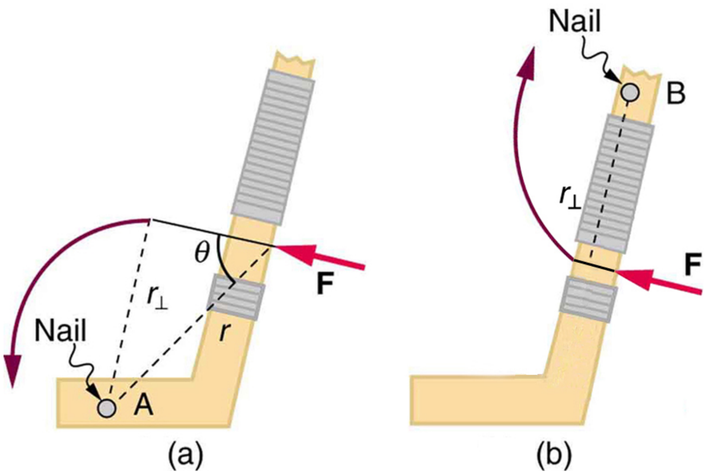
Torque is the turning or twisting effectiveness of a force, illustrated here for door rotation on its hinges (as viewed from overhead). Torque has both magnitude and direction. (a) Counterclockwise torque is produced by this force, which means that the door will rotate in a counterclockwise due to $\mathbf{F}$. Note that $r_\perp$ is the perpendicular distance of the pivot from the line of action of the force. (b) A smaller counterclockwise torque is produced by a smaller force $\mathbf{F}'$ acting at the same distance from the hinges (the pivot point). (c) The same force as in (a) produces a smaller counterclockwise torque when applied at a smaller distance from the hinges. (d) The same force as in (a), but acting in the opposite direction, produces a clockwise torque. (e) A smaller counterclockwise torque is produced by the same magnitude force acting at the same point but in a different direction. Here, θ is less than 90º. (f) Torque is zero here since the force just pulls on the hinges, producing no rotation. In this case, $\theta = 0$º.

The magnitude, direction, and point of application of the force are incorporated into the definition of the physical quantity called torque. **Torque** is the rotational equivalent of a force. It is a measure of the effectiveness of a force in changing or accelerating a rotation (changing the angular velocity over a period of time). In equation form, the magnitude of torque is defined to be

where τ (the Greek letter tau) is the symbol for torque, r is the distance from the pivot point to the point where the force is applied, F is the magnitude of the force, and θ is the angle between the force and the vector directed from the point of application to the pivot point, as seen in [link] and [link]. An alternative expression for torque is given in terms of the **perpendicular lever arm** $r_\perp$ as shown in [link] and [link], which is defined as

Equation:

$$\tau = r_\perp F.$$



(a) (b)

A force applied to an object can

produce a torque, which depends on the location of the pivot point. (a) The three factors $\mathbf{r}$, $\mathbf{F}$, and θ for pivot point A on a body are shown here—$\mathbf{r}$ is the distance from the chosen pivot point to the point where the force $\mathbf{F}$ is applied, and θ is the angle between $\mathbf{F}$ and the vector directed from the point of application to the pivot point. If the object can rotate around point A, it will rotate counterclockwise. This means that torque is counterclockwise relative to pivot A. (b) In this case, point B is the pivot point. The torque from the applied force will cause a clockwise rotation around point B, and so it is a clockwise torque relative to B.

The **SI unit of torque** is newtons times meters, usually written as $\text{N} \cdot \text{m}$. For example, if you push perpendicular to the door with a force of 40 N at a distance of 0.800 m from the hinges, you exert a torque of 32 N·m (0.800 m × 40 N) relative to the hinges. If you reduce the force to 20 N, the torque is reduced to $16\ \text{N}{\cdot}\text{m}$, and so on.

Note that for rotation in a plane, torque has two possible directions. Torque is either clockwise or counterclockwise relative to the chosen pivot point, as illustrated for points B and A, respectively, in [link]. If the object can rotate about point A, it will rotate counterclockwise, which means that the torque for the force is shown as counterclockwise relative to A. But if the object can rotate about point B, it will rotate clockwise, which means the torque for the force shown is clockwise relative to B. Also, the magnitude of the torque is greater when the lever arm is longer.

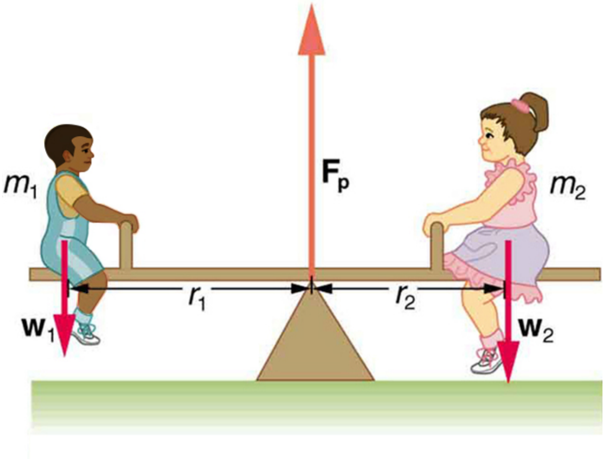
Now, *the second condition necessary to achieve equilibrium* is that *the net external torque on a system must be zero*. An external torque is one that is created by an external force. You can choose the point around which the torque is calculated. The point can be the physical pivot point of a system or any other point in space—but it must be the same point for all torques. If the second condition (net external torque on a system is zero) is satisfied for one choice of pivot point, it will also hold true for any other choice of pivot point in or out of the system of interest. (This is true only in an inertial frame of reference.) The second condition necessary to achieve equilibrium is stated in equation form as

Equation:

$$\text{net } \boldsymbol{\tau} = 0$$

where net means total. Torques, which are in opposite directions are assigned opposite signs. A common convention is to call counterclockwise (ccw) torques positive and clockwise (cw) torques negative.

When two children balance a seesaw as shown in [link], they satisfy the two conditions for equilibrium. Most people have perfect intuition about seesaws, knowing that the lighter child must sit farther from the pivot and that a heavier child can keep a lighter one off the ground indefinitely.



Two children balancing a seesaw satisfy both conditions for equilibrium. The lighter child sits farther from the pivot to create a torque equal in magnitude to that of the heavier child.

Example:

She Saw Torques On A Seesaw

The two children shown in [link] are balanced on a seesaw of negligible mass. (This assumption is made to keep the example simple—more involved examples will follow.) The first child has a mass of 26.0 kg and sits 1.60 m from the pivot.(a) If the second child has a mass of 32.0 kg, how far is she from the pivot? (b) What is F_{p}, the supporting force exerted by the pivot?

Strategy

Both conditions for equilibrium must be satisfied. In part (a), we are asked for a distance; thus, the second condition (regarding torques) must be used, since the first (regarding only forces) has no distances in it. To apply the second condition for equilibrium, we first identify the system of interest to be the seesaw plus the two children. We take the supporting pivot to be the point about which the torques are calculated. We then identify all external forces acting on the system.

Solution (a)

The three external forces acting on the system are the weights of the two children and the supporting force of the pivot. Let us examine the torque produced by each. The first torque is

Equation:

$$\tau_1 = r_1 w_1$$

second,

Equation:

$$\tau_2 = -r_2 w_2$$

and third,

Equation:

$$\begin{aligned}\tau_{\text{p}} &= r_{\text{p}}F_{\text{p}}\\ &= 0\cdot F_{\text{p}}\\ &= 0.\end{aligned}$$

Note that a minus sign has been inserted into the second equation because this torque is clockwise and is therefore negative by convention. Since F_{p} acts directly on the pivot point, the distance r_{p} is zero. A force acting on the pivot cannot cause a rotation, just as pushing directly on the hinges of a door will not cause it to rotate. Now, the second condition for equilibrium is that the sum of the torques on both children is zero. Therefore

Equation:

$$\tau_2 = -\tau_1,$$

or

Equation:

$$r_2 w_2 = r_1 w_1.$$

Weight is mass times the acceleration due to gravity. Entering mg for w, we get

Equation:

$$r_2 m_2 g = r_1 m_1 g.$$

Solve this for the unknown r_2:

Equation:

$$r_2 = r_1\frac{m_1}{m_2}.$$

The quantities on the right side of the equation are known; thus, r_2 is

Equation:

$$r_2 = (1.60\text{ m})\frac{26.0\text{ kg}}{32.0\text{ kg}} = 1.30\text{ m}.$$

As expected, the heavier child must sit closer to the pivot (1.30 m versus 1.60 m) to balance the seesaw.

Solution (b)

This part asks for a force F_{p}. The easiest way to find it is to use the first condition for equilibrium, which is

Equation:

$$\text{net }\mathbf{F}=0.$$

The forces are all vertical, so that we are dealing with a one-dimensional problem along the vertical axis; hence, the condition can be written as

Equation:

$$\text{net }F_y=0$$

where we again call the vertical axis the *y*-axis. Choosing upward to be the positive direction, and using plus and minus signs to indicate the directions of the forces, we see that

Equation:

$$F_{\text{p}}-w_1-w_2=0.$$

This equation yields what might have been guessed at the beginning:

Equation:

$$F_{\text{p}}=w_1+w_2.$$

So, the pivot supplies a supporting force equal to the total weight of the system:

Equation:

$$F_{\text{p}}=m_1g+m_2g.$$

Entering known values gives

Equation:

$$\begin{aligned} F_{\text{p}} &= (26.0\text{ kg})\left(9.80\text{ m/s}^2\right) + (32.0\text{ kg})\left(9.80\text{ m/s}^2\right) \\ &= 568\text{ N.} \end{aligned}$$

Discussion

The two results make intuitive sense. The heavier child sits closer to the pivot. The pivot supports the weight of the two children. Part (b) can also be solved using the second condition for equilibrium, since both distances are known, but only if the pivot point is chosen to be somewhere other than the location of the seesaw’s actual pivot!

Finally, note that the concept of torque has an importance beyond static equilibrium. *Torque plays the same role in rotational motion that force plays in linear motion.* We will examine this in the next chapter.

Note:

Take-Home Experiment

Take a piece of modeling clay and put it on a table, then mash a cylinder down into it so that a ruler can balance on the round side of the cylinder while everything remains still. Put a penny 8 cm away from the pivot. Where would you need to put two pennies to balance? Three pennies?

Section Summary

- The second condition assures those torques are also balanced. Torque is the rotational equivalent of a force in producing a rotation and is defined to be

  Equation:

  $$\tau = r_{\perp}F.$$

- The perpendicular lever arm $r_{\perp}$ is the shortest distance from the pivot point to the line along which F acts. The SI unit for torque is newton-meter (N·m). The second condition necessary to achieve equilibrium is that the net external torque on a system must be zero:
  Equation:

  $$\text{net }\boldsymbol{\tau}=0$$

  By convention, counterclockwise torques are positive, and clockwise torques are negative.

Conceptual Questions

Exercise:

Problem:

What three factors affect the torque created by a force relative to a specific pivot point?

Exercise:

Problem:

A wrecking ball is being used to knock down a building. One tall unsupported concrete wall remains standing. If the wrecking ball hits the wall near the top, is the wall more likely to fall over by rotating at its base or by falling straight down? Explain your answer. How is it most likely to fall if it is struck with the same force at its base? Note that this depends on how firmly the wall is attached at its base.

Exercise:

Problem:

Mechanics sometimes put a length of pipe over the handle of a wrench when trying to remove a very tight bolt. How does this help? (It is also hazardous since it can break the bolt.)

Problems & Exercises

Exercise:

Problem:

(a) When opening a door, you push on it perpendicularly with a force of 55.0 N at a distance of 0.850m from the hinges. What torque are you exerting relative to the hinges? (b) Does it matter if you push at the same height as the hinges?

Solution:

a) 46.8 N·m

b) It does not matter at what height you push. The torque depends on only the magnitude of the force applied and the perpendicular distance of the force's application from the hinges. (Children don't have a tougher time opening a door because they push lower than adults, they have a tougher time because they don't push far enough from the hinges.)

Exercise:

Problem:

When tightening a bolt, you push perpendicularly on a wrench with a force of 165 N at a distance of 0.140 m from the center of the bolt. (a) How much torque are you exerting in newton × meters (relative to the center of the bolt)? (b) Convert this torque to footpounds.

Exercise:

Problem:

Two children push on opposite sides of a door during play. Both push horizontally and perpendicular to the door. One child pushes with a force of 17.5 N at a distance of 0.600 m from the hinges, and the second child pushes at a distance of 0.450 m. What force must the second child exert to keep the door from moving? Assume friction is negligible.

Solution:

23.3 N

Exercise:

Problem:

Use the second condition for equilibrium $(\text{net } \tau = 0)$ to calculate F_p in [link], employing any data given or solved for in part (a) of the example.

Exercise:

Problem:

Repeat the seesaw problem in [link] with the center of mass of the seesaw 0.160 m to the left of the pivot (on the side of the lighter child) and assuming a mass of 12.0 kg for the seesaw. The other data given in the example remain unchanged. Explicitly show how you follow the steps in the Problem-Solving Strategy for static equilibrium.

Solution:

Given:

Equation:

$$\begin{array}{rcl} m_1 & = & 26.0\text{ kg}, m_2 = 32.0\text{ kg}, m_\text{s} = 12.0\text{ kg}, \\ r_1 & = & 1.60\text{ m}, r_\text{s} = 0.160\text{ m}, \text{find (a) } r_{2,} \text{ (b) } F_\text{p} \end{array}$$

a) Since children are balancing:

Equation:

$$\begin{array}{c} \text{net } \tau_\text{cw} = -\text{net } \tau_\text{ccw} \\ \Rightarrow w_1 r_1 + m_\text{s} g r_\text{s} = w_2 r_2 \end{array}$$

So, solving for r_2 gives:
Equation:

$$\begin{aligned} r_2 &= \frac{w_1 r_1 + m_s g r_s}{w_2} = \frac{m_1 g r_1 + m_s g r_s}{m_2 g} = \frac{m_1 r_1 + m_s r_s}{m_2} \\ &= \frac{(26.0\text{ kg})(1.60\text{ m}) + (12.0\text{ kg})(0.160\text{ m})}{32.0\text{ kg}} \\ &= 1.36\text{ m} \end{aligned}$$

b) Since the children are not moving:
Equation:

$$\text{net } F = 0 = F_p - w_1 - w_2 - w_s$$
$$\Rightarrow F_p = w_1 + w_2 + w_s$$

So that
Equation:

$$\begin{aligned} F_p &= (26.0\text{ kg} + 32.0\text{ kg} + 12.0\text{ kg})(9.80\text{ m/s}^2) \\ &= 686\text{ N} \end{aligned}$$

Glossary

torque	turning or twisting effectiveness of a force
perpendicular lever arm	the shortest distance from the pivot point to the line along which **F** lies
SI units of torque	newton times meters, usually written as N·m
center of gravity	the point where the total weight of the body is assumed to be concentrated

Centripetal Force

- Calculate coefficient of friction on a car tire.
- Calculate ideal speed and angle of a car on a turn.

Any force or combination of forces can cause a centripetal or radial acceleration. Just a few examples are the tension in the rope on a tether ball, the force of Earth’s gravity on the Moon, friction between roller skates and a rink floor, a banked roadway’s force on a car, and forces on the tube of a spinning centrifuge.

Any net force causing uniform circular motion is called a **centripetal force**. The direction of a centripetal force is toward the center of curvature, the same as the direction of centripetal acceleration. According to Newton’s second law of motion, net force is mass times acceleration: net $\mathrm{F} = \mathrm{ma}$. For uniform circular motion, the acceleration is the centripetal acceleration — $a = a_c$. Thus, the magnitude of centripetal force $\mathrm{F_c}$ is

Equation:

$$\mathrm{F_c} = ma_{\mathrm{c}}.$$

By using the expressions for centripetal acceleration a_c from $a_c = \frac{v^2}{r};$ we get an expression for the centripetal force $\mathrm{F_c}$ in terms of mass, velocity, and radius of curvature:

Equation:

$$F_c = m\frac{v^2}{r};$$

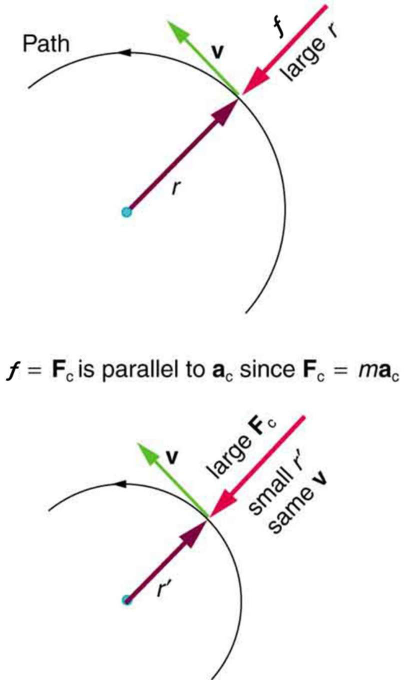
You may use whichever expression for centripetal force is more convenient. Centripetal force F_{c} is always perpendicular to the path and pointing to the center of curvature, because $\mathbf{a}_c$ is perpendicular to the velocity and pointing to the center of curvature.

Note that if you solve the first expression for r, you get

Equation:

$$r = \frac{mv^2}{F_c}.$$

This implies that for a given mass and velocity, a large centripetal force causes a small radius of curvature—that is, a tight curve.


Path
v
f
large r
r
$f = \mathbf{F}_c$ is parallel to $\mathbf{a}_c$ since $\mathbf{F}_c = m\mathbf{a}_c$
v
large $\mathbf{F}_c$
small r′
same v
r′

The frictional force supplies the centripetal force and is numerically equal to it. Centripetal force is perpendicular to velocity and causes uniform circular motion. The larger the $\mathrm{F_c}$, the smaller the radius of curvature r and the sharper the curve. The second curve has the same v, but a larger $\mathrm{F_c}$ produces a smaller $r\prime$.

Note:
Take-Home Experiment
Ask a friend or relative to swing a golf club or a tennis racquet. Take appropriate measurements to estimate the centripetal acceleration of the end of the club or racquet. You may choose to do this in slow motion.

Note:
PhET Explorations: Gravity and Orbits
Move the sun, earth, moon and space station to see how it affects their gravitational forces and orbital paths. Visualize the sizes and distances between different heavenly bodies, and turn off gravity to see what would happen without it!
https://phet.colorado.edu/sims/html/gravity-and-orbits/latest/gravity-and-orbits_en.html

Section Summary

- Centripetal force $\mathrm{F_c}$ is any force causing uniform circular motion. It is a “center-seeking” force that always points toward the center of rotation. It is perpendicular to linear velocity v and has magnitude
  Equation:

$$F_\mathrm{c} = ma_\mathrm{c},$$

which can also be expressed as
Equation:

$$\left. \begin{array}{r} F_{\text{c}} = m\frac{v^2}{r} \\ , \\ \end{array} \right\}$$

Conceptual Questions

Exercise:

Problem:

If you wish to reduce the stress (which is related to centripetal force) on high-speed tires, would you use large- or small-diameter tires? Explain.

Exercise:

Problem:

Define centripetal force. Can any type of force (for example, tension, gravitational force, friction, and so on) be a centripetal force? Can any combination of forces be a centripetal force?

Exercise:

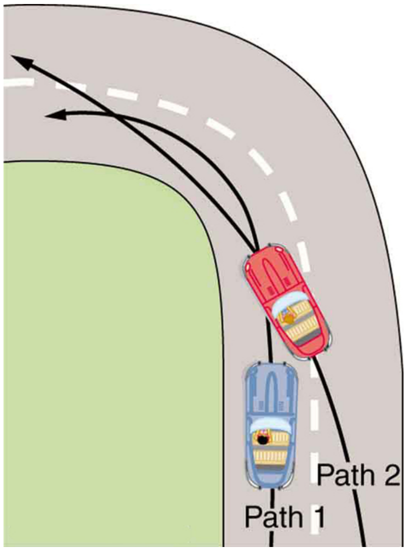
Problem:

If centripetal force is directed toward the center, why do you feel that you are ‘thrown’ away from the center as a car goes around a curve? Explain.

Exercise:

Problem:

Race car drivers routinely cut corners as shown in [link]. Explain how this allows the curve to be taken at the greatest speed.



Two paths around a race track curve are shown. Race car drivers will take the inside path (called cutting the corner) whenever possible because it allows them to take the curve at the highest speed.

Exercise:

Problem:

What is the direction of the force exerted by the car on the passenger as the car goes over the top of the amusement ride pictured in [link] under the following circumstances:

(a) The car goes over the top at such a speed that the gravitational force is the only force acting?

(b) The car goes over the top faster than this speed?

(c) The car goes over the top slower than this speed?

Exercise:

Problem:

As a skater forms a circle, what force is responsible for making her turn? Use a free body diagram in your answer.

Exercise:

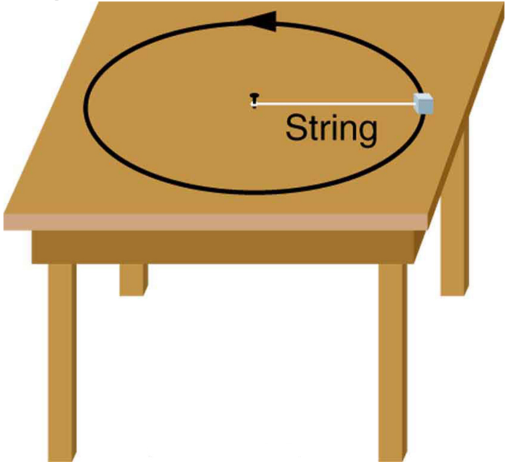
Problem:

Do you feel yourself thrown to either side when you negotiate a curve that is ideally banked for your car's speed? What is the direction of the force exerted on you by the car seat?

Exercise:

Problem:

Suppose a mass is moving in a circular path on a frictionless table as shown in figure. In the Earth's frame of reference, there is no centrifugal force pulling the mass away from the centre of rotation, yet there is a very real force stretching the string attaching the mass to the nail. Using concepts related to centripetal force and Newton's third law, explain what force stretches the string, identifying its physical origin.



A mass attached to a nail
on a frictionless table

moves in a circular path.
The force stretching the
string is real and not
fictional. What is the
physical origin of the
force on the string?

Problems Exercise

Exercise:

Problem:

(a) A 22.0 kg child is riding a playground merry-go-round that is rotating at 40.0 rev/min. What centripetal force must she exert to stay on if she is 1.25 m from its center?

(b) What centripetal force does she need to stay on an amusement park merry-go-round that rotates at 3.00 rev/min if she is 8.00 m from its center?

(c) Compare each force with her weight.

Solution:

a) 483 N

b) 17.4 N

c) 2.24 times her weight, 0.0807 times her weight

Glossary

centripetal force
any net force causing uniform circular motion

ideal banking

the sloping of a curve in a road, where the angle of the slope allows the vehicle to negotiate the curve at a certain speed without the aid of friction between the tires and the road; the net external force on the vehicle equals the horizontal centripetal force in the absence of friction

ideal speed

the maximum safe speed at which a vehicle can turn on a curve without the aid of friction between the tire and the road

ideal angle

the angle at which a car can turn safely on a steep curve, which is in proportion to the ideal speed

banked curve

the curve in a road that is sloping in a manner that helps a vehicle negotiate the curve

Work: The Scientific Definition

LEARNING OBJECTIVES

By the end of this section, you will be able to:

- Explain how an object must be displaced for a force on it to do work.
- Explain how relative directions of force and displacement of an object determine whether the work done on the object is positive, negative, or zero.

The information presented in this section supports the following AP® learning objectives and science practices:

- **5.B.5.1** The student is able to design an experiment and analyze data to examine how a force exerted on an object or system does work on the object or system as it moves through a distance. **(S.P. 4.2, 5.1)**
- **5.B.5.2** The student is able to design an experiment and analyze graphical data in which interpretations of the area under a force-distance curve are needed to determine the work done on or by the object or system. **(S.P. 4.5, 5.1)**
- **5.B.5.3** The student is able to predict and calculate from graphical data the energy transfer to or work done on an object or system from information about a force exerted on the object or system through a distance. **(S.P. 1.5, 2.2, 6.4)**

What It Means to Do Work

The scientific definition of work differs in some ways from its everyday meaning. Certain things we think of as hard work, such as writing an exam or carrying a heavy load on level ground, are not work as defined by a scientist. The scientific definition of work reveals its relationship to energy—whenever work is done, energy is transferred.

For work, in the scientific sense, to be done on an object, a force must be exerted on that object and there must be displacement of that object in the direction of the force.

Formally, the **work** done on a system by a constant force is defined to be *the product of the component of the force in the direction of motion and the distance through which the force acts*. For a constant force, this is expressed in equation form as

Read cosθ = parallel or horizontal components
Equation:

$$W = \mid \mathbf{F} \mid (\cos\theta) \mid \mathbf{d} \mid,$$

where W is work, $\mathbf{d}$ is the displacement of the system, and θ is the angle between the force vector $\mathbf{F}$ and the displacement vector $\mathbf{d}$, as in [link]. We can also write this as
Equation:

$$W = \text{Fd}\cos\theta.$$

To find the work done on a system that undergoes motion that is not one-way or that is in two or three dimensions, we divide the motion into one-way one-dimensional segments and add up the work done over each segment.

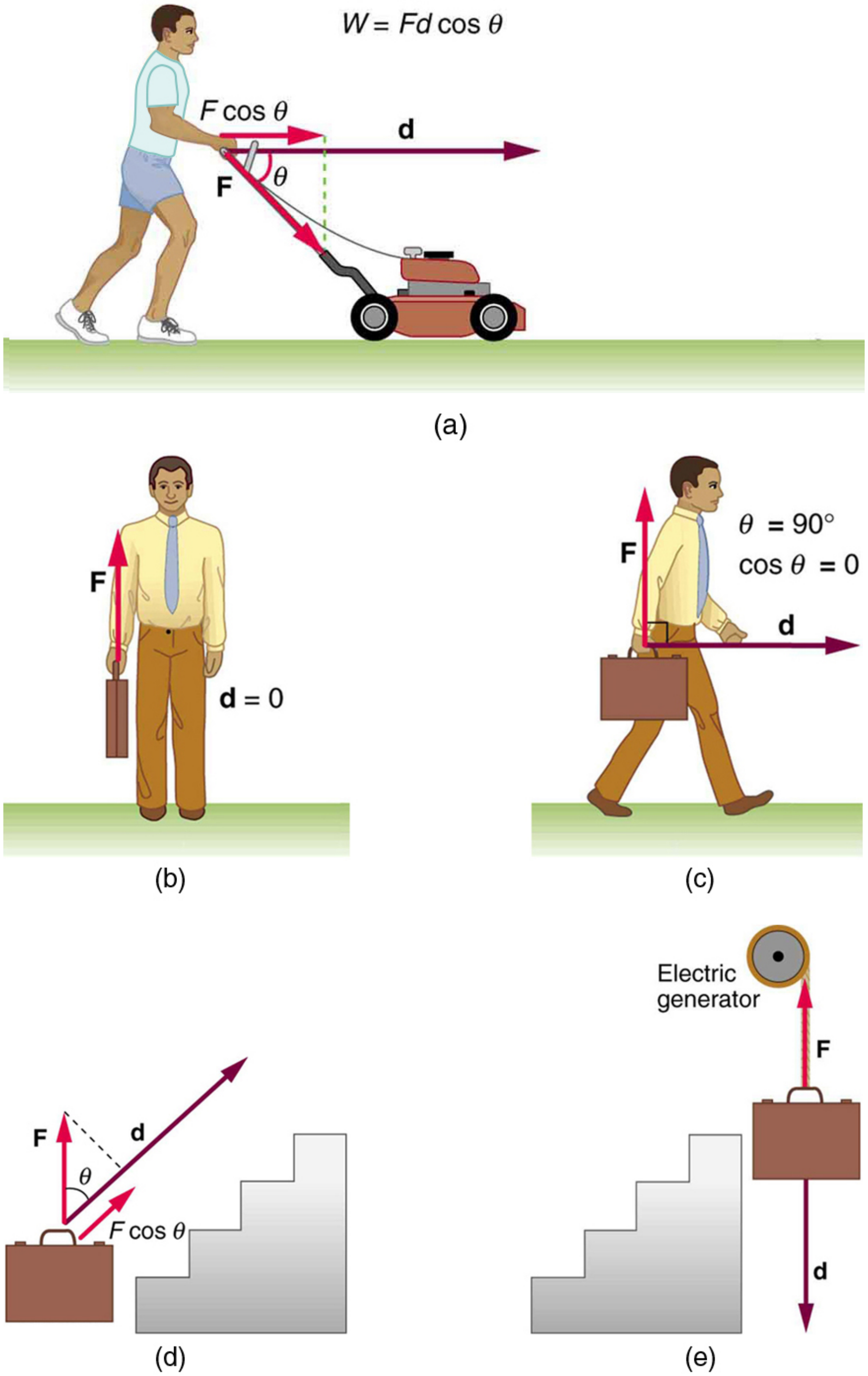
Note:
What is Work?

The work done on a system by a constant force is *the product of the component of the force in the direction of motion times the distance through which the force acts*. For one-way motion in one dimension, this is expressed in equation form as

Equation:

$$W = \mathrm{Fd}\cos\theta,$$

where W is work, F is the magnitude of the force on the system, d is the magnitude of the displacement of the system, and θ is the angle between the force vector $\mathbf{F}$ and the displacement vector $\mathbf{d}$.



Examples of work. (a) The work done by the force $\mathbf{F}$ on this lawn mower is $\mathrm{Fd}\cos\theta$. Note that $F\cos\theta$ is the component of the force in the direction of motion. (b) A person holding a briefcase does no work on it, because there is no displacement. No energy is transferred to or from the briefcase. (c) The person moving the briefcase horizontally at a constant speed does no work on it, and transfers no energy to it. (d) Work *is* done on the briefcase by carrying it up stairs at constant speed, because there is necessarily a component of force $\mathbf{F}$ in the direction of the motion. Energy is transferred to the briefcase and could in turn be used to do work. (e) When the briefcase is lowered, energy is transferred out of the briefcase and into an electric generator. Here the work done on the briefcase by the generator is negative, removing energy from the briefcase, because $\mathbf{F}$ and $\mathbf{d}$ are in opposite directions.

Section Summary

- Work is the transfer of energy by a force acting on an object as it is displaced.
- The work W that a force $\mathbf{F}$ does on an object is the product of the magnitude F of the force, times the magnitude d of the displacement, times the cosine of the angle θ between them. In symbols,
  Equation:

$$W = \text{Fd}\cos\theta.$$

- The SI unit for work and energy is the joule (J), where $1\ \text{J} = 1\ \text{N}\cdot\text{m} = 1\ \text{kg}\cdot\text{m}^2/\text{s}^2$.
- The work done by a force is zero if the displacement is either zero or perpendicular to the force.
- The work done is positive if the force and displacement have the same direction, and negative if they have opposite direction.

Conceptual Questions

Exercise:

Problem:

Give an example of something we think of as work in everyday circumstances that is not work in the scientific sense. Is energy transferred or changed in form in your example? If so, explain how this is accomplished without doing work.

Exercise:

Problem:

Give an example of a situation in which there is a force and a displacement, but the force does no work. Explain why it does no work.

Exercise:

Problem:

Describe a situation in which a force is exerted for a long time but does no work. Explain.

Problems & Exercises

Exercise:

Problem:

How much work does a supermarket checkout attendant do on a can of soup he pushes 0.600 m horizontally with a force of 5.00 N? Express your answer in joules and kilocalories.

Solution:

$3.00 \text{ J} = 7.17 \times 10^{-4} \text{ kcal}$

Exercise:

Problem:

A 75.0-kg person climbs stairs, gaining 2.50 meters in height. Find the work done to accomplish this task.

Exercise:

Problem:

(a) Calculate the work done on a 1500-kg elevator car by its cable to lift it 40.0 m at constant speed, assuming friction averages 100 N. (b) What is the work done on the lift by the gravitational force in this process? (c) What is the total work done on the lift?

Solution:

(a) $5.92 \times 10^5 \text{ J}$

(b) $-5.88 \times 10^5 \text{ J}$

(c) The net force is zero.

Exercise:

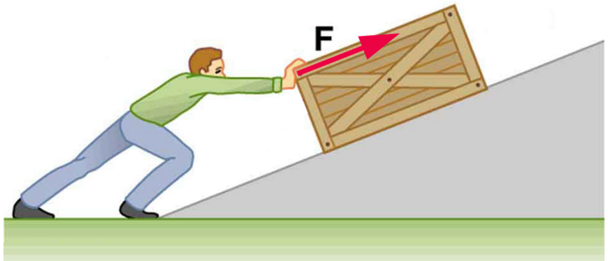
Problem:

Suppose a car travels 108 km at a speed of 30.0 m/s, and uses 2.0 gal of gasoline. Only 30% of the gasoline goes into useful work by the force that keeps the car moving at constant speed despite friction. (See [link] for the energy content of gasoline.) (a) What is the magnitude of the force exerted to keep the car moving at constant speed? (b) If the required force is directly proportional to speed, how many gallons will be used to drive 108 km at a speed of 28.0 m/s?

Exercise:

Problem:

Calculate the work done by an 85.0-kg man who pushes a crate 4.00 m up along a ramp that makes an angle of 20.0° with the horizontal. (See [link].) He exerts a force of 500 N on the crate parallel to the ramp and moves at a constant speed. Be certain to include the work he does on the crate *and* on his body to get up the ramp.



A man pushes a crate up a ramp.

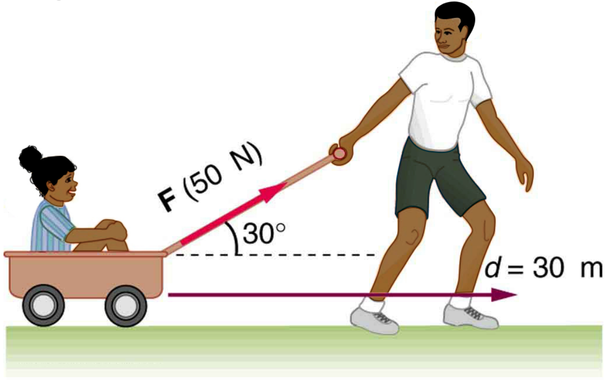
Solution:

3.14×10^3 J

Exercise:

Problem:

How much work is done by the boy pulling his sister 30.0 m in a wagon as shown in [link]? Assume no friction acts on the wagon.



The boy does work on the system of the wagon and the child when he pulls them as shown.

Exercise:

Problem:

A shopper pushes a grocery cart 20.0 m at constant speed on level ground, against a 35.0 N frictional force. He pushes in a direction 25.0° below the horizontal. (a) What is the work done on the cart by friction? (b) What is the work done on the cart by the gravitational force? (c) What is the work done on the cart by the shopper? (d) Find the force the shopper exerts, using energy considerations. (e) What is the total work done on the cart?

Solution:

(a) -700 J

(b) 0

(c) 700 J

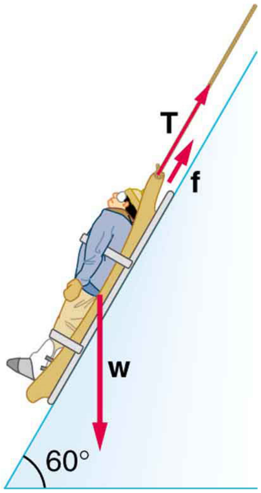
(d) 38.6 N

(e) 0

Exercise:

Problem:

Suppose the ski patrol lowers a rescue sled and victim, having a total mass of 90.0 kg, down a 60.0º slope at constant speed, as shown in [link]. The coefficient of friction between the sled and the snow is 0.100. (a) How much work is done by friction as the sled moves 30.0 m along the hill? (b) How much work is done by the rope on the sled in this distance? (c) What is the work done by the gravitational force on the sled? (d) What is the total work done?



A rescue sled and victim are lowered down a steep slope.

Glossary

energy
the ability to do work

work

the transfer of energy by a force that causes an object to be displaced; the product of the component of the force in the direction of the displacement and the magnitude of the displacement

joule

SI unit of work and energy, equal to one newton-meter

Kinetic Energy and the Work-Energy Theorem

LEARNING OBJECTIVES

By the end of this section, you will be able to:

- Explain work as a transfer of energy and net work as the work done by the net force.
- Explain and apply the work-energy theorem.

The information presented in this section supports the following AP® learning objectives and science practices:

- **3.E.1.1** The student is able to make predictions about the changes in kinetic energy of an object based on considerations of the direction of the net force on the object as the object moves. **(S.P. 6.4, 7.2)**
- **3.E.1.2** The student is able to use net force and velocity vectors to determine qualitatively whether kinetic energy of an object would increase, decrease, or remain unchanged. **(S.P. 1.4)**
- **3.E.1.3** The student is able to use force and velocity vectors to determine qualitatively or quantitatively the net force exerted on an object and qualitatively whether kinetic energy of that object would increase, decrease, or remain unchanged. **(S.P. 1.4, 2.2)**
- **3.E.1.4** The student is able to apply mathematical routines to determine the change in kinetic energy of an object given the forces on the object and the displacement of the object. **(S.P. 2.2)**
- **4.C.1.1** The student is able to calculate the total energy of a system and justify the mathematical routines used in the

calculation of component types of energy within the system whose sum is the total energy. **(S.P. 1.4, 2.1, 2.2)**
- **4.C.2.1** The student is able to make predictions about the changes in the mechanical energy of a system when a component of an external force acts parallel or antiparallel to the direction of the displacement of the center of mass. **(S.P. 6.4)**
- **4.C.2.2** The student is able to apply the concepts of conservation of energy and the work-energy theorem to determine qualitatively and/or quantitatively that work done on a two-object system in linear motion will change the kinetic energy of the center of mass of the system, the potential energy of the systems, and/or the internal energy of the system. **(S.P. 1.4, 2.2, 7.2)**
- **5.B.5.3** The student is able to predict and calculate from graphical data the energy transfer to or work done on an object or system from information about a force exerted on the object or system through a distance. **(S.P. 1.5, 2.2, 6.4)**

Net Work and the Work-Energy Theorem

We know from the study of Newton’s laws in Dynamics: Force and Newton's Laws of Motion that net force causes acceleration. We will see in this section that work done by the net force gives a system energy of motion, and in the process we will also find an expression for the energy of motion.

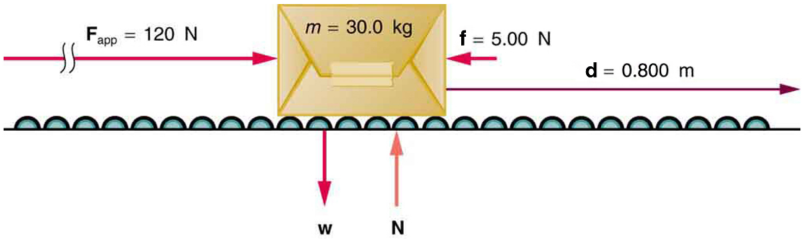
Let us start by considering the total, or net, work done on a system. Net work is defined to be the sum of work done by all external forces—that is, **net work** is the work done by the net external force $\mathbf{F}_{\text{net}}$. In equation form, this is $W_{\text{net}} = F_{\text{net}} d \cos \theta$ where θ is the angle between the force vector and the displacement vector.

Note:

Real World Connections: Work and Direction

Consider driving in a car. While moving, you have forward velocity and therefore kinetic energy. When you hit the brakes, they exert a force opposite to your direction of motion (acting through the wheels). The brakes do work on your car and reduce the kinetic energy. Similarly, when you accelerate, the engine (acting through the wheels) exerts a force in the direction of motion. The engine does work on your car, and increases the kinetic energy. Finally, if you go around a corner at a constant speed, you have the same kinetic energy both before and after the corner. The force exerted by the engine was perpendicular to the direction of motion, and therefore did no work and did not change the kinetic energy.

Net work will be simpler to examine if we consider a one-dimensional situation where a force is used to accelerate an object in a direction parallel to its initial velocity. Such a situation occurs for the package on the roller belt conveyor system shown in [link].



A package on a roller belt is pushed horizontally through a distance **d**.

The force of gravity and the normal force acting on the package are perpendicular to the displacement and do no work. Moreover, they are also equal in magnitude and opposite in direction so they cancel in calculating the net force. The net force arises solely from the horizontal applied force

$\mathbf{F}_{\text{app}}$ and the horizontal friction force $\mathbf{f}$. Thus, as expected, the net force is parallel to the displacement, so that $\theta = 0^{\text{o}}$ and $\cos\theta = 1$, and the net work is given by

Equation:

$$W_{\text{net}} = F_{\text{net}}d.$$

The effect of the net force $\mathbf{F}_{\text{net}}$ is to accelerate the package from v_0 to v. The kinetic energy of the package increases, indicating that the net work done on the system is positive. (See [link].) By using Newton’s second law, and doing some algebra, we can reach an interesting conclusion. Substituting $F_{\text{net}} = \text{ma}$ from Newton’s second law gives

Equation:

$$W_{\text{net}} = \text{mad}.$$

To get a relationship between net work and the speed given to a system by the net force acting on it, we take $d = x - x_0$ and use the equation studied in Motion Equations for Constant Acceleration in One Dimension for the change in speed over a distance d if the acceleration has the constant value a; namely, $v^2 = {v_0}^2 + 2\text{ad}$ (note that a appears in the expression for the net work). Solving for acceleration gives $a = \frac{v^2 - {v_0}^2}{2d}$. When a is substituted into the preceding expression for W_{net}, we obtain

Equation:

$$W_{\text{net}} = m\left(\frac{v^2 - {v_0}^2}{2d}\right)d.$$

The d cancels, and we rearrange this to obtain

Equation:

$$W_{\text{ net}} = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2.$$

This expression is called the **work-energy theorem**, and it actually applies *in general* (even for forces that vary in direction and magnitude), although we have derived it for the special case of a constant force parallel to the displacement. The theorem implies that the net work on a system equals the change in the quantity $\frac{1}{2}mv^2$. This quantity is our first example of a form of energy.

Note:
The Work-Energy Theorem
The net work on a system equals the change in the quantity $\frac{1}{2}mv^2$.
Equation:

$$W_{\text{net}} = \frac{1}{2}mv^2 - \frac{1}{2}\text{mv}_0^2$$

The quantity $\frac{1}{2}mv^2$ in the work-energy theorem is defined to be the translational **kinetic energy** (KE) of a mass m moving at a speed v. (*Translational* kinetic energy is distinct from *rotational* kinetic energy, which is considered later.) In equation form, the translational kinetic energy,
Equation:

$$\text{KE} = \frac{1}{2}mv^2,$$

is the energy associated with translational motion. Kinetic energy is a form of energy associated with the motion of a particle, single body, or system of objects moving together.

We are aware that it takes energy to get an object, like a car or the package in [link], up to speed, but it may be a bit surprising that kinetic energy is proportional to speed squared. This proportionality means, for example, that a car traveling at 100 km/h has four times the kinetic energy it has at 50 km/h, helping to explain why high-speed collisions are so devastating. We will now consider a series of examples to illustrate various aspects of work and energy.

Example:
Calculating the Kinetic Energy of a Package

Suppose a 30.0-kg package on the roller belt conveyor system in [link] is moving at 0.500 m/s. What is its kinetic energy?

Strategy

Because the mass m and speed v are given, the kinetic energy can be calculated from its definition as given in the equation $\mathrm{KE} = \frac{1}{2}mv^2$.

Solution

The kinetic energy is given by

Equation:

$$\mathrm{KE} = \frac{1}{2}mv^2.$$

Entering known values gives

Equation:

$$\mathrm{KE} = 0.5(30.0\ \mathrm{kg})(0.500\ \mathrm{m/s})^2,$$

which yields

Equation:

$$\mathrm{KE} = 3.75\ \mathrm{kg \cdot m^2/s^2} = 3.75\ \mathrm{J}.$$

Discussion

Note that the unit of kinetic energy is the joule, the same as the unit of work, as mentioned when work was first defined. It is also interesting that, although this is a fairly massive package, its kinetic energy is not large at this relatively low speed. This fact is consistent with the observation that people can move packages like this without exhausting themselves.

Example:

Determining Speed from Work and Energy

Find the speed of the package in [link] at the end of the push, using work and energy concepts.

Strategy

Here the work-energy theorem can be used, because we have just calculated the net work, W_{net}, and the initial kinetic energy, $\frac{1}{2}m{v_0}^2$. These calculations allow us to find the final kinetic energy, $\frac{1}{2}mv^2$, and thus the final speed v.

Solution

The work-energy theorem in equation form is

Equation:

$$W_{\text{net}} = \frac{1}{2}mv^2 - \frac{1}{2}m{v_0}^2.$$

Solving for $\frac{1}{2}mv^2$ gives

Equation:

$$\frac{1}{2}\text{mv}^2 = W_{\text{net}} + \frac{1}{2}m{v_0}^2.$$

Thus,

Equation:

$$\frac{1}{2}mv^2 = 92.0\text{ J} + 3.75\text{ J} = 95.75\text{ J}.$$

Solving for the final speed as requested and entering known values gives

Equation:

$$\begin{aligned} v &= \sqrt{\frac{2(95.75\text{ J})}{m}} = \sqrt{\frac{191.5\text{ kg}\cdot\text{m}^2/\text{s}^2}{30.0\text{ kg}}} \\ &= 2.53\text{ m/s}. \end{aligned}$$

Discussion

Using work and energy, we not only arrive at an answer, we see that the final kinetic energy is the sum of the initial kinetic energy and the net work done on the package. This means that the work indeed adds to the energy of the package.

Some of the examples in this section can be solved without considering energy, but at the expense of missing out on gaining insights about what work and energy are doing in this situation. On the whole, solutions involving energy are generally shorter and easier than those using kinematics and dynamics alone.

Section Summary

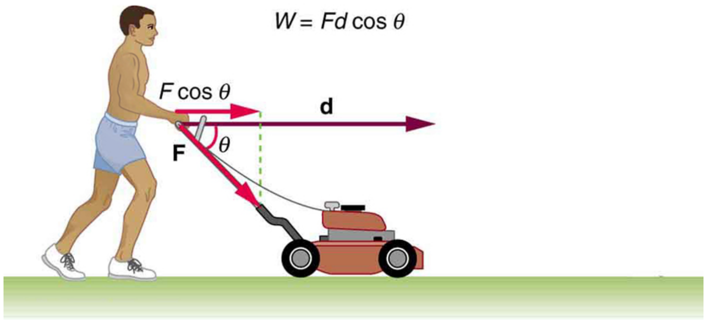
- The net work W_{net} is the work done by the net force acting on an object.
- Work done on an object transfers energy to the object.
- The translational kinetic energy of an object of mass m moving at speed v is $\text{KE} = \frac{1}{2}mv^2$.
- The work-energy theorem states that the net work W_{net} on a system changes its kinetic energy, $W_{\text{net}} = \frac{1}{2}mv^2 - \frac{1}{2}m{v_0}^2$.

Conceptual Questions

Exercise:

Problem:

The person in [link] does work on the lawn mower. Under what conditions would the mower gain energy? Under what conditions would it lose energy?



Exercise:

Problem:

Work done on a system puts energy into it. Work done by a system removes energy from it. Give an example for each statement.

Exercise:

Problem:

When solving for speed in [link], we kept only the positive root. Why?

Problems & Exercises

Exercise:

Problem:

Compare the kinetic energy of a 20,000-kg truck moving at 110 km/h with that of an 80.0-kg astronaut in orbit moving at 27,500 km/h.

Solution:

$1/250$

Exercise:

Problem:

(a) How fast must a 3000-kg elephant move to have the same kinetic energy as a 65.0-kg sprinter running at 10.0 m/s? (b) Discuss how the larger energies needed for the movement of larger animals would relate to metabolic rates.

Exercise:

Problem:

Confirm the value given for the kinetic energy of an aircraft carrier in [link]. You will need to look up the definition of a nautical mile (1 knot = 1 nautical mile/h).

Solution:

1.1×10^{10} J

Exercise:

Problem:

(a) Calculate the force needed to bring a 950-kg car to rest from a speed of 90.0 km/h in a distance of 120 m (a fairly typical distance for a non-panic stop). (b) Suppose instead the car hits a concrete abutment at full speed and is brought to a stop in 2.00 m. Calculate the force exerted on the car and compare it with the force found in part (a).

Exercise:

Problem:

A car’s bumper is designed to withstand a 4.0-km/h (1.1-m/s) collision with an immovable object without damage to the body of the car. The bumper cushions the shock by absorbing the force over a distance. Calculate the magnitude of the average force on a bumper that collapses 0.200 m while bringing a 900-kg car to rest from an initial speed of 1.1 m/s.

Solution:

2.8×10^3 N

Exercise:

Problem:

Boxing gloves are padded to lessen the force of a blow. (a) Calculate the force exerted by a boxing glove on an opponent’s face, if the glove and face compress 7.50 cm during a blow in which the 7.00-kg arm and glove are brought to rest from an initial speed of 10.0 m/s. (b) Calculate the force exerted by an identical blow in the gory old days when no gloves were used and the knuckles and face would compress only 2.00 cm. (c) Discuss the magnitude of the force with glove on. Does it seem high enough to cause damage even though it is lower than the force with no glove?

Exercise:

Problem:

Using energy considerations, calculate the average force a 60.0-kg sprinter exerts backward on the track to accelerate from 2.00 to 8.00 m/s in a distance of 25.0 m, if he encounters a headwind that exerts an average force of 30.0 N against him.

Solution:

102 N

Glossary

net work
work done by the net force, or vector sum of all the forces, acting on an object

work-energy theorem
the result, based on Newton’s laws, that the net work done on an object is equal to its change in kinetic energy

kinetic energy

the energy an object has by reason of its motion, equal to $\frac{1}{2}mv^2$ for the translational (i.e., non-rotational) motion of an object of mass m moving at speed v

Gravitational Potential Energy

- Explain gravitational potential energy in terms of work done against gravity.
- Show that the gravitational potential energy of an object of mass m at height h on Earth is given by $\text{PE}_\text{g} = \text{mgh}$.
- Show how knowledge of the potential energy as a function of position can be used to simplify calculations and explain physical phenomena.

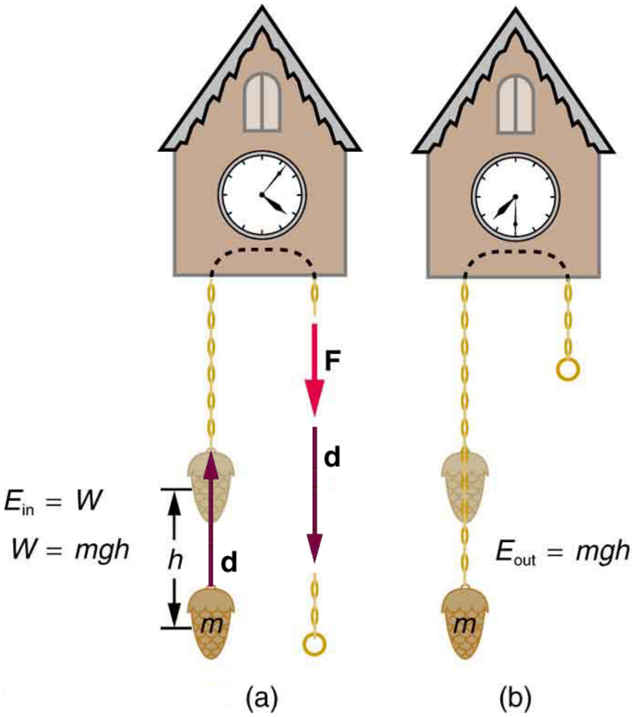
Work Done Against Gravity

Climbing stairs and lifting objects is work in both the scientific and everyday sense—it is work done against the gravitational force. When there is work, there is a transformation of energy. The work done against the gravitational force goes into an important form of stored energy that we will explore in this section.

Let us calculate the work done in lifting an object of mass m through a height h, such as in [link]. If the object is lifted straight up at constant speed, then the force needed to lift it is equal to its weight mg. The work done on the mass is then $\text{W} = \text{Fd} = \text{mgh}$. We define this to be the **gravitational potential energy** (PE_g) put into (or gained by) the object-Earth system. This energy is associated with the state of separation between two objects that attract each other by the gravitational force. For convenience, we refer to this as the PE_g gained by the object, recognizing that this is energy stored in the gravitational field of Earth. Why do we use the word “system”? Potential energy is a property of a system rather than of a single object—due to its physical position. An object’s gravitational potential is due to its position relative to the surroundings within the Earth-object system. The force applied to the object is an external force, from outside the system. When it does positive work it increases the gravitational potential energy of the system. Because gravitational potential energy depends on relative position, we need a reference level at which to set the potential energy equal to 0. We usually choose this point to be Earth’s surface, but this point is arbitrary; what is important is the *difference* in gravitational potential energy, because this difference is what relates to the work done. The difference in gravitational potential energy of an object (in the Earth-object system) between two rungs of a ladder will be the same for the first two rungs as for the last two rungs.

Converting Between Potential Energy and Kinetic Energy

Gravitational potential energy may be converted to other forms of energy, such as kinetic energy. If we release the mass, gravitational force will do an amount of work equal to mgh on it, thereby increasing its kinetic energy by that same amount (by the work-energy theorem). We will find it more useful to consider just the conversion of PE_g to KE without explicitly considering the intermediate step of work. (See [link].) This shortcut makes it is easier to solve problems using energy (if possible) rather than explicitly using forces.



(a) The work done to lift the weight is stored in the mass-Earth system as gravitational potential energy. (b) As the weight moves downward, this gravitational potential energy is transferred to the cuckoo clock.

More precisely, we define the *change* in gravitational potential energy ΔPE_g to be

Equation:

$$\Delta\text{PE}_\text{g} = \text{mgh},$$

where, for simplicity, we denote the change in height by h rather than the usual Δh. Note that h is positive when the final height is greater than the initial height, and vice versa. For example, if a 0.500-kg mass hung from a cuckoo clock is raised 1.00 m, then its change in gravitational potential energy is

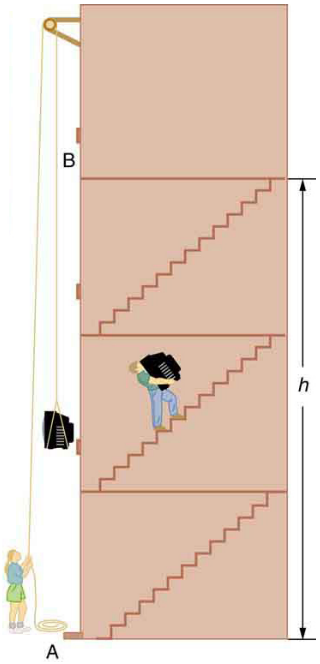
Equation:

$$\begin{aligned} mgh &= (0.500\text{ kg})\left(9.80\text{ m/s}^2\right)(1.00\text{ m}) \\ &= 4.90\text{ kg}\cdot\text{m}^2/\text{s}^2 = 4.90\text{ J}. \end{aligned}$$

Note that the units of gravitational potential energy turn out to be joules, the same as for work and other forms of energy. As the clock runs, the mass is lowered. We can think of the mass as gradually giving up its 4.90 J of gravitational potential energy, *without directly considering the force of gravity that does the work*.

Using Potential Energy to Simplify Calculations

The equation $\Delta\text{PE}_\text{g} = \text{mgh}$ applies for any path that has a change in height of h, not just when the mass is lifted straight up. (See [link].) It is much easier to calculate mgh (a simple multiplication) than it is to calculate the work done along a complicated path. The idea of gravitational potential energy has the double advantage that it is very broadly applicable and it makes calculations easier. From now on, we will consider that any change in vertical position h of a mass m is accompanied by a change in gravitational potential energy mgh, and we will avoid the equivalent but more difficult task of calculating work done by or against the gravitational force.



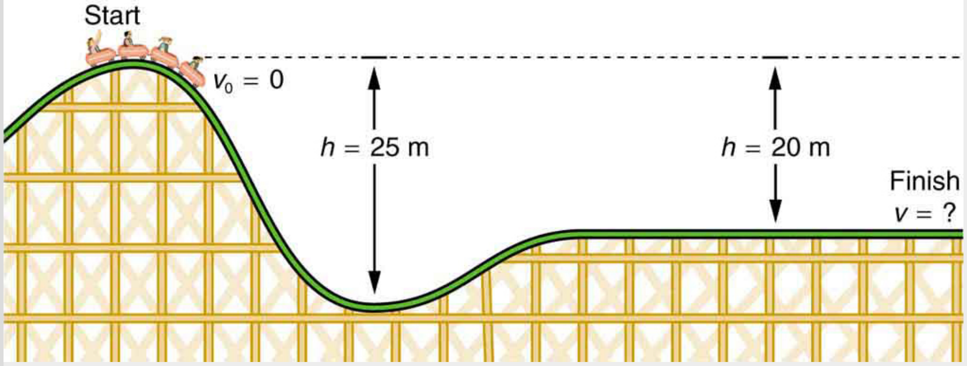
The change in gravitational potential energy $(\Delta \mathrm{PE_g})$ between points A and B is independent of the path. $\Delta \mathrm{PE_g} = \mathrm{mgh}$ for any path between the two points. Gravity is one of a small class of forces where the work done by or against the force depends only on the starting and ending points, not on the path between them.

The work done by the ground upon the kangaroo reduces its kinetic energy to zero as it lands. However, by applying the force of the ground on the hind legs over a longer distance, the impact on the bones is reduced. (credit: Chris Samuel, Flickr)

Example:

Finding the Speed of a Roller Coaster from its Height

(a) What is the final speed of the roller coaster shown in [link] if it starts from rest at the top of the 20.0 m hill and work done by frictional forces is negligible? (b) What is its final speed (again assuming negligible friction) if its initial speed is 5.00 m/s?



The speed of a roller coaster increases as gravity pulls it downhill and is greatest at its lowest point. Viewed in terms of energy, the roller-coaster-Earth system's gravitational potential energy is converted to kinetic energy. If work done by friction is negligible, all ΔPE_g is converted to KE.

Strategy

The roller coaster loses potential energy as it goes downhill. We neglect friction, so that the remaining force exerted by the track is the normal force, which is perpendicular to the direction of motion and does no work. The net work on the roller coaster is then done by gravity alone. The *loss* of gravitational potential energy from moving *downward* through a distance h equals the *gain* in kinetic energy. This can be written in equation form as $-\Delta\text{PE}_\text{g} = \Delta\text{KE}$. Using the equations for PE_g and KE, we can solve for the final speed v, which is the desired quantity.

Solution for (a)

Here the initial kinetic energy is zero, so that $\Delta\text{KE} = \frac{1}{2}mv^2$. The equation for change in potential energy states that $\Delta\text{PE}_\text{g} = mgh$. Since h is negative in this case, we will rewrite this as $\Delta\text{PE}_\text{g} = -mg \mid h \mid$ to show the minus sign clearly. Thus,

Equation:

$$-\Delta\text{PE}_\text{g} = \Delta\text{KE}$$

becomes

Equation:

$$\text{mg} \mid h \mid = \frac{1}{2}mv^2.$$

Solving for v, we find that mass cancels and that

Equation:

$$v=\sqrt{2g\mid h\mid}.$$

Substituting known values,

Equation:

$$\begin{aligned} v &= \sqrt{2\left(9.80\ \text{m/s}^2\right)(20.0\ \text{m})} \\ &= 19.8\ \text{m/s}. \end{aligned}$$

Solution for (b)

Again $-\Delta\text{PE}_\text{g}=\Delta\text{KE}$. In this case there is initial kinetic energy, so $\Delta\text{KE}=\frac{1}{2}mv^2-\frac{1}{2}m{v_0}^2$. Thus,

Equation:

$$\text{mg}\mid h\mid=\frac{1}{2}mv^2-\frac{1}{2}m{v_0}^2.$$

Rearranging gives

Equation:

$$\frac{1}{2}mv^2=\text{mg}\mid h\mid+\frac{1}{2}m{v_0}^2.$$

This means that the final kinetic energy is the sum of the initial kinetic energy and the gravitational potential energy. Mass again cancels, and

Equation:

$$v=\sqrt{2g\mid h\mid+{v_0}^2}.$$

This equation is very similar to the kinematics equation $v=\sqrt{{v_0}^2+2\text{ad}}$, but it is more general—the kinematics equation is valid only for constant acceleration, whereas our equation above is valid for any path regardless of whether the object moves with a constant acceleration. Now, substituting known values gives

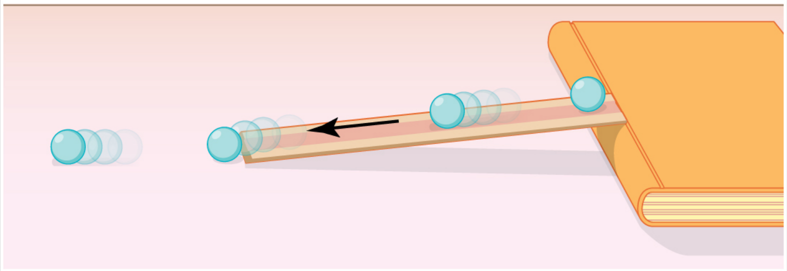
Equation:

$$\begin{aligned} v &= \sqrt{2(9.80\ \text{m/s}^2)(20.0\ \text{m}) + (5.00\ \text{m/s})^2} \\ &= 20.4\ \text{m/s}. \end{aligned}$$

Note:

Making Connections: Take-Home Investigation—Converting Potential to Kinetic Energy

One can study the conversion of gravitational potential energy into kinetic energy in this experiment. On a smooth, level surface, use a ruler of the kind that has a groove running along its length and a book to make an incline (see [link]). Place a marble at the 10-cm position on the ruler and let it roll down the ruler. When it hits the level surface, measure the time it takes to roll one meter. Now place the marble at the 20-cm and the 30-cm positions and again measure the times it takes to roll 1 m on the level surface. Find the velocity of the marble on the level surface for all three positions. Plot velocity squared versus the distance traveled by the marble. What is the shape of each plot? If the shape is a straight line, the plot shows that the marble's kinetic energy at the bottom is proportional to its potential energy at the release point.



A marble rolls down a ruler, and its speed on the level surface is measured.

Section Summary

- Work done against gravity in lifting an object becomes potential energy of the object-Earth system.
- The change in gravitational potential energy, ΔPE_g, is $\Delta\text{PE}_\text{g} = \text{mgh}$, with h being the increase in height and g the acceleration due to gravity.

- The gravitational potential energy of an object near Earth’s surface is due to its position in the mass-Earth system. Only differences in gravitational potential energy, $\Delta \mathrm{PE}_\mathrm{g}$, have physical significance.
- As an object descends without friction, its gravitational potential energy changes into kinetic energy corresponding to increasing speed, so that $\Delta \mathrm{KE} = -\Delta \mathrm{PE}_\mathrm{g}$.

Conceptual Questions

Exercise:

Problem:

In [link], we calculated the final speed of a roller coaster that descended 20 m in height and had an initial speed of 5 m/s downhill. Suppose the roller coaster had had an initial speed of 5 m/s *uphill* instead, and it coasted uphill, stopped, and then rolled back down to a final point 20 m below the start. We would find in that case that its final speed is the same as its initial speed. Explain in terms of conservation of energy.

Exercise:

Problem:

Does the work you do on a book when you lift it onto a shelf depend on the path taken? On the time taken? On the height of the shelf? On the mass of the book?

Problems & Exercises

Exercise:

Problem:

A hydroelectric power facility (see [link]) converts the gravitational potential energy of water behind a dam to electric energy. (a) What is the gravitational potential energy relative to the generators of a lake of volume $50.0\ \mathrm{km}^3$ ($\mathrm{mass} = 5.00 \times 10^{13}\ \mathrm{kg}$), given that the lake has an average height of 40.0 m above the generators? (b) Compare this with the energy stored in a 9-megaton fusion bomb.

Hydroelectric facility (credit: Denis Belevich, Wikimedia Commons)

Solution:

(a) 1.96×10^{16} J

(b) The ratio of gravitational potential energy in the lake to the energy stored in the bomb is 0.52. That is, the energy stored in the lake is approximately half that in a 9-megaton fusion bomb.

Exercise:

Problem:

(a) How much gravitational potential energy (relative to the ground on which it is built) is stored in the Great Pyramid of Cheops, given that its mass is about 7×10^9 kg and its center of mass is 36.5 m above the surrounding ground? (b) How does this energy compare with the daily food intake of a person?

Exercise:

Problem:

Suppose a 350-g kookaburra (a large kingfisher bird) picks up a 75-g snake and raises it 2.5 m from the ground to a branch. (a) How much work did the bird do on the snake? (b) How much work did it do to raise its own center of mass to the branch?

Solution:

(a) 1.8 J

(b) 8.6 J

Exercise:

Problem:

In [link], we found that the speed of a roller coaster that had descended 20.0 m was only slightly greater when it had an initial speed of 5.00 m/s than when it started from rest. This implies that $\Delta\text{PE} >> \text{KE}_\text{i}$. Confirm this statement by taking the ratio of ΔPE to KE_i. (Note that mass cancels.)

Exercise:

Problem:

A 100-g toy car is propelled by a compressed spring that starts it moving. The car follows the curved track in [link]. Show that the final speed of the toy car is 0.687 m/s if its initial speed is 2.00 m/s and it coasts up the frictionless slope, gaining 0.180 m in altitude.

A toy car moves up a sloped track.
(credit: Leszek Leszczynski, Flickr)

Solution:
Equation:

$$v_f = \sqrt{2\text{gh} + {v_0}^2} = \sqrt{2(9.80\ \text{m/s}^2)(-0.180\ \text{m}) + (2.00\ \text{m/s})^2} = 0.687\ \text{m/s}$$

Exercise:

Problem:

In a downhill ski race, surprisingly, little advantage is gained by getting a running start. (This is because the initial kinetic energy is small compared with the gain in gravitational potential energy on even small hills.) To demonstrate this, find the final speed and the time taken for a skier who skies 70.0 m along a 30º slope neglecting friction: (a) Starting from rest. (b) Starting with an initial speed of 2.50 m/s. (c) Does the answer surprise you? Discuss why it is still advantageous to get a running start in very competitive events.

Glossary

gravitational potential energy	the energy an object has due to its position in a gravitational field

Conservative Forces and Potential Energy

- Define conservative force, potential energy, and mechanical energy.
- Explain the potential energy of a spring in terms of its compression when Hooke’s law applies.
- Use the work-energy theorem to show how having only conservative forces implies conservation of mechanical energy.

Potential Energy and Conservative Forces

Work is done by a force, and some forces, such as weight, have special characteristics. A **conservative force** is one, like the gravitational force, for which work done by or against it depends only on the starting and ending points of a motion and not on the path taken. We can define a **potential energy** (PE) for any conservative force, just as we did for the gravitational force. For example, when you wind up a toy, an egg timer, or an old-fashioned watch, you do work against its spring and store energy in it. (We treat these springs as ideal, in that we assume there is no friction and no production of thermal energy.) This stored energy is recoverable as work, and it is useful to think of it as potential energy contained in the spring. Indeed, the reason that the spring has this characteristic is that its force is *conservative*. That is, a conservative force results in stored or potential energy. Gravitational potential energy is one example, as is the energy stored in a spring. We will also see how conservative forces are related to the conservation of energy.

Note:
Potential Energy and Conservative Forces
Potential energy is the energy a system has due to position, shape, or configuration. It is stored energy that is completely recoverable.
A conservative force is one for which work done by or against it depends only on the starting and ending points of a motion and not on the path taken.
We can define a potential energy (PE) for any conservative force. The work done against a conservative force to reach a final configuration

depends on the configuration, not the path followed, and is the potential energy added.

Conservation of Mechanical Energy

Let us now consider what form the work-energy theorem takes when only conservative forces are involved. This will lead us to the conservation of energy principle. The work-energy theorem states that the net work done by all forces acting on a system equals its change in kinetic energy. In equation form, this is

Equation:

$$W_{\text{net}} = \frac{1}{2}mv^2 - \frac{1}{2}m{v_0}^2 = \Delta\text{KE}.$$

If only conservative forces act, then

Equation:

$$W_{\text{net}} = W_{\text{c}},$$

where W_{c} is the total work done by all conservative forces. Thus,

Equation:

$$W_{\text{c}} = \Delta\text{KE}.$$

Now, if the conservative force, such as the gravitational force or a spring force, does work, the system loses potential energy. That is, $W_{\text{c}} = -\Delta\text{PE}$. Therefore,

Equation:

$$-\Delta\text{PE} = \Delta\text{KE}$$

or

Equation:

$$\Delta\text{KE} + \Delta\text{PE} = 0.$$

This equation means that the total kinetic and potential energy is constant for any process involving only conservative forces. That is,

Equation:

$$\left.\begin{array}{ll} & \text{KE} + \text{PE} = \text{constant} \\ \text{or} & \\ & \text{KE}_\text{i} + \text{PE}_\text{i} = \text{KE}_\text{f} + \text{PE}_\text{f} \end{array}\right\}(\text{conservative forces only}),$$

where i and f denote initial and final values. This equation is a form of the work-energy theorem for conservative forces; it is known as the **conservation of mechanical energy** principle. Remember that this applies to the extent that all the forces are conservative, so that friction is negligible. The total kinetic plus potential energy of a system is defined to be its **mechanical energy**, $(\text{KE} + \text{PE})$. In a system that experiences only conservative forces, there is a potential energy associated with each force, and the energy only changes form between KE and the various types of PE, with the total energy remaining constant.

Note that, for conservative forces, we do not directly calculate the work they do; rather, we consider their effects through their corresponding potential energies, just as we did in [link]. Note also that we do not consider details of the path taken—only the starting and ending points are important (as long as the path is not impossible). This assumption is usually a tremendous simplification, because the path may be complicated and forces may vary along the way.

Note:
PhET Explorations: Energy Skate Park
Learn about conservation of energy with a skater dude! Build tracks, ramps and jumps for the skater and view the kinetic energy, potential energy and

friction as he moves. You can also take the skater to different planets or even space! https://phet.colorado.edu/sims/html/energy-skate-park-basics/latest/energy-skate-park-basics_en.html

Section Summary

- A conservative force is one for which work depends only on the starting and ending points of a motion, not on the path taken.
- We can define potential energy (PE) for any conservative force, just as we defined PE_g for the gravitational force.
- The potential energy of a spring is $\text{PE}_\text{s} = \frac{1}{2}kx^2$, where k is the spring’s force constant and x is the displacement from its undeformed position.
- Mechanical energy is defined to be $\text{KE} + \text{PE}$ for a conservative force.
- When only conservative forces act on and within a system, the total mechanical energy is constant. In equation form,

Equation:

$$\left.\begin{array}{c} \text{KE} + \text{PE} = \text{constant} \\ \text{or} \\ \text{KE}_\text{i} + \text{PE}_\text{i} = \text{KE}_\text{f} + \text{PE}_\text{f} \end{array}\right\}$$

where i and f denote initial and final values. This is known as the conservation of mechanical energy.

Conceptual Questions

Exercise:

Problem: What is a conservative force?

Exercise:

Problem:

The force exerted by a diving board is conservative, provided the internal friction is negligible. Assuming friction is negligible, describe changes in the potential energy of a diving board as a swimmer dives from it, starting just before the swimmer steps on the board until just after his feet leave it.

Exercise:

Problem:

Define mechanical energy. What is the relationship of mechanical energy to nonconservative forces? What happens to mechanical energy if only conservative forces act?

Exercise:

Problem:

What is the relationship of potential energy to conservative force?

Problems & Exercises

Exercise:

Problem:

A 5.00×10^5-kg subway train is brought to a stop from a speed of 0.500 m/s in 0.400 m by a large spring bumper at the end of its track. What is the force constant k of the spring?

Solution:
Equation:

$$7.81{\times}10^5\ \text{N/m}$$

Exercise:

Problem:

A pogo stick has a spring with a force constant of $2.50 \times 10^4\ \mathrm{N/m}$, which can be compressed 12.0 cm. To what maximum height can a child jump on the stick using only the energy in the spring, if the child and stick have a total mass of 40.0 kg? Explicitly show how you follow the steps in the Problem-Solving Strategies for Energy.

Glossary

conservative force
: a force that does the same work for any given initial and final configuration, regardless of the path followed

potential energy
: energy due to position, shape, or configuration

potential energy of a spring
: the stored energy of a spring as a function of its displacement; when Hooke’s law applies, it is given by the expression $\frac{1}{2}kx^2$ where x is the distance the spring is compressed or extended and k is the spring constant

conservation of mechanical energy
: the rule that the sum of the kinetic energies and potential energies remains constant if only conservative forces act on and within a system

mechanical energy
: the sum of kinetic energy and potential energy

Conservation of Energy

LEARNING OBJECTIVES

By the end of this section, you will be able to:

- Explain the law of the conservation of energy.
- Describe some of the many forms of energy.
- Define efficiency of an energy conversion process as the fraction left as useful energy or work, rather than being transformed, for example, into thermal energy.

The information presented in this section supports the following AP® learning objectives and science practices:

- **4.C.1.2** The student is able to predict changes in the total energy of a system due to changes in position and speed of objects or frictional interactions within the system. **(S.P. 6.4)**
- **4.C.2.1** The student is able to make predictions about the changes in the mechanical energy of a system when a component of an external force acts parallel or antiparallel to the direction of the displacement of the center of mass. **(S.P. 6.4)**
- **4.C.2.2** The student is able to apply the concepts of conservation of energy and the work-energy theorem to determine qualitatively and/or quantitatively that work done on a two-object system in linear motion will change the kinetic energy of the center of mass of the system, the potential energy of the systems, and/or the internal energy of the system. **(S.P. 1.4, 2.2, 7.2)**
- **5.A.2.1** The student is able to define open and closed systems for everyday situations and apply conservation concepts for energy, charge, and linear momentum to those situations. **(S.P. 6.4, 7.2)**
- **5.B.5.4** The student is able to make claims about the interaction between a system and its environment in which the environment exerts a force on the system, thus doing work on the system and

changing the energy of the system (kinetic energy plus potential energy). **(S.P. 6.4, 7.2)**
- **5.B.5.5** The student is able to predict and calculate the energy transfer to (i.e., the work done on) an object or system from information about a force exerted on the object or system through a distance. **(S.P. 2.2, 6.4)**

Law of Conservation of Energy

Energy, as we have noted, is conserved, making it one of the most important physical quantities in nature. The **law of conservation of energy** can be stated as follows:

Total energy is constant in any process. It may change in form or be transferred from one system to another, but the total remains the same.

We have explored some forms of energy and some ways it can be transferred from one system to another. This exploration led to the definition of two major types of energy—mechanical energy $(\mathrm{KE}+\mathrm{PE})$ and energy transferred via work done by nonconservative forces (W_{nc}). But energy takes *many* other forms, manifesting itself in *many* different ways, and we need to be able to deal with all of these before we can write an equation for the above general statement of the conservation of energy.

Other Forms of Energy than Mechanical Energy

At this point, we deal with all other forms of energy by lumping them into a single group called other energy (OE). Then we can state the conservation of energy in equation form as

Equation:

$$\mathrm{KE_i}+\mathrm{PE_i}+W_{\mathrm{nc}}+\mathrm{OE_i}=\mathrm{KE_f}+\mathrm{PE_f}+\mathrm{OE_f}.$$

All types of energy and work can be included in this very general statement of conservation of energy. Kinetic energy is KE, work done by a conservative force is represented by PE, work done by nonconservative forces is W_{nc}, and all other energies are included as OE. This equation applies to all previous examples; in those situations OE was constant, and so it subtracted out and was not directly considered.

Note:

Making Connections: Usefulness of the Energy Conservation Principle

The fact that energy is conserved and has many forms makes it very important. You will find that energy is discussed in many contexts, because it is involved in all processes. It will also become apparent that many situations are best understood in terms of energy and that problems are often most easily conceptualized and solved by considering energy.

When does OE play a role? One example occurs when a person eats. Food is oxidized with the release of carbon dioxide, water, and energy. Some of this chemical energy is converted to kinetic energy when the person moves, to potential energy when the person changes altitude, and to thermal energy (another form of OE).

Some of the Many Forms of Energy

What are some other forms of energy? You can probably name a number of forms of energy not yet discussed. Many of these will be covered in later chapters, but let us detail a few here. **Electrical energy** is a common form that is converted to many other forms and does work in a wide range of practical situations. Fuels, such as gasoline and food, carry **chemical energy** that can be transferred to a system through oxidation. Chemical fuel can also produce electrical energy, such as in batteries. Batteries can in turn produce light, which is a very pure form of energy. Most energy sources on Earth are in fact stored energy from the energy we receive from the Sun. We sometimes refer to this as **radiant energy**, or electromagnetic radiation, which includes visible light, infrared, and ultraviolet radiation. **Nuclear energy** comes from processes that convert measurable amounts of mass into energy. Nuclear energy is transformed into the energy of sunlight, into electrical energy in power plants, and into the energy of the heat transfer and blast in weapons. Atoms and molecules inside all objects are in random motion. This internal mechanical energy from the random motions is called **thermal energy**, because it is related to the temperature of the object. These and all other forms of energy can be converted into one another and can do work.

[link] gives the amount of energy stored, used, or released from various objects and in various phenomena. The range of energies and the variety of types and situations is impressive.

Note:
Problem-Solving Strategies for Energy
If you know the potential energies for the forces that enter into the problem, then forces are all conservative, and you can apply conservation of mechanical energy simply in terms of potential and kinetic energy. The equation expressing conservation of energy is
Equation:

$$\mathrm{KE_i} + \mathrm{PE_i} = \mathrm{KE_f} + \mathrm{PE_f}.$$

If you know the potential energy for only some of the forces, possibly because some of them are nonconservative and do not have a potential energy, or if there are other energies that are not easily treated in terms of force and work, then the conservation of energy law in its most general form must be used.
Equation:

$$\mathrm{KE_i} + \mathrm{PE_i} + W_{\mathrm{nc}} + \mathrm{OE_i} = \mathrm{KE_f} + \mathrm{PE_f} + \mathrm{OE_f}.$$

Transformation of Energy

The transformation of energy from one form into others is happening all the time. The chemical energy in food is converted into thermal energy through metabolism; light energy is converted into chemical energy through photosynthesis. In a larger example, the chemical energy contained in coal is converted into thermal energy as it burns to turn water into steam in a boiler. This thermal energy in the steam in turn is converted to mechanical energy as it spins a turbine, which is connected to a generator to produce electrical energy. (In all of these examples, not all of the initial energy is converted into the forms mentioned. This important point is discussed later in this section.)

Another example of energy conversion occurs in a solar cell. Sunlight impinging on a solar cell (see [link]) produces electricity, which in turn can be used to run an electric motor. Energy is converted from the primary source of solar energy into electrical energy and then into mechanical energy.

Solar energy is converted into electrical energy by solar cells, which is used to run a motor in this solar-power aircraft. (credit: NASA)

Object/phenomenon	Energy in joules

Object/phenomenon	**Energy in joules**
Big Bang	10^{68}
Energy released in a supernova	10^{44}
Fusion of all the hydrogen in Earth’s oceans	10^{34}
Annual world energy use	4×10^{20}
Large fusion bomb (9 megaton)	3.8×10^{16}
1 kg hydrogen (fusion to helium)	6.4×10^{14}
1 kg uranium (nuclear fission)	8.0×10^{13}
Hiroshima-size fission bomb (10 kiloton)	4.2×10^{13}
90,000-ton aircraft carrier at 30 knots	1.1×10^{10}
1 barrel crude oil	5.9×10^{9}

Object/phenomenon	**Energy in joules**
1 ton TNT	4.2×10^9
1 gallon of gasoline	1.2×10^8
Daily home electricity use (developed countries)	7×10^7
Daily adult food intake (recommended)	1.2×10^7
1000-kg car at 90 km/h	3.1×10^5
1 g fat (9.3 kcal)	3.9×10^4
ATP hydrolysis reaction	3.2×10^4
1 g carbohydrate (4.1 kcal)	1.7×10^4
1 g protein (4.1 kcal)	1.7×10^4

Object/phenomenon	**Energy in joules**
Tennis ball at 100 km/h	22
Mosquito $\left(10^{-2}\text{ g at }0.5\text{ m/s}\right)$	1.3×10^{-6}
Single electron in a TV tube beam	4.0×10^{-15}
Energy to break one DNA strand	10^{-19}

Energy of Various Objects and Phenomena

Efficiency

Even though energy is conserved in an energy conversion process, the output of *useful energy* or work will be less than the energy input. The **efficiency** Eff of an energy conversion process is defined as

Equation:

$$\text{Efficiency}(\text{Eff})=\frac{\text{useful energy or work output}}{\text{total energy input}}=\frac{W_{\text{out}}}{E_{\text{in}}}.$$

[link] lists some efficiencies of mechanical devices and human activities. In a coal-fired power plant, for example, about 40% of the chemical energy in the coal becomes useful electrical energy. The other 60% transforms into other (perhaps less useful) energy forms, such as thermal energy, which is then released to the environment through combustion gases and cooling towers.

Activity/device	Efficiency (%)[footnote] Representative values
Cycling and climbing	20
Swimming, surface	2
Swimming, submerged	4
Shoveling	3
Weightlifting	9
Steam engine	17
Gasoline engine	30
Diesel engine	35
Nuclear power plant	35
Coal power plant	42
Electric motor	98
Compact fluorescent light	20
Gas heater (residential)	90
Solar cell	10

Efficiency of the Human Body and Mechanical Devices

Note:
PhET Explorations: Masses and Springs

A realistic mass and spring laboratory. Hang masses from springs and adjust the spring stiffness and damping. You can even slow time. Transport the lab to different planets. A chart shows the kinetic, potential, and thermal energies for each spring.

Masses and Springs

Section Summary

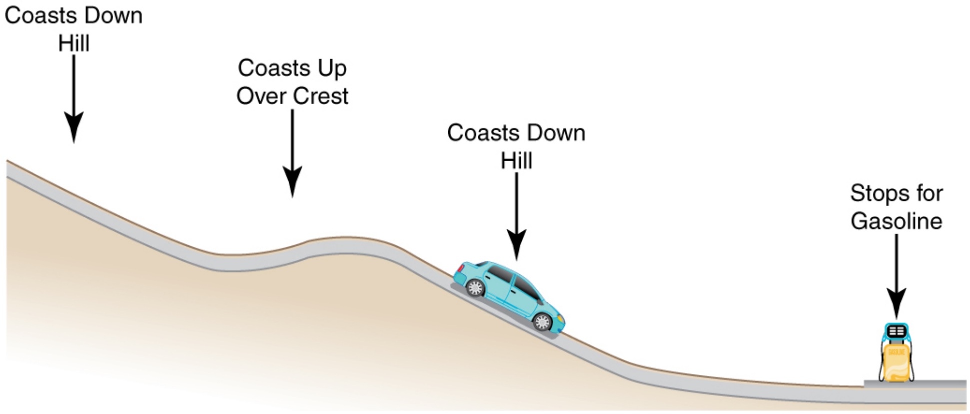
- The law of conservation of energy states that the total energy is constant in any process. Energy may change in form or be transferred from one system to another, but the total remains the same.
- When all forms of energy are considered, conservation of energy is written in equation form as $\mathrm{KE_i} + \mathrm{PE_i} + W_{\mathrm{nc}} + \mathrm{OE_i} = \mathrm{KE_f} + \mathrm{PE_f} + \mathrm{OE_f}$, where OE is all **other forms of energy** besides mechanical energy.
- Commonly encountered forms of energy include electric energy, chemical energy, radiant energy, nuclear energy, and thermal energy.
- Energy is often utilized to do work, but it is not possible to convert all the energy of a system to work.
- The efficiency Eff of a machine or human is defined to be $\mathrm{Eff} = \frac{W_{\mathrm{out}}}{E_{\mathrm{in}}}$, where W_{out} is useful work output and E_{in} is the energy consumed.

Conceptual Questions

Exercise:

Problem:

Consider the following scenario. A car for which friction is *not* negligible accelerates from rest down a hill, running out of gasoline after a short distance. The driver lets the car coast farther down the hill, then up and over a small crest. He then coasts down that hill into a gas station, where he brakes to a stop and fills the tank with gasoline. Identify the forms of energy the car has, and how they are changed and transferred in this series of events. (See [link].)



A car experiencing non-negligible friction coasts down a hill, over a small crest, then downhill again, and comes to a stop at a gas station.

Exercise:

Problem:

Describe the energy transfers and transformations for a javelin, starting from the point at which an athlete picks up the javelin and ending when the javelin is stuck into the ground after being thrown.

Exercise:

Problem:

Do devices with efficiencies of less than one violate the law of conservation of energy? Explain.

Exercise:

Problem:

List four different forms or types of energy. Give one example of a conversion from each of these forms to another form.

Exercise:

Problem: List the energy conversions that occur when riding a bicycle.

Problems & Exercises

Exercise:

Problem:

Using values from [link], how many DNA molecules could be broken by the energy carried by a single electron in the beam of an old-fashioned TV tube? (These electrons were not dangerous in themselves, but they did create dangerous x rays. Later model tube TVs had shielding that absorbed x rays before they escaped and exposed viewers.)

Solution:

4×10^4 molecules

Exercise:

Problem:

Using energy considerations and assuming negligible air resistance, show that a rock thrown from a bridge 20.0 m above water with an initial speed of 15.0 m/s strikes the water with a speed of 24.8 m/s independent of the direction thrown.

Solution:

Equating ΔPE_g and ΔKE, we obtain

$$v=\sqrt{2\text{gh}+{v_0}^2}=\sqrt{2(9.80\text{ m/s}^2)(20.0\text{ m})+(15.0\text{ m/s})^2}=24.8\text{ m/s}$$

Exercise:

Problem:

If the energy in fusion bombs were used to supply the energy needs of the world, how many of the 9-megaton variety would be needed for a year's supply of energy (using data from [link])? This is not as far-fetched as it may sound—there are thousands of nuclear bombs, and their energy can be trapped in underground explosions and converted to electricity, as natural geothermal energy is.

Exercise:

Problem:

(a) Use of hydrogen fusion to supply energy is a dream that may be realized in the next century. Fusion would be a relatively clean and almost limitless supply of energy, as can be seen from [link]. To illustrate this, calculate how many years the present energy needs of the world could be supplied by one millionth of the oceans' hydrogen fusion energy. (b) How does this time compare with historically significant events, such as the duration of stable economic systems?

Solution:

(a) 25×10^6 years

(b) This is much, much longer than human time scales.

Glossary

law of conservation of energy	the general law that total energy is constant in any process; energy may change in form or be transferred from one system to another, but the total remains the same
electrical energy	the energy carried by a flow of charge
chemical energy	

the energy in a substance stored in the bonds between atoms and molecules that can be released in a chemical reaction

radiant energy
the energy carried by electromagnetic waves

nuclear energy
energy released by changes within atomic nuclei, such as the fusion of two light nuclei or the fission of a heavy nucleus

thermal energy
the energy within an object due to the random motion of its atoms and molecules that accounts for the object's temperature

efficiency
a measure of the effectiveness of the input of energy to do work; useful energy or work divided by the total input of energy

Power

By the end of this section, you will be able to:

- Relate the work done during a time interval to the power delivered
- Find the power expended by a force acting on a moving body

The concept of work involves force and displacement; the work-energy theorem relates the net work done on a body to the difference in its kinetic energy, calculated between two points on its trajectory. None of these quantities or relations involves time explicitly, yet we know that the time available to accomplish a particular amount of work is frequently just as important to us as the amount itself. In the chapter-opening figure, several sprinters may have achieved the same velocity at the finish, and therefore did the same amount of work, but the winner of the race did it in the least amount of time.

We express the relation between work done and the time interval involved in doing it, by introducing the concept of power. Since work can vary as a function of time, we first define **average power** as the work done during a time interval, divided by the interval,

Equation:

$$P_{\text{ave}} = \frac{\Delta W}{\Delta t}.$$

Note:

Power

Power is defined as the rate of doing work, or the limit of the average power for time intervals approaching zero,

Equation:

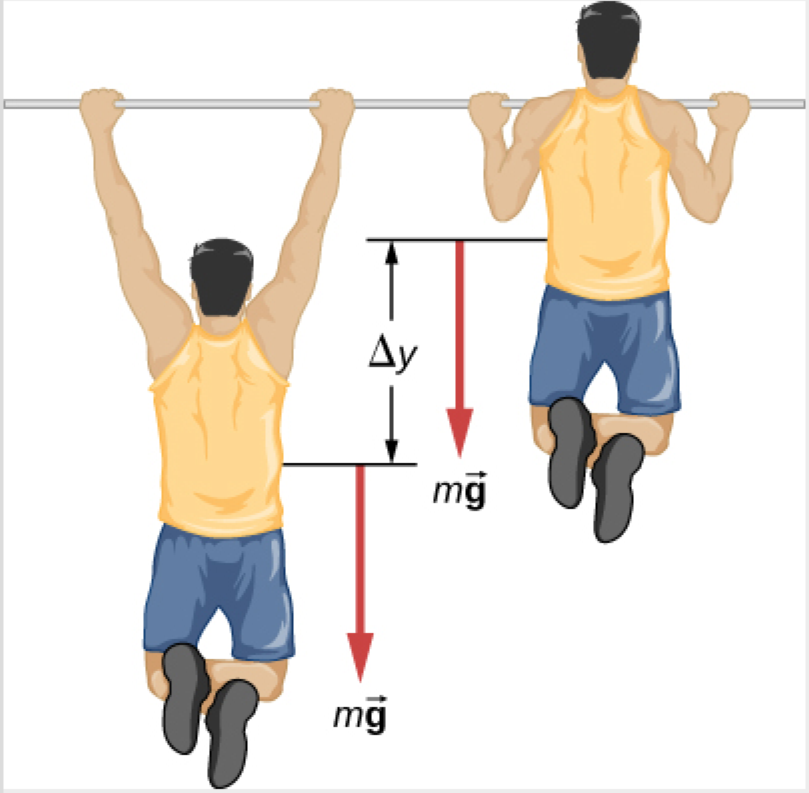
$$P = \frac{\Delta W}{\Delta t}.$$

The work-energy theorem relates how work can be transformed into kinetic energy. Since there are other forms of energy as well, as we discuss in the next chapter, we can also define power as the rate of transfer of energy. Work and energy are measured in units of joules, so power is measured in units of joules per second, which has been given the SI name watts, abbreviation W: $1\ \text{J/s} = 1\ \text{W}$. Another common unit for expressing the power capability of everyday devices is horsepower: $1\ \text{hp} = 746\ \text{W}$.

Example:

Pull-Up Power

An 80-kg army trainee does 10 pull-ups in 10 s ([link]). How much average power do the trainee's muscles supply moving his body? (*Hint:* Make reasonable estimates for any quantities needed.)



What is the power expended in doing ten pull-ups in ten seconds?

Strategy

The work done against gravity, going up or down a distance Δy, is $mg\Delta y$. (If you lift and lower yourself at constant speed, the force you exert cancels gravity over the whole pull-up cycle.) Thus, the work done by the trainee's muscles (moving, but not accelerating, his body) for a complete repetition (up and down) is $2mg\Delta y$. Let's assume that $\Delta y = 2\text{ft} \approx 60\,\text{cm}$. Also, assume that the arms comprise 10% of the body mass and are not included in the moving mass. With these assumptions, we can calculate the work done for 10 pull-ups and divide by 10 s to get the average power.

Solution

The result we get, applying our assumptions, is

Equation:

$$P_{\text{ave}} = \frac{10\ \times\ 2(0.9\ \times\ 80\,\text{kg})(9.8\,\text{m/s}^2)(0.6\,\text{m})}{10\,\text{s}} = 850\ \text{W}.$$

Significance

This is typical for power expenditure in strenuous exercise; in everyday units, it's somewhat more than one horsepower $(1\ \text{hp} = 746\ \text{W})$.

Note:

Exercise:

Problem:

Check Your Understanding Estimate the power expended by a weightlifter raising a 150-kg barbell 2 m in 3 s.

Solution:

980 W

The power involved in moving a body can also be expressed in terms of the forces acting on it. If a force $\vec{\mathbf{F}}$ acts on a body that is displaced $d\vec{\mathbf{r}}$ in a time *dt*, the power expended by the force is

Note:
Equation:

$$P = \frac{\Delta W}{\Delta t} = \frac{\vec{\mathbf{F}} \cdot \Delta\vec{\mathbf{r}}}{\Delta t} = \overrightarrow{\mathbf{F}} \cdot \left(\frac{\Delta\vec{\mathbf{r}}}{\Delta t}\right) = \vec{\mathbf{F}} \cdot \vec{\mathbf{v}},$$

where $\vec{\mathbf{v}}$ is the velocity of the body. The fact that the limits implied by the derivatives exist, for the motion of a real body, justifies the rearrangement of the infinitesimals.

Summary

- Power is the rate of doing work; that is, the derivative of work with respect to time.
- Alternatively, the work done, during a time interval, is the integral of the power supplied over the time interval.
- The power delivered by a force, acting on a moving particle, is the dot product of the force and the particle’s velocity.

Key Equations

Work done by a force over an infinitesimal displacement	$\Delta W = \overrightarrow{\mathbf{F}} \cdot \overrightarrow{\Delta\mathbf{r}} = \left|\overrightarrow{\mathbf{F}}\right| \left|\overrightarrow{\Delta\mathbf{r}}\right| \cos\theta$
Work done by a force acting	

along a path from *A* to *B*	$W_{AB} = \int\limits_{\text{path}AB} \vec{\mathbf{F}} \cdot \mathit{\Delta}\vec{\mathbf{r}}$
Work done by a constant force of kinetic friction	$W_{\text{fr}} = -f_k \left\vert l_{AB} \right\vert$
Work done going from *A* to *B* by Earth's gravity, near its surface	$W_{\text{grav},AB} = -mg\left(y_B - y_A\right)$
Work done going from *A* to *B* by one-dimensional spring force	$W_{\text{spring},AB} = -\left(\frac{1}{2}k\right)\left(x_B^2 - x_A^2\right)$
Kinetic energy of a non-relativistic particle	$K = \frac{1}{2}mv^2 = \frac{p^2}{2m}$
Work-energy theorem	$W_{\text{net}} = K_B - K_A$
Power as rate of doing work	$P = \frac{\mathit{\Delta}W}{\mathit{\Delta}t}$
Power as the dot product of force and velocity	$P = \vec{\mathbf{F}} \cdot \vec{\mathbf{v}}$

Conceptual Questions

Exercise:

Problem:

Most electrical appliances are rated in watts. Does this rating depend on how long the appliance is on? (When off, it is a zero-watt device.) Explain in terms of the definition of power.

Solution:

Appliances are rated in terms of the energy consumed in a relatively small time interval. It does not matter how long the appliance is on, only the rate of change of energy per unit time.

Exercise:

Problem:

Explain, in terms of the definition of power, why energy consumption is sometimes listed in kilowatt-hours rather than joules. What is the relationship between these two energy units?

Exercise:

Problem:

A spark of static electricity, such as that you might receive from a doorknob on a cold dry day, may carry a few hundred watts of power. Explain why you are not injured by such a spark.

Solution:

The spark occurs over a relatively short time span, thereby delivering a very low amount of energy to your body.

Exercise:

Problem:

Does the work done in lifting an object depend on how fast it is lifted? Does the power expended depend on how fast it is lifted?

Exercise:

Problem: Can the power expended by a force be negative?

Solution:

If the force is antiparallel or points in an opposite direction to the velocity, the power expended can be negative.

Exercise:

Problem:

How can a 50-W light bulb use more energy than a 1000-W oven?

Problems

Exercise:

Problem:

A person in good physical condition can put out 100 W of useful power for several hours at a stretch, perhaps by pedaling a mechanism that drives an electric generator. Neglecting any problems of generator efficiency and practical considerations such as resting time: (a) How many people would it take to run a 4.00-kW electric clothes dryer? (b) How many people would it take to replace a large electric power plant that generates 800 MW?

Solution:

a. 40; b. 8 million

Exercise:

Problem:

What is the cost of operating a 3.00-W electric clock for a year if the cost of electricity is \$0.0900 per $\mathrm{kW \cdot h}$?

Exercise:

Problem:

A large household air conditioner may consume 15.0 kW of power. What is the cost of operating this air conditioner 3.00 h per day for 30.0 d if the cost of electricity is \$0.110 per $\mathrm{kW \cdot h}$?

Solution:

$149

Exercise:

Problem:

(a) What is the average power consumption in watts of an appliance that uses $5.00\ \text{kW} \cdot \text{h}$ of energy per day? (b) How many joules of energy does this appliance consume in a year?

Exercise:

Problem:

(a) What is the average useful power output of a person who does $6.00\ \times\ 10^{6}\ \text{J}$ of useful work in 8.00 h? (b) Working at this rate, how long will it take this person to lift 2000 kg of bricks 1.50 m to a platform? (Work done to lift his body can be omitted because it is not considered useful output here.)

Solution:

a. 208 W; b. 141 s

Exercise:

Problem:

A 500-kg dragster accelerates from rest to a final speed of 110 m/s in 400 m (about a quarter of a mile) and encounters an average frictional force of 1200 N. What is its average power output in watts and horsepower if this takes 7.30 s?

Exercise:

Problem:

(a) How long will it take an 850-kg car with a useful power output of 40.0 hp (1 hp equals 746 W) to reach a speed of 15.0 m/s, neglecting friction? (b) How long will this acceleration take if the car also climbs a 3.00-m high hill in the process?

Solution:

a. 3.20 s; b. 4.04 s

Exercise:

Problem:

(a) Find the useful power output of an elevator motor that lifts a 2500-kg load a height of 35.0 m in 12.0 s, if it also increases the speed from rest to 4.00 m/s. Note that the total mass of the counterbalanced system is 10,000 kg—so that only 2500 kg is raised in height, but the full 10,000 kg is accelerated. (b) What does it cost, if electricity is $0.0900 per $\mathrm{kW} \cdot \mathrm{h}$?

Exercise:

Problem:

(a) How long would it take a 1.50×10^5-kg airplane with engines that produce 100 MW of power to reach a speed of 250 m/s and an altitude of 12.0 km if air resistance were negligible? (b) If it actually takes 900 s, what is the power? (c) Given this power, what is the average force of air resistance if the airplane takes 1200 s? (*Hint:* You must find the distance the plane travels in 1200 s assuming constant acceleration.)

Solution:

a. 224 s; b. 24.8 MW; c. 49.7 kN

Exercise:

Problem:

Calculate the power output needed for a 950-kg car to climb a 2.00° slope at a constant 30.0 m/s while encountering wind resistance and friction totaling 600 N.

Exercise:

Problem:

A man of mass 80 kg runs up a flight of stairs 20 m high in 10 s. (a) how much power is used to lift the man? (b) If the man’s body is 25% efficient, how much power does he expend?

Solution:

a. 1.57 kW; b. 6.28 kW

Exercise:

Problem:

The man of the preceding problem consumes approximately $1.05 \times 10^7 \text{ J}$ (2500 food calories) of energy per day in maintaining a constant weight. What is the average power he produces over a day? Compare this with his power production when he runs up the stairs.

Exercise:

Problem:

An electron in a television tube is accelerated uniformly from rest to a speed of $8.4 \times 10^7 \text{ m/s}$ over a distance of 2.5 cm. What is the power delivered to the electron at the instant that its displacement is 1.0 cm?

Solution:

$6.83 \mu\text{W}$

Exercise:

Problem:

Coal is lifted out of a mine a vertical distance of 50 m by an engine that supplies 500 W to a conveyer belt. How much coal per minute can be brought to the surface? Ignore the effects of friction.

Exercise:

Problem:

A girl pulls her 15-kg wagon along a flat sidewalk by applying a 10-N force at 37° to the horizontal. Assume that friction is negligible and that the wagon starts from rest. (a) How much work does the girl do on the wagon in the first 2.0 s. (b) How much instantaneous power does she exert at $t = 2.0$ s?

Solution:

a. 8.51 J; b. 8.51 W

Exercise:

Problem:

A typical automobile engine has an efficiency of 25%. Suppose that the engine of a 1000-kg automobile has a maximum power output of 140 hp. What is the maximum grade that the automobile can climb at 50 km/h if the frictional retarding force on it is 300 N?

Exercise:

Problem:

When jogging at 13 km/h on a level surface, a 70-kg man uses energy at a rate of approximately 850 W. Using the facts that the “human engine” is approximately 25% efficient, determine the rate at which this man uses energy when jogging up a 5.0° slope at this same speed. Assume that the frictional retarding force is the same in both cases.

Solution:

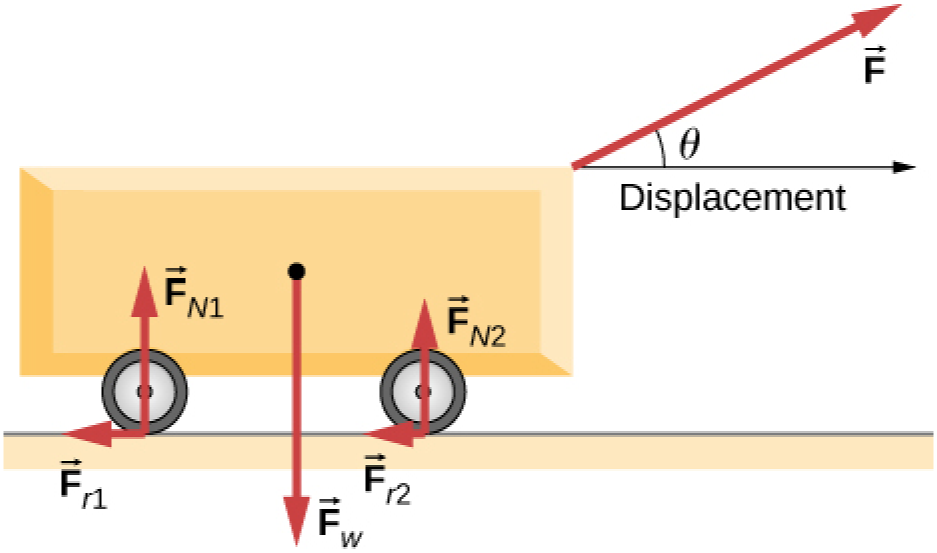
1.7 kW

Additional Problems

Exercise:

Problem:

A cart is pulled a distance D on a flat, horizontal surface by a constant force F that acts at an angle θ with the horizontal direction. The other forces on the object during this time are gravity (F_w), normal forces (F_{N1}) and (F_{N2}), and rolling frictions F_{r1} and F_{r2}, as shown below. What is the work done by each force?



Exercise:

Problem:

Consider a particle on which several forces act, one of which is known to be constant in time: $\vec{\mathbf{F}}_1 = (3\text{ N})\hat{\mathbf{i}} + (4\text{ N})\hat{\mathbf{j}}$. As a result, the particle moves along the *x*-axis from $x = 0$ to $x = 5\text{ m}$ in some time interval. What is the work done by $\vec{\mathbf{F}}_1$?

Solution:

$15\text{ N}\cdot\text{m}$

Exercise:

Problem:

Consider a particle on which several forces act, one of which is known to be constant in time: $\vec{\mathbf{F}}_1 = (3\,\mathrm{N})\hat{\mathbf{i}} + (4\,\mathrm{N})\hat{\mathbf{j}}$. As a result, the particle moves first along the *x*-axis from $x = 0$ to $x = 5\,\mathrm{m}$ and then parallel to the *y*-axis from $y = 0$ to $y = 6\,\mathrm{m}$. What is the work done by $\vec{\mathbf{F}}_1$?

Exercise:

Problem:

Consider a particle on which several forces act, one of which is known to be constant in time: $\vec{\mathbf{F}}_1 = (3\,\mathrm{N})\hat{\mathbf{i}} + (4\,\mathrm{N})\hat{\mathbf{j}}$. As a result, the particle moves along a straight path from a Cartesian coordinate of (0 m, 0 m) to (5 m, 6 m). What is the work done by $\vec{\mathbf{F}}_1$?

Solution:

$39\,\mathrm{N}\cdot\mathrm{m}$

Exercise:

Problem:

Consider a particle on which a force acts that depends on the position of the particle. This force is given by $\vec{\mathbf{F}}_1 = (2y)\hat{\mathbf{i}} + (3x)\hat{\mathbf{j}}$. Find the work done by this force when the particle moves from the origin to a point 5 meters to the right on the *x*-axis.

Exercise:

Problem:

A boy pulls a 5-kg cart with a 20-N force at an angle of 30° above the horizontal for a length of time. Over this time frame, the cart moves a distance of 12 m on the horizontal floor. (a) Find the work done on the cart by the boy. (b) What will be the work done by the boy if he pulled with the same force horizontally instead of at an angle of 30° above the horizontal over the same distance?

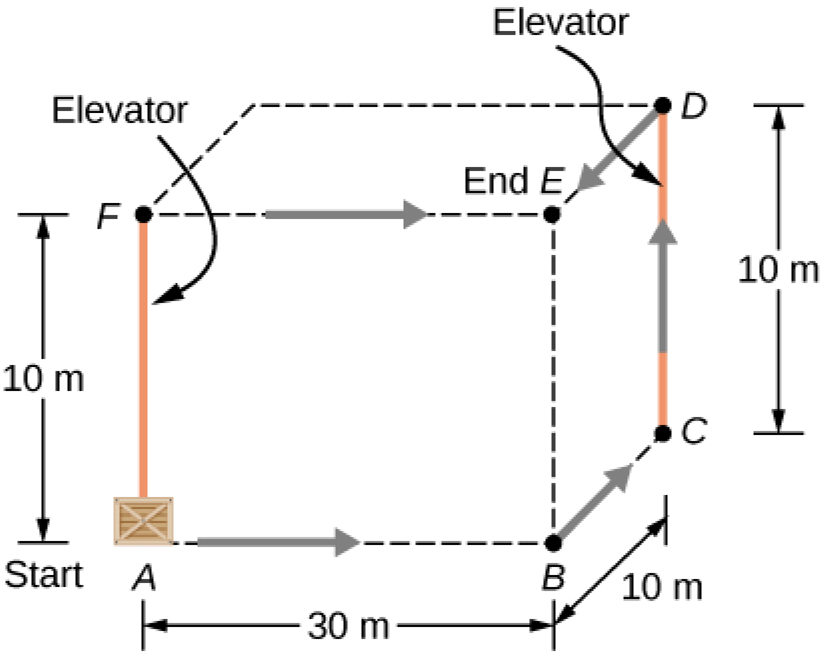
Solution:

a. $208\ \text{N}\cdot\text{m}$; b. $240\ \text{N}\cdot\text{m}$

Exercise:

Problem:

A crate of mass 200 kg is to be brought from a site on the ground floor to a third floor apartment. The workers know that they can either use the elevator first, then slide it along the third floor to the apartment, or first slide the crate to another location marked C below, and then take the elevator to the third floor and slide it on the third floor a shorter distance. The trouble is that the third floor is very rough compared to the ground floor. Given that the coefficient of kinetic friction between the crate and the ground floor is 0.100 and between the crate and the third floor surface is 0.300, find the work needed by the workers for each path shown from *A* to *E*. Assume that the force the workers need to do is just enough to slide the crate at constant velocity (zero acceleration). *Note:* The work by the elevator against the force of gravity is not done by the workers.



Exercise:

Problem:

A hockey puck of mass 0.17 kg is shot across a rough floor with the roughness different at different places, which can be described by a position-dependent coefficient of kinetic friction. For a puck moving along the *x*-axis, the coefficient of kinetic friction is the following function of *x*, where *x* is in m: $\mu(x) = 0.1 + 0.05x$. Find the work done by the kinetic frictional force on the hockey puck when it has moved (a) from $x = 0$ to $x = 2\,\text{m}$, and (b) from $x = 2\,\text{m}$ to $x = 4\,\text{m}$.

Solution:

a. $-0.9\,\text{N} \cdot \text{m}$; b. $-0.83\,\text{N} \cdot \text{m}$

Exercise:

Problem:

A horizontal force of 20 N is required to keep a 5.0 kg box traveling at a constant speed up a frictionless incline for a vertical height change of 3.0 m. (a) What is the work done by gravity during this change in height? (b) What is the work done by the normal force? (c) What is the work done by the horizontal force?

Exercise:

Problem:

A 7.0-kg box slides along a horizontal frictionless floor at 1.7 m/s and collides with a relatively massless spring that compresses 23 cm before the box comes to a stop. (a) How much kinetic energy does the box have before it collides with the spring? (b) Calculate the work done by the spring. (c) Determine the spring constant of the spring.

Solution:

a. 10. J; b. 10. J; c. 380 N/m

Exercise:

Problem:

You are driving your car on a straight road with a coefficient of friction between the tires and the road of 0.55. A large piece of debris falls in front of your view and you immediate slam on the brakes, leaving a skid mark of 30.5 m (100-feet) long before coming to a stop. A policeman sees your car stopped on the road, looks at the skid mark, and gives you a ticket for traveling over the 13.4 m/s (30 mph) speed limit. Should you fight the speeding ticket in court?

Exercise:

Problem:

A crate is being pushed across a rough floor surface. If no force is applied on the crate, the crate will slow down and come to a stop. If the crate of mass 50 kg moving at speed 8 m/s comes to rest in 10 seconds, what is the rate at which the frictional force on the crate takes energy away from the crate?

Solution:

160 J/s

Exercise:

Problem:

Suppose a horizontal force of 20 N is required to maintain a speed of 8 m/s of a 50 kg crate. (a) What is the power of this force? (b) Note that the acceleration of the crate is zero despite the fact that 20 N force acts on the crate horizontally. What happens to the energy given to the crate as a result of the work done by this 20 N force?

Exercise:

Problem:

Grains from a hopper falls at a rate of 10 kg/s vertically onto a conveyor belt that is moving horizontally at a constant speed of 2 m/s. (a) What force is needed to keep the conveyor belt moving at the constant velocity? (b) What is the minimum power of the motor driving the conveyor belt?

Solution:

a. 10 N; b. 20 W

Exercise:

Problem:

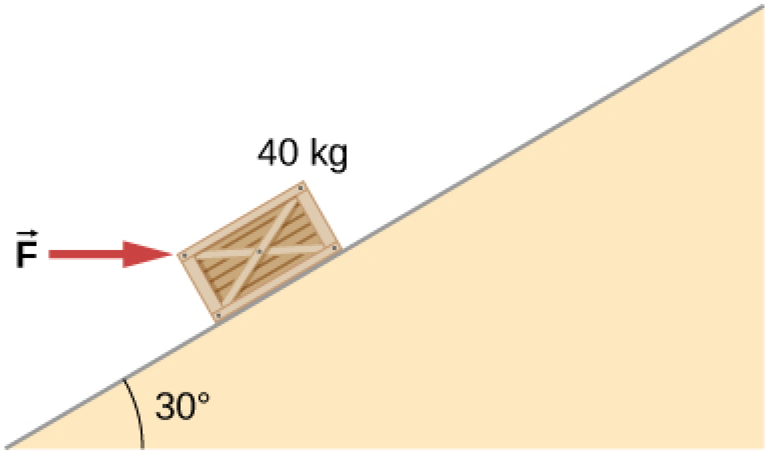
A cyclist in a race must climb a 5° hill at a speed of 8 m/s. If the mass of the bike and the biker together is 80 kg, what must be the power output of the biker to achieve the goal?

Challenge Problems

Exercise:

Problem:

Shown below is a 40-kg crate that is pushed at constant velocity a distance 8.0 m along a 30° incline by the horizontal force $\vec{\mathbf{F}}$. The coefficient of kinetic friction between the crate and the incline is $\mu_k = 0.40$. Calculate the work done by (a) the applied force, (b) the frictional force, (c) the gravitational force, and (d) the net force.



Solution:

If crate goes up: a. 3.46 kJ; b. −1.89 kJ; c. −1.57 kJ; d. 0; If crate goes down: a. −0.39 kJ; b. −1.18 kJ; c. 1.57 kJ; d. 0

Exercise:

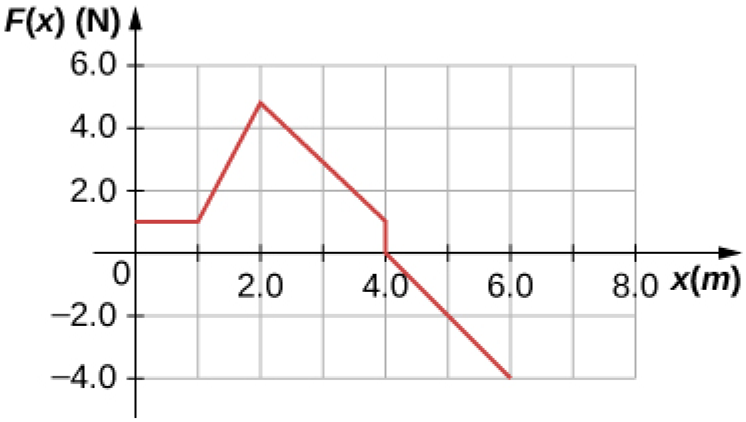
Problem:

The surface of the preceding problem is modified so that the coefficient of kinetic friction is decreased. The same horizontal force is applied to the crate, and after being pushed 8.0 m, its speed is 5.0 m/s. How much work is now done by the force of friction? Assume that the crate starts at rest.

Exercise:

Problem:

The force $F(x)$ varies with position, as shown below. Find the work done by this force on a particle as it moves from $x = 1.0\,\text{m}$ to $x = 5.0\,\text{m}$.



Solution:

8.0 J

Exercise:

Problem:

Find the work done by the same force in [link], between the same points, $A=(0,0)$ and $B=(2\text{ m},2\text{ m})$, over a circular arc of radius 2 m, centered at (0, 2 m). Evaluate the path integral using Cartesian coordinates. (*Hint:* You will probably need to consult a table of integrals.)

Exercise:

Problem: Answer the preceding problem using polar coordinates.

Solution:

35.7 J

Exercise:

Problem:

Find the work done by the same force in [link], between the same points, $A=(0,0)$ and $B=(2\text{ m},2\text{ m})$, over a circular arc of radius 2 m, centered at (2 m, 0). Evaluate the path integral using Cartesian coordinates. (*Hint:* You will probably need to consult a table of integrals.)

Exercise:

Problem: Answer the preceding problem using polar coordinates.

Solution:

24.3 J

Exercise:

Problem:

Constant power *P* is delivered to a car of mass *m* by its engine. Show that if air resistance can be ignored, the distance covered in a time *t* by the car, starting from rest, is given by $s=(8P/9m)^{1/2}t^{3/2}$.

Exercise:

Problem:

Suppose that the air resistance a car encounters is independent of its speed. When the car travels at 15 m/s, its engine delivers 20 hp to its wheels. (a) What is the power delivered to the wheels when the car travels at 30 m/s? (b) How much energy does the car use in covering 10 km at 15 m/s? At 30 m/s? Assume that the engine is 25% efficient. (c) Answer the same questions if the force of air resistance is proportional to the speed of the automobile. (d) What do these results, plus your experience with gasoline consumption, tell you about air resistance?

Solution:

a. 40 hp; b. 39.8 MJ, independent of speed; c. 80 hp, 79.6 MJ at 30 m/s; d. If air resistance is proportional to speed, the car gets about 22 mpg at 34 mph and half that at twice the speed, closer to actual driving experience.

Exercise:

Problem:

Consider a linear spring, as in [link](a), with mass *M* uniformly distributed along its length. The left end of the spring is fixed, but the right end, at the equilibrium position $x = 0$, is moving with speed *v* in the *x*-direction. What is the total kinetic energy of the spring? (*Hint:* First express the kinetic energy of an infinitesimal element of the spring *dm* in terms of the total mass, equilibrium length, speed of the right-hand end, and position along the spring; then integrate.)

Glossary

average power	work done in a time interval divided by the time interval
power	(or instantaneous power) rate of doing work

Work, Energy, and Power in Humans

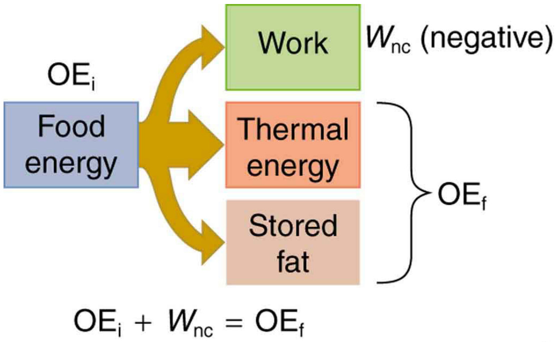
LEARNING OBJECTIVES

By the end of this section, you will be able to:

- Explain the human body’s consumption of energy when at rest versus when engaged in activities that do useful work.
- Calculate the conversion of chemical energy in food into useful work.

Energy Conversion in Humans

Our own bodies, like all living organisms, are energy conversion machines. Conservation of energy implies that the chemical energy stored in food is converted into work, thermal energy, and/or stored as chemical energy in fatty tissue. (See [link].) The fraction going into each form depends both on how much we eat and on our level of physical activity. If we eat more than is needed to do work and stay warm, the remainder goes into body fat.



Energy consumed by humans is converted to work, thermal energy, and stored fat. By far the largest fraction goes to thermal energy, although the fraction varies depending on the type of physical activity.

Power Consumed at Rest

The *rate* at which the body uses food energy to sustain life and to do different activities is called the **metabolic rate**. The total energy conversion rate of a person *at rest* is called the **basal metabolic rate** (BMR) and is divided among various systems in the body, as shown in [link]. The largest fraction goes to the liver and spleen, with the brain coming next. Of course, during vigorous exercise, the energy consumption of the skeletal muscles and heart increase markedly. About 75% of the calories burned in a day go into these basic functions. The BMR is a function of age, gender, total body weight, and amount of muscle mass (which burns more calories than body fat). Athletes have a greater BMR due to this last factor.

Organ	Power consumed at rest (W)	Oxygen consumption (mL/min)	Percent of BMR
Liver & spleen	23	67	27

Organ	Power consumed at rest (W)	Oxygen consumption (mL/min)	Percent of BMR
Brain	16	47	19
Skeletal muscle	15	45	18
Kidney	9	26	10
Heart	6	17	7
Other	16	48	19
Totals	85 W	250 mL/min	100%

Basal Metabolic Rates (BMR)

Energy consumption is directly proportional to oxygen consumption because the digestive process is basically one of oxidizing food. We can measure the energy people use during various activities by measuring their oxygen use. (See [link].) Approximately 20 kJ of energy are produced for each liter of oxygen consumed, independent of the type of food. [link] shows energy and oxygen consumption rates (power expended) for a variety of activities.

Power of Doing Useful Work

Work done by a person is sometimes called **useful work**, which is *work done on the outside world*, such as lifting weights. Useful work requires a force exerted through a distance on the outside world, and so it excludes internal work, such as that done by the heart when pumping blood. Useful work does include that done in climbing stairs or accelerating to a full run, because these are accomplished by exerting forces on the outside world. Forces exerted by the body are nonconservative, so that they can change the mechanical energy $(\mathrm{KE}+\mathrm{PE})$ of the system worked upon, and this is often the goal. A baseball player throwing a ball, for example, increases both the ball’s kinetic and potential energy.

If a person needs more energy than they consume, such as when doing vigorous work, the body must draw upon the chemical energy stored in fat. So exercise can be helpful in losing fat. However, the amount of exercise needed to produce a loss in fat, or to burn off extra calories consumed that day, can be large, as [link] illustrates.

Example:

Calculating Weight Loss from Exercising

If a person who normally requires an average of 12,000 kJ (3000 kcal) of food energy per day consumes 13,000 kJ per day, he will steadily gain weight. How much bicycling per day is required to work off this extra 1000 kJ?

Solution

[link] states that 400 W are used when cycling at a moderate speed. The time required to work off 1000 kJ at this rate is then

Equation:

$$\text{Time} = \frac{\text{energy}}{\left(\frac{\text{energy}}{\text{time}}\right)} = \frac{1000\text{ kJ}}{400\text{ W}} = 2500\text{ s} = 42\text{ min.}$$

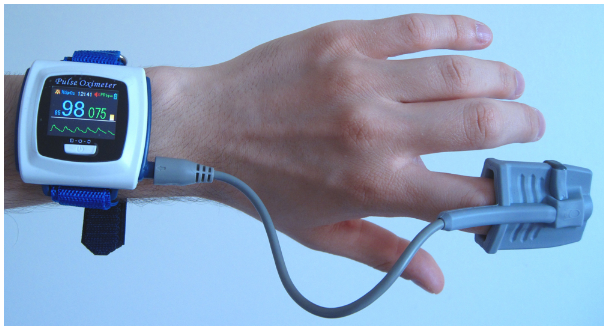
Discussion

If this person uses more energy than he or she consumes, the person’s body will obtain the needed energy by metabolizing body fat. If the person uses 13,000 kJ but consumes only 12,000 kJ, then the amount of fat loss will be

Equation:

$$\text{Fat loss} = (1000\text{ kJ})\left(\frac{1.0\text{ g fat}}{39\text{ kJ}}\right) = 26\text{ g},$$

assuming the energy content of fat to be 39 kJ/g.



A pulse oxymeter is an apparatus that measures the amount of oxygen in blood. Oxymeters can be used to determine a person's metabolic rate, which is the rate at which food energy is converted to another form. Such measurements can indicate the level of athletic conditioning as well as certain medical problems. (credit: UusiAjaja, Wikimedia Commons)

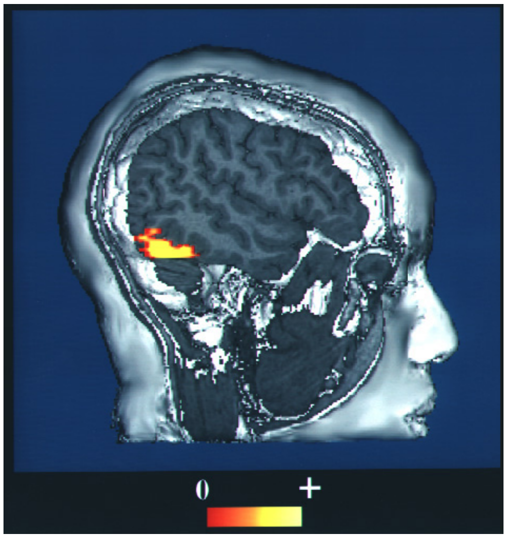
Activity	**Energy consumption in watts**	**Oxygen consumption in liters O_2/min**
Sleeping	83	0.24
Sitting at rest	120	0.34

Activity	Energy consumption in watts	Oxygen consumption in liters O_2/min
Standing relaxed	125	0.36
Sitting in class	210	0.60
Walking (5 km/h)	280	0.80
Cycling (13–18 km/h)	400	1.14
Shivering	425	1.21
Playing tennis	440	1.26
Swimming breaststroke	475	1.36
Ice skating (14.5 km/h)	545	1.56
Climbing stairs (116/min)	685	1.96
Cycling (21 km/h)	700	2.00
Running cross-country	740	2.12
Playing basketball	800	2.28

Activity	Energy consumption in watts	Oxygen consumption in liters O_2/min
Cycling, professional racer	1855	5.30
Sprinting	2415	6.90

Energy and Oxygen Consumption Rates[footnote] (Power) for an average 76-kg male

All bodily functions, from thinking to lifting weights, require energy. (See [link].) The many small muscle actions accompanying all quiet activity, from sleeping to head scratching, ultimately become thermal energy, as do less visible muscle actions by the heart, lungs, and digestive tract. Shivering, in fact, is an involuntary response to low body temperature that pits muscles against one another to produce thermal energy in the body (and do no work). The kidneys and liver consume a surprising amount of energy, but the biggest surprise of all it that a full 25% of all energy consumed by the body is used to maintain electrical potentials in all living cells. (Nerve cells use this electrical potential in nerve impulses.) This bioelectrical energy ultimately becomes mostly thermal energy, but some is utilized to power chemical processes such as in the kidneys and liver, and in fat production.



This fMRI scan shows an increased level of energy consumption in the vision center of the brain. Here, the patient was being asked to recognize faces. (credit: NIH via Wikimedia Commons)

Section Summary

- The human body converts energy stored in food into work, thermal energy, and/or chemical energy that is stored in fatty tissue.
- The *rate* at which the body uses food energy to sustain life and to do different activities is called the metabolic rate, and the corresponding rate when at rest is called the basal metabolic rate (BMR)
- The energy included in the basal metabolic rate is divided among various systems in the body, with the largest fraction going to the liver and spleen, and the brain coming next.
- About 75% of food calories are used to sustain basic body functions included in the basal metabolic rate.

- The energy consumption of people during various activities can be determined by measuring their oxygen use, because the digestive process is basically one of oxidizing food.

Conceptual Questions

Exercise:

Problem:

Explain why it is easier to climb a mountain on a zigzag path rather than one straight up the side. Is your increase in gravitational potential energy the same in both cases? Is your energy consumption the same in both?

Exercise:

Problem:

Do you do work on the outside world when you rub your hands together to warm them? What is the efficiency of this activity?

Exercise:

Problem:

Shivering is an involuntary response to lowered body temperature. What is the efficiency of the body when shivering, and is this a desirable value?

Exercise:

Problem:

Discuss the relative effectiveness of dieting and exercise in losing weight, noting that most athletic activities consume food energy at a rate of 400 to 500 W, while a single cup of yogurt can contain 1360 kJ (325 kcal). Specifically, is it likely that exercise alone will be sufficient to lose weight? You may wish to consider that regular exercise may increase the metabolic rate, whereas protracted dieting may reduce it.

Problems & Exercises

Exercise:

Problem:

(a) How long can you rapidly climb stairs (116/min) on the 93.0 kcal of energy in a 10.0-g pat of butter? (b) How many flights is this if each flight has 16 stairs?

Solution:

(a) 9.5 min

(b) 69 flights of stairs

Exercise:

Problem:

(a) What is the power output in watts and horsepower of a 70.0-kg sprinter who accelerates from rest to 10.0 m/s in 3.00 s? (b) Considering the amount of power generated, do you think a well-trained athlete could do this repetitively for long periods of time?

Exercise:

Problem:

Calculate the power output in watts and horsepower of a shot-putter who takes 1.20 s to accelerate the 7.27-kg shot from rest to 14.0 m/s, while raising it 0.800 m. (Do not include the power produced to accelerate his body.)

Shot putter at the Dornoch Highland Gathering in 2007. (credit: John Haslam, Flickr)

Solution:

641 W, 0.860 hp

Exercise:

Problem:

(a) What is the efficiency of an out-of-condition professor who does 2.10×10^5 J of useful work while metabolizing 500 kcal of food energy? (b) How many food calories would a well-conditioned athlete metabolize in doing the same work with an efficiency of 20%?

Exercise:

Problem:

Energy that is not utilized for work or heat transfer is converted to the chemical energy of body fat containing about 39 kJ/g. How many grams of fat will you gain if you eat 10,000 kJ (about 2500 kcal) one day and do nothing but sit relaxed for 16.0 h and sleep for the other 8.00 h? Use data from [link] for the energy consumption rates of these activities.

Solution:

31 g

Exercise:

Problem:

Using data from [link], calculate the daily energy needs of a person who sleeps for 7.00 h, walks for 2.00 h, attends classes for 4.00 h, cycles for 2.00 h, sits relaxed for 3.00 h, and studies for 6.00 h. (Studying consumes energy at the same rate as sitting in class.)

Exercise:

Problem:

What is the efficiency of a subject on a treadmill who puts out work at the rate of 100 W while consuming oxygen at the rate of 2.00 L/min? (Hint: See [link].)

Solution:

14.3%

Exercise:

Problem:

Shoveling snow can be extremely taxing because the arms have such a low efficiency in this activity. Suppose a person shoveling a footpath metabolizes food at the rate of 800 W. (a) What is her useful power output? (b) How long will it take her to lift 3000 kg of snow 1.20 m? (This could be the amount of heavy snow on 20 m of footpath.) (c) How much waste heat transfer in kilojoules will she generate in the process?

Exercise:

Problem:

Very large forces are produced in joints when a person jumps from some height to the ground. (a) Calculate the magnitude of the force produced if an 80.0-kg person jumps from a 0.600–m-high ledge and lands stiffly, compressing joint material 1.50 cm as a result. (Be certain to include the weight of the person.) (b) In practice the knees bend almost involuntarily to help extend the distance over which you stop. Calculate the magnitude of the force produced if the stopping distance is 0.300 m. (c) Compare both forces with the weight of the person.

Solution:

(a) 3.21×10^{4} N

(b) 2.35×10^{3} N

(c) Ratio of net force to weight of person is 41.0 in part (a); 3.00 in part (b)

Exercise:

Problem:

Jogging on hard surfaces with insufficiently padded shoes produces large forces in the feet and legs. (a) Calculate the magnitude of the force needed to stop the downward motion of a jogger’s leg, if his leg has a mass of 13.0 kg, a speed of 6.00 m/s, and stops in a distance of 1.50 cm. (Be certain to include the weight of the 75.0-kg jogger’s body.) (b) Compare this force with the weight of the jogger.

Exercise:

Problem:

(a) Calculate the energy in kJ used by a 55.0-kg woman who does 50 deep knee bends in which her center of mass is lowered and raised 0.400 m. (She does work in both directions.) You may assume her efficiency is 20%. (b) What is the average power consumption rate in watts if she does this in 3.00 min?

Solution:

(a) 108 kJ

(b) 599 W

Exercise:

Problem:

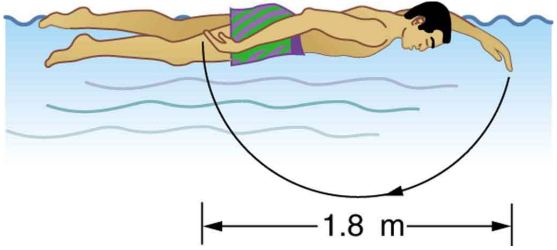
Kanellos Kanellopoulos flew 119 km from Crete to Santorini, Greece, on April 23, 1988, in the *Daedalus 88*, an aircraft powered by a bicycle-type drive mechanism (see [link]). His useful power output for the 234-min trip was about 350 W. Using the efficiency for cycling from [link], calculate the food energy in kilojoules he metabolized during the flight.

The Daedalus 88 in flight. (credit: NASA photo by Beasley)

Exercise:

Problem:

The swimmer shown in [link] exerts an average horizontal backward force of 80.0 N with his arm during each 1.80 m long stroke. (a) What is his work output in each stroke? (b) Calculate the power output of his arms if he does 120 strokes per minute.



Solution:

(a) 144 J

(b) 288 W

Exercise:

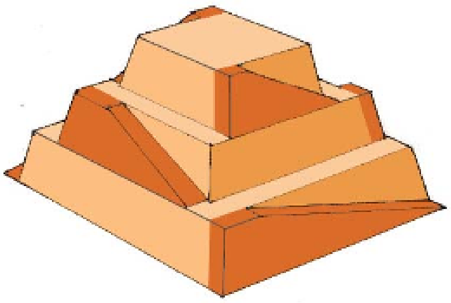
Problem:

Mountain climbers carry bottled oxygen when at very high altitudes. (a) Assuming that a mountain climber uses oxygen at twice the rate for climbing 116 stairs per minute (because of low air temperature and winds), calculate how many liters of oxygen a climber would need for 10.0 h of climbing. (These are liters at sea level.) Note that only 40% of the inhaled oxygen is utilized; the rest is exhaled. (b) How much useful work does the climber do if he and his equipment have a mass of 90.0 kg and he gains 1000 m of altitude? (c) What is his efficiency for the 10.0-h climb?

Exercise:

Problem:

The awe-inspiring Great Pyramid of Cheops was built more than 4500 years ago. Its square base, originally 230 m on a side, covered 13.1 acres, and it was 146 m high, with a mass of about 7×10^9 kg. (The pyramid's dimensions are slightly different today due to quarrying and some sagging.) Historians estimate that 20,000 workers spent 20 years to construct it, working 12-hour days, 330 days per year. (a) Calculate the gravitational potential energy stored in the pyramid, given its center of mass is at one-fourth its height. (b) Only a fraction of the workers lifted blocks; most were involved in support services such as building ramps (see [link]), bringing food and water, and hauling blocks to the site. Calculate the efficiency of the workers who did the lifting, assuming there were 1000 of them and they consumed food energy at the rate of 300 kcal/h. What does your answer imply about how much of their work went into block-lifting, versus how much work went into friction and lifting and lowering their own bodies? (c) Calculate the mass of food that had to be supplied each day, assuming that the average worker required 3600 kcal per day and that their diet was 5% protein, 60% carbohydrate, and 35% fat. (These proportions neglect the mass of bulk and nondigestible materials consumed.)



Ancient pyramids were probably constructed using ramps as simple machines. (credit: Franck Monnier, Wikimedia Commons)

Solution:

(a) 2.50×10^{12} J

(b) 2.52%

(c) 1.4×10^{4} kg (14 metric tons)

Exercise:

Problem:

(a) How long can you play tennis on the 800 kJ (about 200 kcal) of energy in a candy bar? (b) Does this seem like a long time? Discuss why exercise is necessary but may not be sufficient to cause a person to lose weight.

Glossary

metabolic rate
the rate at which the body uses food energy to sustain life and to do different activities

basal metabolic rate
the total energy conversion rate of a person at rest

useful work
work done on an external system

Temperature

- Define temperature.
- Convert temperatures between the Celsius, Fahrenheit, and Kelvin scales.
- Define thermal equilibrium.
- State the zeroth law of thermodynamics.

The concept of temperature has evolved from the common concepts of hot and cold. Human perception of what feels hot or cold is a relative one. For example, if you place one hand in hot water and the other in cold water, and then place both hands in tepid water, the tepid water will feel cool to the hand that was in hot water, and warm to the one that was in cold water. The scientific definition of temperature is less ambiguous than your senses of hot and cold. **Temperature** is operationally defined to be what we measure with a thermometer. (Many physical quantities are defined solely in terms of how they are measured. We shall see later how temperature is related to the kinetic energies of atoms and molecules, a more physical explanation.) Two accurate thermometers, one placed in hot water and the other in cold water, will show the hot water to have a higher temperature. If they are then placed in the tepid water, both will give identical readings (within measurement uncertainties). In this section, we discuss temperature, its measurement by thermometers, and its relationship to thermal equilibrium. Again, temperature is the quantity measured by a thermometer.

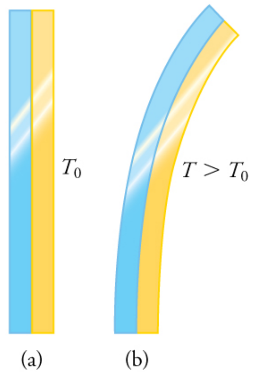
Note:

Misconception Alert: Human Perception vs. Reality

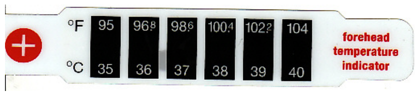
On a cold winter morning, the wood on a porch feels warmer than the metal of your bike. The wood and bicycle are in thermal equilibrium with the outside air, and are thus the same temperature. They *feel* different because of the difference in the way that they conduct heat away from your skin. The metal conducts heat away from your body faster than the wood does (see more about conductivity in Conduction). This is just one example demonstrating that the human sense of hot and cold is not determined by temperature alone.

Another factor that affects our perception of temperature is humidity. Most people feel much hotter on hot, humid days than on hot, dry days. This is because on humid days, sweat does not evaporate from the skin as efficiently as it does on dry days. It is the evaporation of sweat (or water from a sprinkler or pool) that cools us off.

Any physical property that depends on temperature, and whose response to temperature is reproducible, can be used as the basis of a thermometer. Because many physical properties depend on temperature, the variety of thermometers is remarkable. For example, volume increases with temperature for most substances. This property is the basis for the common alcohol thermometer, the old mercury thermometer, and the bimetallic strip ([link]). Other properties used to measure temperature include electrical resistance and color, as shown in [link], and the emission of infrared radiation, as shown in [link].



The curvature of a bimetallic strip depends on temperature. (a) The strip is straight at the starting temperature, where its two components have the same length. (b) At a higher temperature, this strip bends to the right, because the metal on the left has expanded more than the metal on the right.



Each of the six squares on this plastic (liquid crystal) thermometer contains a film of a different heat-sensitive liquid crystal material. Below 95ºF, all six squares are black. When the plastic thermometer is exposed to temperature that increases to 95ºF, the first liquid crystal square changes color. When the temperature increases above 96.8ºF the second liquid crystal square also changes color, and so forth. (credit: Arkrishna, Wikimedia Commons)

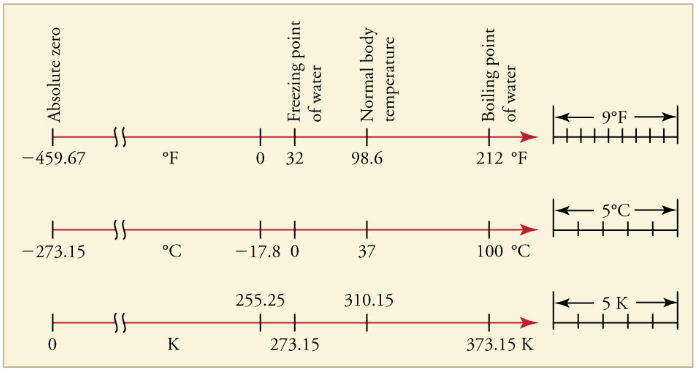
Fireman Jason Ormand uses a pyrometer to check the temperature of an aircraft carrier’s ventilation system. Infrared radiation (whose emission varies with temperature) from the vent is measured and a temperature readout is quickly produced. Infrared measurements are also frequently used as a measure of body temperature. These modern thermometers, placed in the ear canal, are more accurate than alcohol thermometers placed under the tongue or in the armpit. (credit: Lamel J. Hinton/U.S. Navy)

Temperature Scales

Thermometers are used to measure temperature according to well-defined scales of measurement, which use pre-defined reference points to help compare quantities. The three most common temperature scales are the Fahrenheit, Celsius, and Kelvin scales. A temperature scale can be created by identifying two easily reproducible temperatures. The freezing and boiling temperatures of water at standard atmospheric pressure are commonly used.

The **Celsius** scale (which replaced the slightly different *centigrade* scale) has the freezing point of water at 0^{o}C and the boiling point at 100^{o}C. Its unit is the **degree Celsius**$(^{\text{o}}\text{C})$. On the **Fahrenheit** scale (still the most frequently used in the United States), the freezing point of water is at 32^{o}F and the boiling point is at 212^{o}F. The unit of temperature on this scale is the **degree Fahrenheit**$(^{\text{o}}\text{F})$. Note that a temperature difference of one degree Celsius is greater than a temperature difference of one degree Fahrenheit. Only 100 Celsius degrees span the same range as 180 Fahrenheit degrees, thus one degree on the Celsius scale is 1.8 times larger than one degree on the Fahrenheit scale $180/100=9/5$.

The **Kelvin** scale is the temperature scale that is commonly used in science. It is an *absolute temperature* scale defined to have 0 K at the lowest possible temperature, called **absolute zero**. The official temperature unit on this scale is the *kelvin*, which is abbreviated K, and is not accompanied by a degree sign. The freezing and boiling points of water are 273.15 K and 373.15 K, respectively. Thus, the magnitude of temperature differences is the same in units of kelvins and degrees Celsius. Unlike other temperature scales, the Kelvin scale is an absolute scale. It is used extensively in scientific work because a number of physical quantities, such as the volume of an ideal gas, are directly related to absolute temperature. The kelvin is the SI unit used in scientific work.



Relationships between the Fahrenheit, Celsius, and Kelvin temperature scales, rounded to the nearest degree. The relative sizes of the scales are also shown.

The relationships between the three common temperature scales is shown in [link]. Temperatures on these scales can be converted using the equations in [link].

To convert from . . .	**Use this equation . . .**	**Also written as . . .**

To convert from . . .	**Use this equation . . .**	**Also written as . . .**
Celsius to Fahrenheit	$T(^{\circ}\text{F}) = \frac{9}{5}T(^{\circ}\text{C}) + 32$	$T_{^{\circ}\text{F}} = \frac{9}{5}T_{^{\circ}\text{C}} + 32$
Fahrenheit to Celsius	$T(^{\circ}\text{C}) = \frac{5}{9}(T(^{\circ}\text{F}) - 32)$	$T_{^{\circ}\text{C}} = \frac{5}{9}\left(T_{^{\circ}\text{F}} - 32\right)$
Celsius to Kelvin	$T(\text{K}) = T(^{\circ}\text{C}) + 273.15$	$T_{\text{K}} = T_{^{\circ}\text{C}} + 273.15$
Kelvin to Celsius	$T(^{\circ}\text{C}) = T(\text{K}) - 273.15$	$T_{^{\circ}\text{C}} = T_{\text{K}} - 273.15$
Fahrenheit to Kelvin	$T(\text{K}) = \frac{5}{9}(T(^{\circ}\text{F}) - 32) + 273.15$	$T_{\text{K}} = \frac{5}{9}\left(T_{^{\circ}\text{F}} - 32\right) + 273.15$
Kelvin to Fahrenheit	$T(^{\circ}\text{F}) = \frac{9}{5}(T(\text{K}) - 273.15) + 32$	$T_{^{\circ}\text{F}} = \frac{9}{5}(T_{\text{K}} - 273.15) + 32$

Temperature Conversions

Notice that the conversions between Fahrenheit and Kelvin look quite complicated. In fact, they are simple combinations of the conversions between Fahrenheit and Celsius, and the conversions between Celsius and Kelvin.

Example:
Converting between Temperature Scales: Room Temperature
“Room temperature” is generally defined to be 25°C. (a) What is room temperature in ºF? (b) What is it in K?

Strategy
To answer these questions, all we need to do is choose the correct conversion equations and plug in the known values.

Solution for (a)

1. Choose the right equation. To convert from ºC to ºF, use the equation

Equation:

$$T_{\text{ºF}} = \frac{9}{5}T_{\text{ºC}} + 32.$$

2. Plug the known value into the equation and solve:

Equation:

$$T_{\text{ºF}} = \frac{9}{5}25\text{ºC} + 32 = 77\text{ºF}.$$

Solution for (b)

1. Choose the right equation. To convert from ºC to K, use the equation

Equation:

$$T_{\text{K}} = T_{\text{ºC}} + 273.15.$$

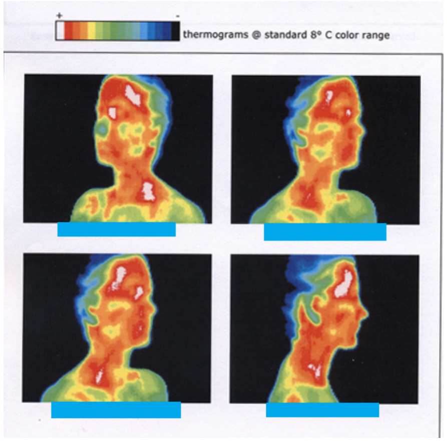
2. Plug the known value into the equation and solve:

Equation:

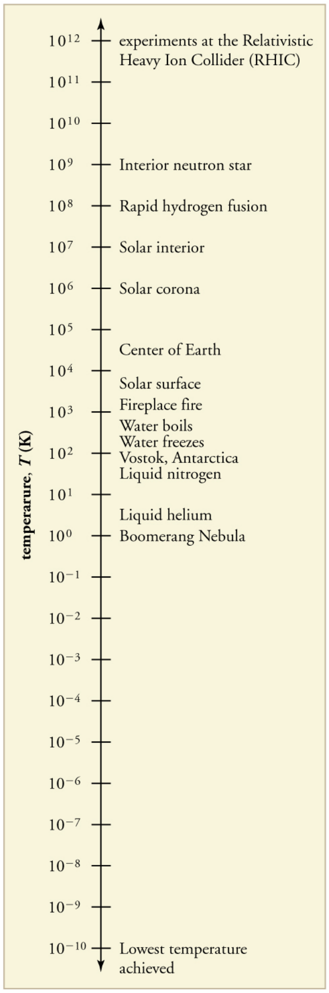
$$T_{\text{K}} = 25\text{ºC} + 273.15 = 298\,\text{K}.$$

Temperature Ranges in the Universe

[link] shows the wide range of temperatures found in the universe. Human beings have been known to survive with body temperatures within a small range, from 24ºC to 44ºC (75ºF to 111ºF). The average normal body temperature is usually given as 37.0ºC (98.6ºF), and variations in this temperature can indicate a medical condition: a fever, an infection, a tumor, or circulatory problems (see [link]).



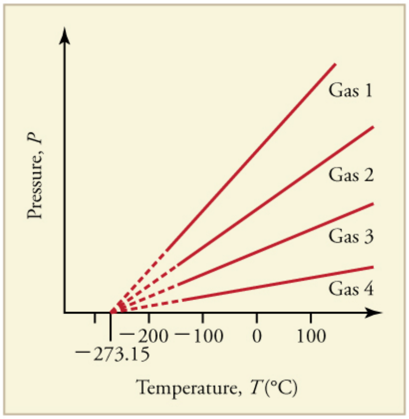
This image of radiation from a person’s body (an infrared thermograph) shows the location of temperature abnormalities in the upper body. Dark blue corresponds to cold areas and red to white corresponds to hot areas. An elevated temperature might be an indication of malignant tissue (a cancerous tumor in the breast, for example), while a depressed temperature might be due to a decline in blood flow from a clot. In this case, the abnormalities are caused by a condition called hyperhidrosis. (credit: Porcelina81, Wikimedia Commons)



Each increment on this logarithmic scale indicates an increase by a factor of ten, and thus illustrates the tremendous range of temperatures in nature. Note that zero on a logarithmic scale would occur off the bottom of the page at infinity.

Note:
Making Connections: Absolute Zero

What is absolute zero? Absolute zero is the temperature at which all molecular motion has ceased. The concept of absolute zero arises from the behavior of gases. [link] shows how the pressure of gases at a constant volume decreases as temperature decreases. Various scientists have noted that the pressures of gases extrapolate to zero at the same temperature, 273.15°C. This extrapolation implies that there is a lowest temperature. This temperature is called *absolute zero*. Today we know that most gases first liquefy and then freeze, and it is not actually possible to reach absolute zero. The numerical value of absolute zero temperature is 273.15°C or 0 K.



Graph of pressure versus temperature for various gases kept at a constant volume. Note that all of the graphs extrapolate to zero pressure at the same temperature.

Section Summary

- Temperature is the quantity measured by a thermometer.
- Temperature is related to the average kinetic energy of atoms and molecules in a system.
- Absolute zero is the temperature at which there is no molecular motion.
- There are three main temperature scales: Celsius, Fahrenheit, and Kelvin.
- Temperatures on one scale can be converted to temperatures on another scale using the following equations:
  Equation:

$$T_{^\circ\text{F}} = \frac{9}{5} T_{^\circ\text{C}} + 32$$

Equation:

$$T_{^\circ\text{C}} = \frac{5}{9}\left(T_{^\circ\text{F}} - 32\right)$$

Equation:

$$T_{\text{K}} = T_{^\circ\text{C}} + 273.15$$

Equation:

$$T_{^\circ\text{C}} = T_{\text{K}} - 273.15$$

- Systems are in thermal equilibrium when they have the same temperature.
- Thermal equilibrium occurs when two bodies are in contact with each other and can freely exchange energy.

Conceptual Questions

Exercise:

Problem: What does it mean to say that two systems are in thermal equilibrium?

Exercise:

Problem:

Give an example of a physical property that varies with temperature and describe how it is used to measure temperature.

Exercise:

Problem:

When a cold alcohol thermometer is placed in a hot liquid, the column of alcohol goes *down* slightly before going up. Explain why.

Problems & Exercises

Exercise:

Problem: What is the Fahrenheit temperature of a person with a 39.0°C fever?

Solution:

102ºF

Exercise:

Problem:

Frost damage to most plants occurs at temperatures of 28.0ºF or lower. What is this temperature on the Kelvin scale?

Exercise:

Problem:

To conserve energy, room temperatures are kept at 68.0ºF in the winter and 78.0ºF in the summer. What are these temperatures on the Celsius scale?

Solution:

20.0ºC and 25.6ºC

Exercise:

Problem:

A tungsten light bulb filament may operate at 2900 K. What is its Fahrenheit temperature? What is this on the Celsius scale?

Exercise:

Problem:

The surface temperature of the Sun is about 5750 K. What is this temperature on the Fahrenheit scale?

Solution:

9890ºF

Exercise:

Problem:

One of the hottest temperatures ever recorded on the surface of Earth was 134ºF in Death Valley, CA. What is this temperature in Celsius degrees? What is this temperature in Kelvin?

Exercise:

Problem:

(a) Suppose a cold front blows into your locale and drops the temperature by 40.0 Fahrenheit degrees. How many degrees Celsius does the temperature decrease when there is a 40.0^{o}F decrease in temperature? (b) Show that any change in temperature in Fahrenheit degrees is nine-fifths the change in Celsius degrees.

Solution:

(a) 22.2^{o}C

(b)

$$\begin{aligned} \Delta T(^{\text{o}}\text{F}) &= T_2(^{\text{o}}\text{F}) - T_1(^{\text{o}}\text{F}) \\ &= \frac{9}{5}T_2(^{\text{o}}\text{C}) + 32.0^{\text{o}} - \left(\frac{9}{5}T_1(^{\text{o}}\text{C}) + 32.0^{\text{o}}\right) \\ &= \frac{9}{5}\left(T_2(^{\text{o}}\text{C}) - T_1(^{\text{o}}\text{C})\right) = \frac{9}{5}\Delta T(^{\text{o}}\text{C}) \end{aligned}$$

Exercise:

Problem:

(a) At what temperature do the Fahrenheit and Celsius scales have the same numerical value? (b) At what temperature do the Fahrenheit and Kelvin scales have the same numerical value?

Glossary

temperature	the quantity measured by a thermometer
Celsius scale	temperature scale in which the freezing point of water is 0^{o}C and the boiling point of water is 100^{o}C
degree Celsius	unit on the Celsius temperature scale
Fahrenheit scale	temperature scale in which the freezing point of water is 32^{o}F and the boiling point of water is 212^{o}F
degree Fahrenheit	unit on the Fahrenheit temperature scale
Kelvin scale	temperature scale in which 0 K is the lowest possible temperature, representing absolute zero

absolute zero
the lowest possible temperature; the temperature at which all molecular motion ceases

thermal equilibrium
the condition in which heat no longer flows between two objects that are in contact; the two objects have the same temperature

zeroth law of thermodynamics
law that states that if two objects are in thermal equilibrium, and a third object is in thermal equilibrium with one of those objects, it is also in thermal equilibrium with the other object

Thermal Expansion of Solids and Liquids

- Define and describe thermal expansion.
- Calculate the linear expansion of an object given its initial length, change in temperature, and coefficient of linear expansion.
- Calculate the volume expansion of an object given its initial volume, change in temperature, and coefficient of volume expansion.
- Calculate thermal stress on an object given its original volume, temperature change, volume change, and bulk modulus.

Thermal expansion joints like these in the Auckland Harbour Bridge in New Zealand allow bridges to change length without buckling. (credit: Ingolfson, Wikimedia Commons)

The expansion of alcohol in a thermometer is one of many commonly encountered examples of **thermal expansion**, the change in size or volume of a given mass with temperature. Hot air rises because its volume increases, which causes the hot air’s density to be smaller than the density of surrounding air, causing a buoyant (upward) force on the hot air. The same happens in all liquids and gases, driving natural heat transfer upwards in homes, oceans, and weather systems. Solids also undergo thermal expansion. Railroad tracks and bridges, for example, have expansion joints to allow them to freely expand and contract with temperature changes.

What are the basic properties of thermal expansion? First, thermal expansion is clearly related to temperature change. The greater the temperature change, the more a bimetallic strip will bend. Second, it depends on the material. In a thermometer, for example, the expansion of alcohol is much greater than the expansion of the glass containing it.

What is the underlying cause of thermal expansion? As is discussed in Kinetic Theory: Atomic and Molecular Explanation of Pressure and Temperature, an increase in temperature implies an increase in the kinetic energy of the individual atoms. In a solid, unlike in a gas, the atoms or molecules are closely packed together, but their kinetic energy (in the form of small, rapid vibrations) pushes neighboring atoms or molecules apart from each other. This neighbor-to-neighbor pushing results in a slightly greater distance, on average, between neighbors, and adds up to a larger size for the whole body. For most substances under ordinary conditions, there is no preferred direction, and an increase in temperature will increase the solid’s size by a certain fraction in each dimension.

Note:
Linear Thermal Expansion—Thermal Expansion in One Dimension
The change in length ΔL is proportional to length L. The dependence of thermal expansion on temperature, substance, and length is summarized in the equation
Equation:

$$\Delta L = \alpha L \Delta T,$$

where ΔL is the change in length L, ΔT is the change in temperature, and α is the **coefficient of linear expansion**, which varies slightly with temperature.

[link] lists representative values of the coefficient of linear expansion, which may have units of $1/^{\circ}\text{C}$ or 1/K. Because the size of a kelvin and a degree Celsius are the same, both α and ΔT can be expressed in units of kelvins or degrees Celsius. The equation $\Delta L = \alpha L \Delta T$ is accurate for small changes in temperature and can be used for large changes in temperature if an average value of α is used.

Material	**Coefficient of linear expansion** $\alpha(1/^{\circ}\text{C})$	**Coefficient of volume expansion** $\beta(1/^{\circ}\text{C})$
Solids		
Aluminum	25×10^{-6}	75×10^{-6}
Brass	19×10^{-6}	56×10^{-6}
Copper	17×10^{-6}	51×10^{-6}

Material	**Coefficient of linear expansion** $\alpha(1/^{\mathrm{o}}\mathrm{C})$	**Coefficient of volume expansion** $\beta(1/^{\mathrm{o}}\mathrm{C})$
Gold	14×10^{-6}	42×10^{-6}
Iron or Steel	12×10^{-6}	35×10^{-6}
Invar (Nickel-iron alloy)	0.9×10^{-6}	2.7×10^{-6}
Lead	29×10^{-6}	87×10^{-6}
Silver	18×10^{-6}	54×10^{-6}
Glass (ordinary)	9×10^{-6}	27×10^{-6}
Glass (Pyrex®)	3×10^{-6}	9×10^{-6}
Quartz	0.4×10^{-6}	1×10^{-6}

Material	**Coefficient of linear expansion** $\alpha(1/^{o}\mathrm{C})$	**Coefficient of volume expansion** $\beta(1/^{o}\mathrm{C})$
Concrete, Brick	~12×10^{-6}	~36×10^{-6}
Marble (average)	2.5×10^{-6}	7.5×10^{-6}
Liquids		
Ether		1650×10^{-6}
Ethyl alcohol		1100×10^{-6}
Petrol		950×10^{-6}
Glycerin		500×10^{-6}
Mercury		180×10^{-6}

Material	**Coefficient of linear expansion** $\alpha(1/{}^{\circ}\text{C})$	**Coefficient of volume expansion** $\beta(1/{}^{\circ}\text{C})$
Water		210×10^{-6}
Gases		
Air and most other gases at atmospheric pressure		3400×10^{-6}

Thermal Expansion Coefficients at 20°C**[footnote]**
Values for liquids and gases are approximate.

Example:
Calculating Linear Thermal Expansion: The Golden Gate Bridge
The main span of San Francisco's Golden Gate Bridge is 1275 m long at its coldest. The bridge is exposed to temperatures ranging from -15°C to 40°C. What is its change in length between these temperatures? Assume that the bridge is made entirely of steel.

Strategy
Use the equation for linear thermal expansion $\Delta L = \alpha L \Delta T$ to calculate the change in length , ΔL. Use the coefficient of linear expansion, α, for steel from [link], and note that the change in temperature, ΔT, is 55°C.

Solution
Plug all of the known values into the equation to solve for ΔL.

Equation:

$$\Delta L = \alpha L \Delta T = \left(\frac{12 \times 10^{-6}}{{}^{\circ}\text{C}}\right)(1275\ \text{m})(55^{\circ}\text{C}) = 0.84\ \text{m}.$$

Discussion
Although not large compared with the length of the bridge, this change in length is observable. It is generally spread over many expansion joints so that the expansion at each joint is small.

Section Summary

- Thermal expansion is the increase, or decrease, of the size (length, area, or volume) of a body due to a change in temperature.
- Thermal expansion is large for gases, and relatively small, but not negligible, for liquids and solids.
- Linear thermal expansion is
  Equation:

  $$\Delta L = \alpha L \Delta T,$$

  where ΔL is the change in length L, ΔT is the change in temperature, and α is the coefficient of linear expansion, which varies slightly with temperature.
- The change in area due to thermal expansion is
  Equation:

  $$\Delta A = 2\alpha A \Delta T,$$

  where ΔA is the change in area.
- The change in volume due to thermal expansion is
  Equation:

  $$\Delta V = \beta V \Delta T,$$

  where β is the coefficient of volume expansion and $\beta \approx 3\alpha$. Thermal stress is created when thermal expansion is constrained.

Glossary

thermal expansion

the change in size or volume of an object with change in temperature

coefficient of linear expansion

α, the change in length, per unit length, per 1ºC change in temperature; a constant used in the calculation of linear expansion; the coefficient of linear expansion depends on the material and to some degree on the temperature of the material

coefficient of volume expansion

β, the change in volume, per unit volume, per 1ºC change in temperature

thermal stress

stress caused by thermal expansion or contraction

Density

LEARNING OBJECTIVES

By the end of this section, you will be able to:

- Define density.
- Calculate the mass of a reservoir from its density.
- Compare and contrast the densities of various substances.

The information presented in this section supports the following AP® learning objectives and science practices:

- **1.E.1.1** The student is able to predict the densities, differences in densities, or changes in densities under different conditions for natural phenomena and design an investigation to verify the prediction. **(S.P. 6.2, 6.4)**
- **1.E.1.2** The student is able to select from experimental data the information necessary to determine the density of an object and/or compare densities of several objects. **(S.P. 4.1, 6.4)**

Which weighs more, a ton of feathers or a ton of bricks? This old riddle plays with the distinction between mass and density. A ton is a ton, of course; but bricks have much greater density than feathers, and so we are tempted to think of them as heavier. (See [link].)

Density, as you will see, is an important characteristic of substances. It is crucial, for example, in determining whether an object sinks or floats in a fluid. Density is the mass per unit volume of a substance or object. In equation form, density is defined as

Equation:

$$\rho = \frac{m}{V},$$

where the Greek letter ρ (rho) is the symbol for density, m is the mass, and V is the volume occupied by the substance.

Note:

Density

Density is mass per unit volume.

Equation:

$$\rho = \frac{m}{V},$$

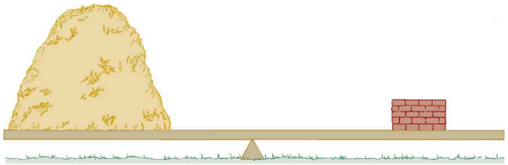
where ρ is the symbol for density, m is the mass, and V is the volume occupied by the substance.

In the riddle regarding the feathers and bricks, the masses are the same, but the volume occupied by the feathers is much greater, since their density is much lower. The SI unit of density is kg/m^3, representative values are given in [link]. The metric system was originally devised so that water would have a density of $1\ \text{g/cm}^3$, equivalent to $10^3\ \text{kg/m}^3$. Thus the basic mass unit, the kilogram, was first devised to be the mass of 1000 mL of water, which has a volume of 1000 cm^3.

Substance	$\rho(10^3\ \text{kg/m}^3\ \text{or g/mL})$	**Substance**	$\rho(10^3\ \text{kg/m}^3\ \text{or g/mL})$	**Substance**	$\rho($
Solids		**Liquids**		**Gases**	
Aluminum	2.7	Water (4ºC)	1.000	Air	
Brass	8.44	Blood	1.05	Carbon dioxide	
Copper (average)	8.8	Sea water	1.025	Carbon monoxide	
Gold	19.32	Mercury	13.6	Hydrogen	
Iron or steel	7.8	Ethyl alcohol	0.79	Helium	
Lead	11.3	Petrol	0.68	Methane	
Polystyrene	0.10	Glycerin	1.26	Nitrogen	
Tungsten	19.30	Olive oil	0.92	Nitrous oxide	
Uranium	18.70			Oxygen	
Concrete	2.30–3.0			Steam $(100^{\text{o}}\ \text{C})$	
Cork	0.24				

Substance	$\rho(10^3\ \mathrm{kg/m^3}\ \text{or}\ \mathrm{g/mL})$	**Substance**	$\rho(10^3\ \mathrm{kg/m^3}\ \text{or}\ \mathrm{g/mL})$	**Substance**	$\rho($
Glass, common (average)	2.6				
Granite	2.7				
Earth's crust	3.3				
Wood	0.3–0.9				
Ice (0°C)	0.917				
Bone	1.7–2.0				

Densities of Various Substances



A ton of feathers and a ton of bricks have the same mass, but the feathers make a much bigger pile because they have a much lower density.

As you can see by examining [link], the density of an object may help identify its composition. The density of gold, for example, is about 2.5 times the density of iron, which is about 2.5 times the density of aluminum. Density also reveals something about the phase of the matter and its substructure. Notice that the densities of liquids and solids are roughly comparable, consistent with the fact that their atoms are in close contact. The densities of gases are much less than those of liquids and solids, because the atoms in gases are separated by large amounts of empty space.

Note:
Take-Home Experiment Sugar and Salt
A pile of sugar and a pile of salt look pretty similar, but which weighs more? If the volumes of both piles are the same, any difference in mass is due to their different densities (including the air space between crystals). Which do you think has the greater density? What values did you find? What method did you use to determine these values?

Example:
Calculating the Mass of a Reservoir From Its Volume
A reservoir has a surface area of $50.0\ \mathrm{km}^2$ and an average depth of 40.0 m. What mass of water is held behind the dam? (See [link] for a view of a large reservoir—the Three Gorges Dam site on the Yangtze River in central

China.)

Strategy

We can calculate the volume V of the reservoir from its dimensions, and find the density of water ρ in [link]. Then the mass m can be found from the definition of density

Equation:

$$\rho = \frac{m}{V}.$$

Solution

Solving equation $\rho = m/V$ for m gives $m = \rho V$.

The volume V of the reservoir is its surface area A times its average depth h:

Equation:

$$\begin{aligned} V &= \text{Ah} = \left(50.0\ \text{km}^2\right)\left(40.0\ \text{m}\right) \\ &= \left[\left(50.0\ \text{km}^2\right)\left(\frac{10^3\ \text{m}}{1\ \text{km}}\right)^2\right]\left(40.0\ \text{m}\right) = 2.00 \times 10^9\ \text{m}^3 \end{aligned}$$

The density of water ρ from [link] is $1.000 \times 10^3\ \text{kg/m}^3$. Substituting V and ρ into the expression for mass gives

Equation:

$$\begin{aligned} m &= \left(1.00 \times 10^3\ \text{kg/m}^3\right)\left(2.00 \times 10^9\ \text{m}^3\right) \\ &= 2.00 \times 10^{12}\ \text{kg}. \end{aligned}$$

Discussion

A large reservoir contains a very large mass of water. In this example, the weight of the water in the reservoir is $\text{mg} = 1.96 \times 10^{13}\ \text{N}$, where g is the acceleration due to the Earth's gravity (about $9.80\ \text{m/s}^2$). It is reasonable to ask whether the dam must supply a force equal to this tremendous weight. The answer is no. As we shall see in the following sections, the force the dam must supply can be much smaller than the weight of the water it holds back.

Three Gorges Dam in central China. When completed in 2008, this became the world's largest hydroelectric plant, generating power equivalent to that generated by 22 average-sized nuclear power plants. The concrete dam is 181 m high and 2.3 km across. The reservoir made by this dam is 660 km long. Over 1 million people were displaced by the creation of the reservoir. (credit: Le Grand Portage)

Section Summary

- Density is the mass per unit volume of a substance or object. In equation form, density is defined as **Equation:**

$$\rho = \frac{m}{V}.$$

- The SI unit of density is kg/m^3.

Conceptual Questions

Exercise:

Problem: Approximately how does the density of air vary with altitude?

Exercise:

Problem:

Give an example in which density is used to identify the substance composing an object. Would information in addition to average density be needed to identify the substances in an object composed of more than one material?

Exercise:

Problem:

[link] shows a glass of ice water filled to the brim. Will the water overflow when the ice melts? Explain your answer.

Problems & Exercises

Exercise:

Problem: Gold is sold by the troy ounce (31.103 g). What is the volume of 1 troy ounce of pure gold?

Solution:

$1.610\ \text{cm}^3$

Exercise:

Problem:

Mercury is commonly supplied in flasks containing 34.5 kg (about 76 lb). What is the volume in liters of this much mercury?

Exercise:

Problem:

(a) What is the mass of a deep breath of air having a volume of 2.00 L? (b) Discuss the effect taking such a breath has on your body's volume and density.

Solution:

(a) 2.58 g

(b) The volume of your body increases by the volume of air you inhale. The average density of your body decreases when you take a deep breath, because the density of air is substantially smaller than the average density of the body before you took the deep breath.

Exercise:

Problem:

A straightforward method of finding the density of an object is to measure its mass and then measure its volume by submerging it in a graduated cylinder. What is the density of a 240-g rock that displaces 89.0 cm^3 of water? (Note that the accuracy and practical applications of this technique are more limited than a variety of others that are based on Archimedes' principle.)

Solution:

2.70 g/cm^3

Exercise:

Problem:

Suppose you have a coffee mug with a circular cross section and vertical sides (uniform radius). What is its inside radius if it holds 375 g of coffee when filled to a depth of 7.50 cm? Assume coffee has the same density as water.

Exercise:

Problem:

(a) A rectangular gasoline tank can hold 50.0 kg of gasoline when full. What is the depth of the tank if it is 0.500-m wide by 0.900-m long? (b) Discuss whether this gas tank has a reasonable volume for a passenger car.

Solution:

(a) 0.163 m

(b) Equivalent to 19.4 gallons, which is reasonable

Exercise:

Problem:

A trash compactor can reduce the volume of its contents to 0.350 their original value. Neglecting the mass of air expelled, by what factor is the density of the rubbish increased?

Exercise:

Problem:

A 2.50-kg steel gasoline can holds 20.0 L of gasoline when full. What is the average density of the full gas can, taking into account the volume occupied by steel as well as by gasoline?

Solution:

$7.9 \times 10^2 \text{ kg/m}^3$

Exercise:

Problem:

What is the density of 18.0-karat gold that is a mixture of 18 parts gold, 5 parts silver, and 1 part copper? (These values are parts by mass, not volume.) Assume that this is a simple mixture having an average density equal to the weighted densities of its constituents.

Solution:

15.6 g/cm^3

Exercise:

Problem:

There is relatively little empty space between atoms in solids and liquids, so that the average density of an atom is about the same as matter on a macroscopic scale—approximately 10^3 kg/m^3. The nucleus of an atom has a radius about 10^{-5} that of the atom and contains nearly all the mass of the entire atom. (a) What is the approximate density of a nucleus? (b) One remnant of a supernova, called a neutron star, can have the density of a nucleus. What would be the radius of a neutron star with a mass 10 times that of our Sun (the radius of the Sun is $7 \times 10^8 \text{ m}$)?

Solution:

(a) 10^{18} kg/m^3

(b) $2 \times 10^4 \text{ m}$

Glossary

density
: the mass per unit volume of a substance or object

Heat, Specific Heat, and Calorimetry

By the end of this section, you will be able to:

- Explain phenomena involving heat as a form of energy transfer
- Solve problems involving heat transfer

We have seen in previous chapters that energy is one of the fundamental concepts of physics. **Heat** is a type of energy transfer that is caused by a temperature difference, and it can change the temperature of an object. As we learned earlier in this chapter, heat transfer is the movement of energy from one place or material to another as a result of a difference in temperature. Heat transfer is fundamental to such everyday activities as home heating and cooking, as well as many industrial processes. It also forms a basis for the topics in the remainder of this chapter.

We also introduce the concept of internal energy, which can be increased or decreased by heat transfer. We discuss another way to change the internal energy of a system, namely doing work on it. Thus, we are beginning the study of the relationship of heat and work, which is the basis of engines and refrigerators and the central topic (and origin of the name) of thermodynamics.

Internal Energy and Heat

A thermal system has *internal energy* (also called thermal energy**)**, which is the sum of the mechanical energies of its molecules. A system's internal energy is proportional to its temperature. As we saw earlier in this chapter, if two objects at different temperatures are brought into contact with each other, energy is transferred from the hotter to the colder object until the bodies reach thermal equilibrium (that is, they are at the same temperature). No work is done by either object because no force acts through a distance (as we discussed in Work and Kinetic Energy). These observations reveal that heat is energy transferred spontaneously due to a temperature difference. [link] shows an example of heat transfer.

The meaning of "heat" in physics is different from its ordinary meaning. For example, in conversation, we may say "the heat was unbearable," but in physics, we would say that the temperature was high. Heat is a form of energy flow, whereas temperature is not. Incidentally, humans are sensitive to *heat flow* rather than to temperature.

Since heat is a form of energy, its SI unit is the joule (J). Another common unit of energy often used for heat is the **calorie** (cal), defined as the energy needed to change the temperature of 1.00 g of water by $1.00\,^\circ\text{C}$ —specifically, between $14.5\,^\circ\text{C}$ and $15.5\,^\circ\text{C}$, since there is a slight temperature dependence. Also commonly used is the **kilocalorie** (kcal), which is the energy needed to change the temperature of 1.00 kg of water by $1.00\,^\circ\text{C}$. Since mass is most often specified in kilograms, the kilocalorie is convenient. Confusingly, food calories (sometimes called "big calories," abbreviated Cal) are actually kilocalories, a fact not easily determined from package labeling.

Mechanical Equivalent of Heat

It is also possible to change the temperature of a substance by doing work, which transfers energy into or out of a system. This realization helped establish that heat is a form of energy. James Prescott Joule (1818–1889) performed many experiments to establish the **mechanical equivalent of heat**—*the work needed to produce the same effects as heat transfer*. In the units used for these two quantities, the value for this equivalence is

Equation:

$$1.000\text{ kcal} = 4186\text{ J}.$$

We consider this equation to represent the conversion between two units of energy. (Other numbers that you may see refer to calories defined for temperature ranges other than $14.5\text{ }^\circ\text{C}$ to $15.5\text{ }^\circ\text{C}$.)

Increasing internal energy by heat transfer gives the same result as increasing it by doing work. Therefore, although a system has a well-defined internal energy, we cannot say that it has a certain “heat content” or “work content.” A well-defined quantity that depends only on the current state of the system, rather than on the history of that system, is known as a *state variable*. Temperature and internal energy are state variables. To sum up this paragraph, *heat and work are not state variables*.

Incidentally, increasing the internal energy of a system does not necessarily increase its temperature. As we’ll see in the next section, the temperature does not change when a substance changes from one phase to another. An example is the melting of ice, which can be accomplished by adding heat or by doing frictional work, as when an ice cube is rubbed against a rough surface.

Temperature Change and Heat Capacity

We have noted that heat transfer often causes temperature change. Experiments show that with no phase change and no work done on or by the system, the transferred heat is typically directly proportional to the change in temperature and to the mass of the system, to a good approximation. (Below we show how to handle situations where the approximation is not valid.) The constant of proportionality depends on the substance and its phase, which may be gas, liquid, or solid. We omit discussion of the fourth phase, plasma, because although it is the most common phase in the universe, it is rare and short-lived on Earth.

Note:
Heat Transfer and Temperature Change
A practical approximation for the relationship between heat transfer and temperature change is:
Equation:

$$Q = mc\Delta T,$$

where Q is the symbol for heat transfer (“quantity of heat”), *m* is the mass of the substance, and ΔT is the change in temperature. The symbol *c* stands for the **specific heat** (also called “*specific heat capacity*”) and depends on the material and phase. The specific heat is numerically equal to the amount of heat necessary to change the temperature of 1.00 kg of mass by $1.00\text{ }^\circ\text{C}$. The SI unit for specific heat is $\text{J}/(\text{kg}\ \times\ \text{K})$ or $\text{J}/(\text{kg}\ \times\ ^\circ\text{C})$. (Recall that the temperature change ΔT is the same in units of kelvin and degrees Celsius.)

Values of specific heat must generally be measured, because there is no simple way to calculate them precisely. [link] lists representative values of specific heat for various substances. We see from this table that the specific heat of water is five times that of glass and 10 times that of iron, which means that it takes five times as much heat to raise the temperature of water a given amount as for glass, and 10 times as much as for iron. In fact, water has one of the largest specific heats of any material, which is important for sustaining life on Earth.

Substances	**Specific Heat (*c*)**	
Solids	$\mathrm{J/kg \cdot {}^\circ C}$	$\mathrm{kcal/kg \cdot {}^\circ C}$[2]
Aluminum	900	0.215
Asbestos	800	0.19
Concrete, granite (average)	840	0.20
Copper	387	0.0924
Glass	840	0.20
Gold	129	0.0308
Human body (average at $37\ {}^\circ\mathrm{C}$)	3500	0.83
Ice (average, $-50\ {}^\circ\mathrm{C}$ to $0\ {}^\circ\mathrm{C}$)	2090	0.50
Iron, steel	452	0.108
Lead	128	0.0305
Silver	235	0.0562
Wood	1700	0.40
Liquids		
Benzene	1740	0.415
Ethanol	2450	0.586
Glycerin	2410	0.576
Mercury	139	0.0333
Water $(15.0\ {}^\circ\mathrm{C})$	4186	1.000
Gases[3]		
Air (dry)	721 (1015)	0.172 (0.242)
Ammonia	1670 (2190)	0.399 (0.523)
Carbon dioxide	638 (833)	0.152 (0.199)
Nitrogen	739 (1040)	0.177 (0.248)

Substances	**Specific Heat (*c*)**	
Oxygen	651 (913)	0.156 (0.218)
Steam ($100\ ^\circ\text{C}$)	1520 (2020)	0.363 (0.482)

Specific Heats of Various Substances[1][1]The values for solids and liquids are at constant volume and $25\ ^\circ\text{C}$, except as noted. [2]These values are identical in units of $\text{cal}/\text{g}\cdot{}^\circ\text{C}$. [3]Specific heats at constant volume and at $20.0\ ^\circ\text{C}$ except as noted, and at 1.00 atm pressure. Values in parentheses are specific heats at a constant pressure of 1.00 atm.

Except for gases, the temperature and volume dependence of the specific heat of most substances is weak at normal temperatures. Therefore, we will generally take specific heats to be constant at the values given in the table.

Example:

Calculating the Required Heat

A 0.500-kg aluminum pan on a stove and 0.250 L of water in it are heated from $20.0\ ^\circ\text{C}$ to $80.0\ ^\circ\text{C}$. (a) How much heat is required? What percentage of the heat is used to raise the temperature of (b) the pan and (c) the water?

Strategy

We can assume that the pan and the water are always at the same temperature. When you put the pan on the stove, the temperature of the water and that of the pan are increased by the same amount. We use the equation for the heat transfer for the given temperature change and mass of water and aluminum. The specific heat values for water and aluminum are given in [link].

Solution

1. Calculate the temperature difference:
   Equation:
   $$\Delta T = T_\text{f} - T_\text{i} = 60.0\ ^\circ\text{C}.$$
2. Calculate the mass of water. Because the density of water is $1000\ \text{kg}/\text{m}^3$, 1 L of water has a mass of 1 kg, and the mass of 0.250 L of water is $m_w = 0.250\ \text{kg}$.
3. Calculate the heat transferred to the water. Use the specific heat of water in [link]:
   Equation:
   $$Q_\text{w} = m_\text{w} c_\text{w} \Delta T = (0.250\ \text{kg})\ (4186\ \text{J}/\text{kg}\ ^\circ\text{C})\ (60.0\ ^\circ\text{C}) = 62.8\ \text{kJ}.$$
4. Calculate the heat transferred to the aluminum. Use the specific heat for aluminum in [link]:
   Equation:
   $$Q_\text{Al} = m_\text{A1} c_\text{A1} \Delta T = (0.500\ \text{kg})\ (900\ \text{J}/\text{kg}\ ^\circ\text{C})\ (60.0\ ^\circ\text{C}) = 27.0\ \text{kJ}.$$
5. Find the total transferred heat:
   Equation:
   $$Q_\text{Total} = Q_\text{W} + Q_\text{Al} = 89.8\ \text{kJ}.$$

Significance
In this example, the heat transferred to the container is a significant fraction of the total transferred heat. Although the mass of the pan is twice that of the water, the specific heat of water is over four times that of aluminum. Therefore, it takes a bit more than twice as much heat to achieve the given temperature change for the water as for the aluminum pan.

[link] illustrates a temperature rise caused by doing work. (The result is the same as if the same amount of energy had been added with a blowtorch instead of mechanically.)

Example:

Calculating the Temperature Increase from the Work Done on a Substance

Truck brakes used to control speed on a downhill run do work, converting gravitational potential energy into increased internal energy (higher temperature) of the brake material ([link]). This conversion prevents the gravitational potential energy from being converted into kinetic energy of the truck. Since the mass of the truck is much greater than that of the brake material absorbing the energy, the temperature increase may occur too fast for sufficient heat to transfer from the brakes to the environment; in other words, the brakes may overheat.

The smoking brakes on a braking truck are visible evidence of the mechanical equivalent of heat.

Calculate the temperature increase of 10 kg of brake material with an average specific heat of $800\ \text{J/kg}\cdot{}^\circ\text{C}$ if the material retains 10% of the energy from a 10,000-kg truck descending 75.0 m (in vertical displacement) at a constant speed.

Strategy
We calculate the gravitational potential energy (*Mgh*) that the entire truck loses in its descent, equate it to the increase in the brakes' internal energy, and then find the temperature increase produced in the brake material alone.

Solution
First we calculate the change in gravitational potential energy as the truck goes downhill:

Equation:

$$Mgh = (10{,}000\,\text{kg})\left(9.80\,\text{m/s}^2\right)(75.0\,\text{m}) = 7.35\ \times\ 10^6\,\text{J}.$$

Because the kinetic energy of the truck does not change, conservation of energy tells us the lost potential energy is dissipated, and we assume that 10% of it is transferred to internal energy of the brakes, so take $Q = Mgh/10$. Then we calculate the temperature change from the heat transferred, using

Equation:

$$\Delta T = \frac{Q}{mc},$$

where *m* is the mass of the brake material. Insert the given values to find

Equation:

$$\Delta T = \frac{7.35\ \times\ 10^5\,\text{J}}{(10\,\text{kg})(800\,\text{J/kg}\ ^\circ\text{C})} = 92\ ^\circ\text{C}.$$

Significance

If the truck had been traveling for some time, then just before the descent, the brake temperature would probably be higher than the ambient temperature. The temperature increase in the descent would likely raise the temperature of the brake material very high, so this technique is not practical. Instead, the truck would use the technique of engine braking. A different idea underlies the recent technology of hybrid and electric cars, where mechanical energy (kinetic and gravitational potential energy) is converted by the brakes into electrical energy in the battery, a process called regenerative braking.

In a common kind of problem, objects at different temperatures are placed in contact with each other but isolated from everything else, and they are allowed to come into equilibrium. A container that prevents heat transfer in or out is called a **calorimeter**, and the use of a calorimeter to make measurements (typically of heat or specific heat capacity) is called **calorimetry**.

We will use the term “calorimetry problem” to refer to any problem in which the objects concerned are thermally isolated from their surroundings. An important idea in solving calorimetry problems is that during a heat transfer between objects isolated from their surroundings, the heat gained by the colder object must equal the heat lost by the hotter object, due to conservation of energy:

Note:

Equation:

$$Q_{\text{cold}} + Q_{\text{hot}} = 0.$$

We express this idea by writing that the sum of the heats equals zero because the heat gained is usually considered positive; the heat lost, negative.

Example:

Calculating the Final Temperature in Calorimetry

Suppose you pour 0.250 kg of $20.0\text{-}^\circ\mathrm{C}$ water (about a cup) into a 0.500-kg aluminum pan off the stove with a temperature of $150\ ^\circ\mathrm{C}$. Assume no heat transfer takes place to anything else: The pan is placed on an insulated pad, and heat transfer to the air is neglected in the short time needed to reach equilibrium. Thus, this is a calorimetry problem, even though no isolating container is specified. Also assume that a negligible amount of water boils off. What is the temperature when the water and pan reach thermal equilibrium?

Strategy

Originally, the pan and water are not in thermal equilibrium: The pan is at a higher temperature than the water. Heat transfer restores thermal equilibrium once the water and pan are in contact; it stops once thermal equilibrium between the pan and the water is achieved. The heat lost by the pan is equal to the heat gained by the water—that is the basic principle of calorimetry.

Solution

1. Use the equation for heat transfer $Q = mc\Delta T$ to express the heat lost by the aluminum pan in terms of the mass of the pan, the specific heat of aluminum, the initial temperature of the pan, and the final temperature:
   Equation:
   $$Q_{\text{hot}} = m_{\text{A1}}c_{\text{A1}}\left(T_{\text{f}} - 150\ ^\circ\mathrm{C}\right).$$
2. Express the heat gained by the water in terms of the mass of the water, the specific heat of water, the initial temperature of the water, and the final temperature:
   Equation:
   $$Q_{\text{cold}} = m_{\text{w}}c_{\text{w}}\left(T_{\text{f}} - 20.0\ ^\circ\mathrm{C}\right).$$
3. Note that $Q_{\text{hot}} < 0$ and $Q_{\text{cold}} > 0$ and that as stated above, they must sum to zero:
   Equation:
   $$\begin{aligned} Q_{\text{cold}} + Q_{\text{hot}} &= 0 \\ Q_{\text{cold}} &= -Q_{\text{hot}} \\ m_{\text{w}}c_{\text{w}}\left(T_{\text{f}} - 20.0\ ^\circ\mathrm{C}\right) &= -m_{\text{A1}}c_{\text{A1}}\left(T_{\text{f}} - 150\ ^\circ\mathrm{C}\right). \end{aligned}$$
4. This a linear equation for the unknown final temperature, T_{f}. Solving for T_{f},
   Equation:
   $$T_{\text{f}} = \frac{m_{\text{A1}}c_{\text{A1}}\left(150\ ^\circ\mathrm{C}\right) + m_{\text{w}}c_{\text{w}}\left(20.0\ ^\circ\mathrm{C}\right)}{m_{\text{A1}}c_{\text{A1}} + m_{\text{w}}c_{\text{w}}},$$

   and insert the numerical values:
   Equation:
   $$T_{\text{f}} = \frac{(0.500\ \text{kg})\left(900\ \text{J/kg}\ ^\circ\mathrm{C}\right)\left(150\ ^\circ\mathrm{C}\right) + (0.250\ \text{kg})\left(4186\ \text{J/kg}\ ^\circ\mathrm{C}\right)\left(20.0\ ^\circ\mathrm{C}\right)}{(0.500\ \text{kg})\left(900\ \text{J/kg}\ ^\circ\mathrm{C}\right) + (0.250\ \text{kg})\left(4186\ \text{J/kg}\ ^\circ\mathrm{C}\right)} = 59.1\ ^\circ\mathrm{C}.$$

Significance

Why is the final temperature so much closer to $20.0\ ^\circ\text{C}$ than to $150\ ^\circ\text{C}$? The reason is that water has a greater specific heat than most common substances and thus undergoes a smaller temperature change for a given heat transfer. A large body of water, such as a lake, requires a large amount of heat to increase its temperature appreciably. This explains why the temperature of a lake stays relatively constant during the day even when the temperature change of the air is large. However, the water temperature does change over longer times (e.g., summer to winter).

Note:
Exercise:

Problem:

Check Your Understanding If 25 kJ is necessary to raise the temperature of a rock from $25\ ^\circ\text{C}$ to $30\ ^\circ\text{C}$, how much heat is necessary to heat the rock from $45\ ^\circ\text{C}$ to $50\ ^\circ\text{C}$?

Solution:

To a good approximation, the heat transfer depends only on the temperature difference. Since the temperature differences are the same in both cases, the same 25 kJ is necessary in the second case. (As we will see in the next section, the answer would have been different if the object had been made of some substance that changes phase anywhere between $30\ ^\circ\text{C}$ and $50\ ^\circ\text{C}$.)

Summary

- Heat and work are the two distinct methods of energy transfer.
- Heat transfer to an object when its temperature changes is often approximated well by $Q = mc\Delta T$, where *m* is the object's mass and *c* is the specific heat of the substance.

Conceptual Questions

Exercise:

Problem: How is heat transfer related to temperature?

Solution:

Temperature differences cause heat transfer.

Exercise:

Problem: Describe a situation in which heat transfer occurs.

Exercise:

Problem: When heat transfers into a system, is the energy stored as heat? Explain briefly.

Solution:

No, it is stored as thermal energy. A thermodynamic system does not have a well-defined quantity of heat.

Exercise:

Problem:

The brakes in a car increase in temperature by ΔT when bringing the car to rest from a speed *v*. How much greater would ΔT be if the car initially had twice the speed? You may assume the car stops fast enough that no heat transfers out of the brakes.

Problems

Exercise:

Problem:

On a hot day, the temperature of an 80,000-L swimming pool increases by $1.50\ ^\circ\text{C}$. What is the net heat transfer during this heating? Ignore any complications, such as loss of water by evaporation.

Solution:

$m = 5.20\ \times\ 10^{8}$ J

Exercise:

Problem:

To sterilize a 50.0-g glass baby bottle, we must raise its temperature from $22.0\ ^\circ\text{C}$ to $95.0\ ^\circ\text{C}$. How much heat transfer is required?

Exercise:

Problem:

The same heat transfer into identical masses of different substances produces different temperature changes. Calculate the final temperature when 1.00 kcal of heat transfers into 1.00 kg of the following, originally at $20.0\ ^\circ\text{C}$: (a) water; (b) concrete; (c) steel; and (d) mercury.

Solution:

$Q = mc\Delta T\ \ \Rightarrow \Delta T = \frac{Q}{mc}$; a. $21.0\ ^\circ\text{C}$; b. $25.0\ ^\circ\text{C}$; c. $29.3\ ^\circ\text{C}$; d. $50.0\ ^\circ\text{C}$

Exercise:

Problem:

Rubbing your hands together warms them by converting work into thermal energy. If a woman rubs her hands back and forth for a total of 20 rubs, at a distance of 7.50 cm per rub, and with an average frictional force of 40.0 N, what is the temperature increase? The mass of tissues warmed is only 0.100 kg, mostly in the palms and fingers.

Exercise:

Problem:

A 0.250-kg block of a pure material is heated from $20.0\,^{\circ}\text{C}$ to $65.0\,^{\circ}\text{C}$ by the addition of 4.35 kJ of energy. Calculate its specific heat and identify the substance of which it is most likely composed.

Solution:

$Q = mc\Delta T \quad \Rightarrow c = \frac{Q}{m\Delta T} = \frac{1.04\,\text{kcal}}{(0.250\,\text{kg})(45.0\,^{\circ}\text{C})} = 0.0924\,\text{kcal/kg}\cdot{}^{\circ}\text{C}$. It is copper.

Exercise:

Problem:

Suppose identical amounts of heat transfer into different masses of copper and water, causing identical changes in temperature. What is the ratio of the mass of copper to water?

Exercise:

Problem:

Following vigorous exercise, the body temperature of an 80.0 kg person is $40.0\,^{\circ}\text{C}$. At what rate in watts must the person transfer thermal energy to reduce the body temperature to $37.0\,^{\circ}\text{C}$ in 30.0 min, assuming the body continues to produce energy at the rate of 150 W?
$(1\text{ watt} = 1\,\text{joule/second or }1\text{ W} = 1\text{ J/s})$

Exercise:

Problem:

In a study of healthy young men[footnote], doing 20 push-ups in 1 minute burned an amount of energy per kg that for a 70.0-kg man corresponds to 8.06 calories (kcal). How much would a 70.0-kg man's temperature rise if he did not lose any heat during that time?
JW Vezina, "An examination of the differences between two methods of estimating energy expenditure in resistance training activities," *Journal of Strength and Conditioning Research*, April 28, 2014, http://www.ncbi.nlm.nih.gov/pubmed/24402448

Solution:

$0.139\,^{\circ}\text{C}$

Exercise:

Problem:

A 1.28-kg sample of water at $10.0\,^{\circ}\text{C}$ is in a calorimeter. You drop a piece of steel with a mass of 0.385 kg at $215\,^{\circ}\text{C}$ into it. After the sizzling subsides, what is the final equilibrium temperature? (Make the reasonable assumptions that any steam produced condenses into liquid water during the process of equilibration and that the evaporation and condensation don't affect the outcome, as we'll see in the next section.)

Exercise:

Problem:

Repeat the preceding problem, assuming the water is in a glass beaker with a mass of 0.200 kg, which in turn is in a calorimeter. The beaker is initially at the same temperature as the water. Before doing the problem, should the answer be higher or lower than the preceding answer? Comparing the mass and specific heat of the beaker to those of the water, do you think the beaker will make much difference?

Solution:

It should be lower. The beaker will not make much difference: $16.3\ ^\circ\text{C}$

Glossary

calorie (cal)	energy needed to change the temperature of 1.00 g of water by $1.00\ ^\circ\text{C}$
calorimeter	container that prevents heat transfer in or out
calorimetry	study of heat transfer inside a container impervious to heat
heat	energy transferred solely due to a temperature difference
kilocalorie (kcal)	energy needed to change the temperature of 1.00 kg of water between $14.5\ ^\circ\text{C}$ and $15.5\ ^\circ\text{C}$
mechanical equivalent of heat	work needed to produce the same effects as heat transfer
specific heat	amount of heat necessary to change the temperature of 1.00 kg of a substance by $1.00\ ^\circ\text{C}$; also called “specific heat capacity”

Phase Changes
By the end of this section, you will be able to:

- Describe phase transitions and equilibrium between phases
- Solve problems involving latent heat
- Solve calorimetry problems involving phase changes

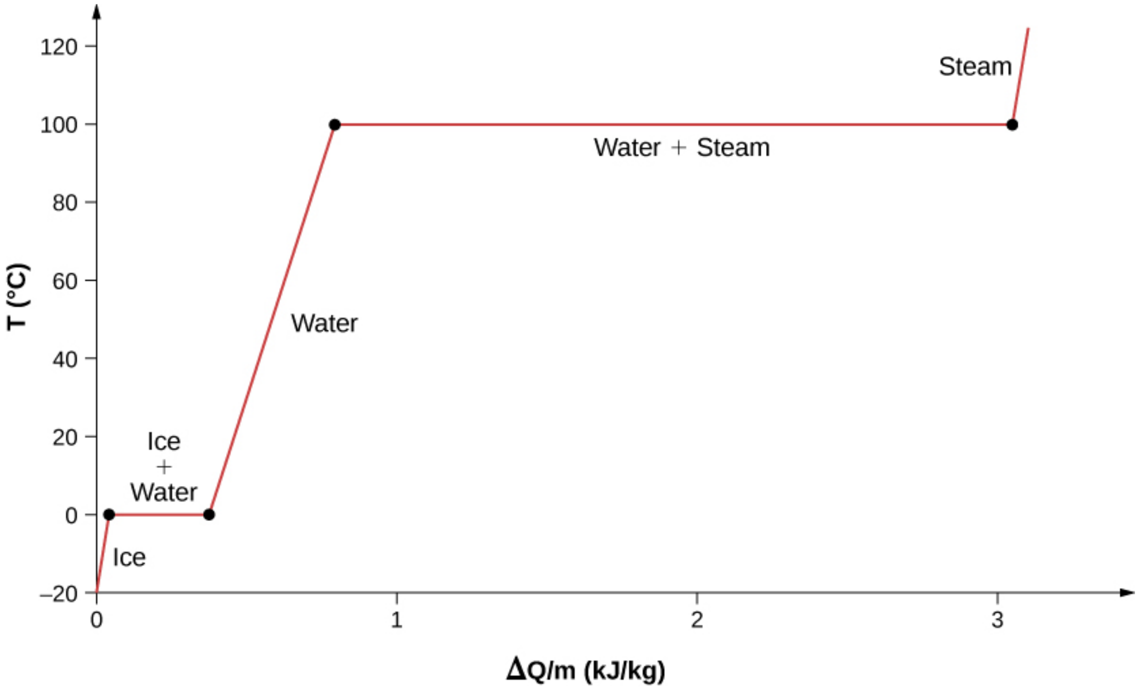
Phase transitions play an important theoretical and practical role in the study of heat flow. In melting (or "fusion"), a solid turns into a liquid; the opposite process is freezing. In evaporation, a liquid turns into a gas; the opposite process is condensation.

A substance melts or freezes at a temperature called its melting point, and boils (evaporates rapidly) or condenses at its boiling point. These temperatures depend on pressure. High pressure favors the denser form, so typically, high pressure raises the melting point and boiling point, and low pressure lowers them. For example, the boiling point of water is $100\ ^\circ\text{C}$ at 1.00 atm. At higher pressure, the boiling point is higher, and at lower pressure, it is lower. The main exception is the melting and freezing of water, discussed in the next section.

Phase Change and Latent Heat

So far, we have discussed heat transfers that cause temperature change. However, in a phase transition, heat transfer does not cause any temperature change.

For an example of phase changes, consider the addition of heat to a sample of ice at $-20\ ^\circ\text{C}$ ([link]) and atmospheric pressure. The temperature of the ice rises linearly, absorbing heat at a constant rate of $2090\ \text{J/kg}\cdot{}^\text{o}\text{C}$ until it reaches $0\ ^\circ\text{C}$. Once at this temperature, the ice begins to melt and continues until it has all melted, absorbing 333 kJ/kg of heat. The temperature remains constant at $0\ ^\circ\text{C}$ during this phase change. Once all the ice has melted, the temperature of the liquid water rises, absorbing heat at a new constant rate of $4186\ \text{kJ/kg}\cdot{}^\text{o}\text{C}$. At $100\ ^\circ\text{C}$, the water begins to boil. The temperature again remains constant during this phase change while the water absorbs 2256 kJ/kg of heat and turns into steam. When all the liquid has become steam, the temperature rises again, absorbing heat at a rate of $2020\ \text{J/kg}\cdot{}^\text{o}\text{C}$. If we started with steam and cooled it to make it condense into liquid water and freeze into ice, the process would exactly reverse, with the temperature again constant during each phase transition.



Temperature versus heat. The system is constructed so that no vapor evaporates while ice warms to become liquid water, and so that, when vaporization occurs, the vapor remains in the system. The long stretches of constant temperatures at $0\ ^\circ C$ and $100\ ^\circ C$ reflect the large amounts of heat needed to cause melting and vaporization, respectively.

Where does the heat added during melting or boiling go, considering that the temperature does not change until the transition is complete? Energy is required to melt a solid, because the attractive forces between the molecules in the solid must be broken apart, so that in the liquid, the molecules can move around at comparable kinetic energies; thus, there is no rise in temperature. Energy is needed to vaporize a liquid for similar reasons. Conversely, work is done by attractive forces when molecules are brought together during freezing and condensation. That energy must be transferred out of the system, usually in the form of heat, to allow the molecules to stay together ([link]). Thus, condensation occurs in association with cold objects—the glass in [link], for example.

Condensation forms on this glass of iced tea because the temperature of the nearby air is reduced. The air cannot hold as much water as it did at room temperature, so water condenses. Energy is released when the water condenses, speeding the melting of the ice in the glass. (credit: Jenny Downing)

The energy released when a liquid freezes is used by orange growers when the temperature approaches $0\ ^\circ\text{C}$. Growers spray water on the trees so that the water freezes and heat is released to the growing oranges. This prevents the temperature inside the orange from dropping below freezing, which would damage the fruit ([link]).

The ice on these trees released large amounts of energy when it froze, helping to prevent the temperature of the trees from dropping below $0\ ^\circ\text{C}$. Water is intentionally sprayed on orchards to help prevent hard frosts. (credit: Hermann Hammer)

The energy per unit mass required to change a substance from the solid phase to the liquid phase, or released when the substance changes from liquid to solid, is known as the **heat of fusion**. The energy per unit mass required to change a substance from the liquid phase to the vapor phase is known as the **heat of vaporization**. The strength of the forces depends on the type of molecules. The heat Q absorbed or released in a phase change in a sample of mass *m* is given by

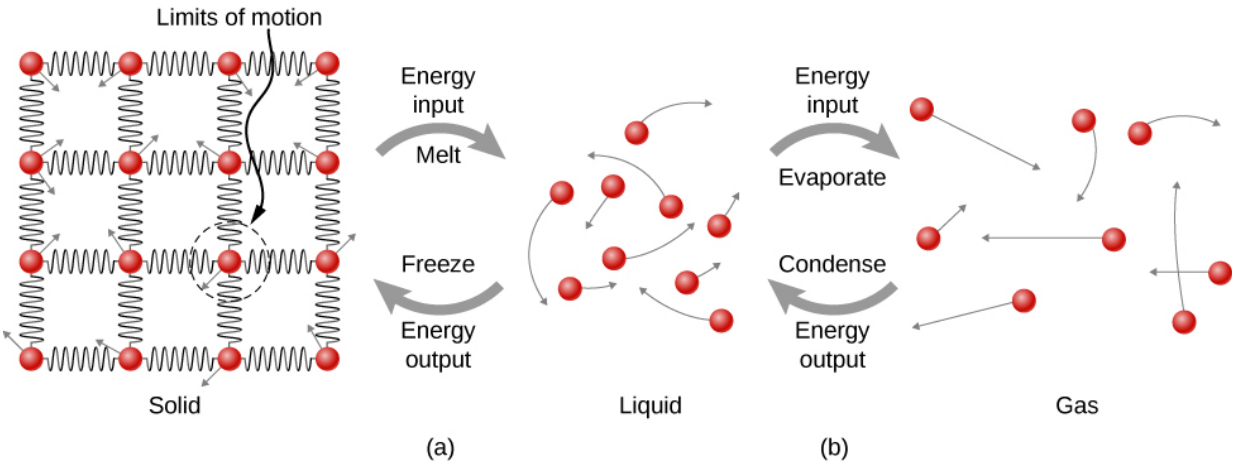
Note:
Equation:

$$Q = mL_\mathrm{f}(\mathrm{melting/freezing})$$

Note:
Equation:

$$Q = mL_\mathrm{v}(\mathrm{vaporization/condensation})$$

where the latent heat of fusion L_f and latent heat of vaporization L_v are material constants that are determined experimentally. (Latent heats are also called **latent heat coefficients** and heats of transformation.) These constants are "latent," or hidden, because in phase changes, energy enters or leaves a system without causing a temperature change in the system, so in effect, the energy is hidden.



(a) Energy is required to partially overcome the attractive forces (modeled as springs) between molecules in a solid to form a liquid. That same energy must be removed from the liquid for freezing to take place. (b) Molecules become separated by large distances when going from liquid to vapor, requiring significant energy to completely overcome molecular attraction. The same energy must be removed from the vapor for condensation to take place.

[link] lists representative values of L_f and L_v in kJ/kg, together with melting and boiling points. Note that in general, $L_\text{v} > L_\text{f}$. The table shows that the amounts of energy involved in phase changes can easily be comparable to or greater than those involved in temperature changes, as [link] and the accompanying discussion also showed.

		L_f			L_v	
Substance	**Melting Point** $(^\circ\text{C})$	**kJ/kg**	**kcal/kg**	**Boiling Point** $(^\circ\text{C})$	**kJ/kg**	**kcal/kg**
Helium[2]	$-272.2\ \ (0.95\text{ K})$	5.23	1.25	$-268.9\ (4.2\text{ K})$	20.9	4.99
Hydrogen	$-259.3\ (13.9\text{ K})$	58.6	14.0	$-252.9\ (20.2\text{ K})$	452	108
Nitrogen	$-210.0\ (63.2\text{ K})$	25.5	6.09	$-195.8\ (77.4\text{ K})$	201	48.0
Oxygen	$-218.8\ (54.4\text{ K})$	13.8	3.30	$-183.0\ (90.2\text{ K})$	213	50.9
Ethanol	–114	104	24.9	78.3	854	204
Ammonia	–75	332	79.3	–33.4	1370	327
Mercury	–38.9	11.8	2.82	357	272	65.0
Water	0.00	334	79.8	100.0	2256[3]	539[4]
Sulfur	119	38.1	9.10	444.6	326	77.9
Lead	327	24.5	5.85	1750	871	208
Antimony	631	165	39.4	1440	561	134
Aluminum	660	380	90	2450	11400	2720
Silver	961	88.3	21.1	2193	2336	558
Gold	1063	64.5	15.4	2660	1578	377
Copper	1083	134	32.0	2595	5069	1211
Uranium	1133	84	20	3900	1900	454
Tungsten	3410	184	44	5900	4810	1150

Heats of Fusion and Vaporization[1]

[1] Values quoted at the normal melting and boiling temperatures at standard atmospheric pressure (1 atm).

[2] Helium has no solid phase at atmospheric pressure. The melting point given is at a pressure of 2.5 MPa.

[3] At $37.0\ ^\circ\text{C}$ (body temperature), the heat of vaporization L_v for water is 2430 kJ/kg or 580 kcal/kg.

[4] At $37.0\ ^\circ\text{C}$ (body temperature), the heat of vaporization, L_v for water is 2430 kJ/kg or 580 kcal/kg.

Phase changes can have a strong stabilizing effect on temperatures that are not near the melting and boiling points, since evaporation and condensation occur even at temperatures below the boiling point. For example, air temperatures in humid climates rarely go above approximately $38.0\ ^\circ\text{C}$ because most heat transfer goes into evaporating water into the air. Similarly, temperatures in humid weather rarely fall below the dew point—the temperature where condensation occurs given the concentration of water vapor in the air—because so much heat is released when water vapor condenses.

More energy is required to evaporate water below the boiling point than at the boiling point, because the kinetic energy of water molecules at temperatures below $100\ ^\circ\text{C}$ is less than that at $100\ ^\circ\text{C}$, so less energy is available from random thermal motions. For example, at body temperature, evaporation of sweat from the skin requires a heat input of 2428 kJ/kg, which is about 10% higher than the latent heat of vaporization at $100\ ^\circ\text{C}$. This heat comes from the skin, and this evaporative cooling effect of sweating helps reduce the body temperature in hot weather. However, high humidity inhibits evaporation, so that body temperature might rise, while unevaporated sweat might be left on your brow.

Example:

Calculating Final Temperature from Phase Change

Three ice cubes are used to chill a soda at $20\ ^\circ\text{C}$ with mass $m_{\text{soda}} = 0.25\ \text{kg}$. The ice is at $0\ ^\circ\text{C}$ and each ice cube has a mass of 6.0 g. Assume that the soda is kept in a foam container so that heat loss can be ignored and that the soda has the same specific heat as water. Find the final temperature when all ice has melted.

Strategy

The ice cubes are at the melting temperature of $0\ ^\circ\text{C}$. Heat is transferred from the soda to the ice for melting. Melting yields water at $0\ ^\circ\text{C}$, so more heat is transferred from the soda to this water until the water plus soda system reaches thermal equilibrium.

The heat transferred to the ice is

Equation:

$$Q_{\text{ice}} = m_{\text{ice}} L_{\text{f}} + m_{\text{ice}} c_{\text{W}} \left(T_{\text{f}} - 0\ ^\circ\text{C}\right).$$

The heat given off by the soda is

Equation:

$$Q_{\text{soda}} = m_{\text{soda}} c_{\text{W}} \left(T_{\text{f}} - 20\ ^\circ\text{C}\right).$$

Since no heat is lost, $Q_{\text{ice}} = -Q_{\text{soda}}$, as in [link], so that

Equation:

$$m_{\text{ice}} L_{\text{f}} + m_{\text{ice}} c_{\text{W}} \left(T_{\text{f}} - 0\ ^\circ\text{C}\right) = -m_{\text{soda}} c_{\text{W}} \left(T_{\text{f}} - 20\ ^\circ\text{C}\right).$$

Solve for the unknown quantity T_{f}:

Equation:

$$T_{\text{f}} = \frac{m_{\text{soda}} c_{\text{w}} \left(20\ ^\circ\text{C}\right) - m_{\text{ice}} L_{\text{f}}}{\left(m_{\text{soda}} + m_{\text{ice}}\right) c_{\text{w}}}.$$

Solution

First we identify the known quantities. The mass of ice is $m_{\text{ice}} = 3\ \times\ 6.0\ \text{g} = 0.018\ \text{kg}$ and the mass of soda is $m_{\text{soda}} = 0.25\ \text{kg}$. Then we calculate the final temperature:

Equation:

$$T_{\text{f}} = \frac{20{,}930\ \text{J} - 6012\ \text{J}}{1122\ \text{J}/^\circ\text{C}} = 13\ ^\circ\text{C}.$$

Significance

This example illustrates the large energies involved during a phase change. The mass of ice is about 7% of the mass of the soda but leads to a noticeable change in the temperature of the soda. Although we assumed that the ice was at the freezing temperature, this is unrealistic for ice straight out of a freezer: The typical temperature is $-6\ ^\circ\text{C}$. However, this correction makes no significant change from the result we found. Can you explain why?

Like solid-liquid and and liquid-vapor transitions, direct solid-vapor transitions or sublimations involve heat. The energy transferred is given by the equation $Q = mL_\text{s}$, where L_s is the **heat of sublimation**, analogous to L_f and L_v. The heat of sublimation at a given temperature is equal to the heat of fusion plus the heat of vaporization at that temperature.

We can now calculate any number of effects related to temperature and phase change. In each case, it is necessary to identify which temperature and phase changes are taking place. Keep in mind that heat transfer and work can cause both temperature and phase changes.

Note:
Problem-Solving Strategy: The Effects of Heat Transfer

1. Examine the situation to determine that there is a change in the temperature or phase. Is there heat transfer into or out of the system? When it is not obvious whether a phase change occurs or not, you may wish to first solve the problem as if there were no phase changes, and examine the temperature change obtained. If it is sufficient to take you past a boiling or melting point, you should then go back and do the problem in steps—temperature change, phase change, subsequent temperature change, and so on.
2. Identify and list all objects that change temperature or phase.
3. Identify exactly what needs to be determined in the problem (identify the unknowns). A written list is useful.
4. Make a list of what is given or what can be inferred from the problem as stated (identify the knowns). If there is a temperature change, the transferred heat depends on the specific heat of the substance ([link]), and if there is a phase change, the transferred heat depends on the latent heat of the substance ([link]).
5. Solve the appropriate equation for the quantity to be determined (the unknown).
6. Substitute the knowns along with their units into the appropriate equation and obtain numerical solutions complete with units. You may need to do this in steps if there is more than one state to the process, such as a temperature change followed by a phase change. However, in a calorimetry problem, each step corresponds to a term in the single equation $Q_\text{hot} + Q_\text{cold} = 0$.
7. Check the answer to see if it is reasonable. Does it make sense? As an example, be certain that any temperature change does not also cause a phase change that you have not taken into account.

Note:
Exercise:

Problem:

Check Your Understanding Why does snow often remain even when daytime temperatures are higher than the freezing temperature?

Solution:

Snow is formed from ice crystals and thus is the solid phase of water. Because enormous heat is necessary for phase changes, it takes a certain amount of time for this heat to be transferred from the air, even if the air is above $0\ ^\circ\text{C}$.

Summary

- Most substances have three distinct phases (under ordinary conditions on Earth), and they depend on temperature and pressure.
- Two phases coexist (i.e., they are in thermal equilibrium) at a set of pressures and temperatures.
- Phase changes occur at fixed temperatures for a given substance at a given pressure, and these temperatures are called boiling, freezing (or melting), and sublimation points.

Conceptual Questions

Exercise:

Problem:

A pressure cooker contains water and steam in equilibrium at a pressure greater than atmospheric pressure. How does this greater pressure increase cooking speed?

Solution:

It raises the boiling point, so the water, which the food gains heat from, is at a higher temperature.

Exercise:

Problem:

Can carbon dioxide be liquefied at room temperature ($20\text{ }^\circ\text{C}$)? If so, how? If not, why not? (See the phase diagram in the preceding problem.)

Solution:

Yes, by raising the pressure above 56 atm.

Exercise:

Problem: What is the distinction between gas and vapor?

Exercise:

Problem: Heat transfer can cause temperature and phase changes. What else can cause these changes?

Solution:

work

Exercise:

Problem:

How does the latent heat of fusion of water help slow the decrease of air temperatures, perhaps preventing temperatures from falling significantly below $0\text{ }^\circ\text{C}$, in the vicinity of large bodies of water?

Exercise:

Problem: What is the temperature of ice right after it is formed by freezing water?

Solution:

$0\text{ }^\circ\text{C}$ (at or near atmospheric pressure)

Exercise:

Problem:

If you place $0\ ^\circ\text{C}$ ice into $0\ ^\circ\text{C}$ water in an insulated container, what will the net result be? Will there be less ice and more liquid water, or more ice and less liquid water, or will the amounts stay the same?

Exercise:

Problem:

What effect does condensation on a glass of ice water have on the rate at which the ice melts? Will the condensation speed up the melting process or slow it down?

Solution:

Condensation releases heat, so it speeds up the melting.

Exercise:

Problem:

In Miami, Florida, which has a very humid climate and numerous bodies of water nearby, it is unusual for temperatures to rise above about $38\ ^\circ\text{C}$ ($100\ ^\circ\text{F}$). In the desert climate of Phoenix, Arizona, however, temperatures rise above that almost every day in July and August. Explain how the evaporation of water helps limit high temperatures in humid climates.

Exercise:

Problem:

In winter, it is often warmer in San Francisco than in Sacramento, 150 km inland. In summer, it is nearly always hotter in Sacramento. Explain how the bodies of water surrounding San Francisco moderate its extreme temperatures.

Solution:

Because of water's high specific heat, it changes temperature less than land. Also, evaporation reduces temperature rises. The air tends to stay close to equilibrium with the water, so its temperature does not change much where there's a lot of water around, as in San Francisco but not Sacramento.

Exercise:

Problem:

Freeze-dried foods have been dehydrated in a vacuum. During the process, the food freezes and must be heated to facilitate dehydration. Explain both how the vacuum speeds up dehydration and why the food freezes as a result.

Exercise:

Problem:

In a physics classroom demonstration, an instructor inflates a balloon by mouth and then cools it in liquid nitrogen. When cold, the shrunken balloon has a small amount of light blue liquid in it, as well as some snow-like crystals. As it warms up, the liquid boils, and part of the crystals sublime, with some crystals lingering for a while and then producing a liquid. Identify the blue liquid and the two solids in the cold balloon. Justify your identifications using data from [link].

Solution:

The liquid is oxygen, whose boiling point is above that of nitrogen but whose melting point is below the boiling point of liquid nitrogen. The crystals that sublime are carbon dioxide, which has no liquid phase at atmospheric pressure. The crystals that melt are water, whose melting point is above carbon dioxide’s sublimation point. The water came from the instructor’s breath.

Problems

Exercise:

Problem:

How much heat transfer (in kilocalories) is required to thaw a 0.450-kg package of frozen vegetables originally at $0\ ^\circ\mathrm{C}$ if their heat of fusion is the same as that of water?

Exercise:

Problem:

A bag containing $0\ ^\circ\mathrm{C}$ ice is much more effective in absorbing energy than one containing the same amount of $0\ ^\circ\mathrm{C}$ water. (a) How much heat transfer is necessary to raise the temperature of 0.800 kg of water from $0\ ^\circ\mathrm{C}$ to $30.0\ ^\circ\mathrm{C}$? (b) How much heat transfer is required to first melt 0.800 kg of $0\ ^\circ\mathrm{C}$ ice and then raise its temperature? (c) Explain how your answer supports the contention that the ice is more effective.

Solution:

a. $1.00\ \times\ 10^5$ J; b. $3.68\ \times\ 10^5$ J; c. The ice is much more effective in absorbing heat because it first must be melted, which requires a lot of energy, and then it gains the same amount of heat as the bag that started with water. The first $2.67\ \times\ 10^5$ J of heat is used to melt the ice, then it absorbs the $1.00\ \times\ 10^5$ J of heat as water.

Exercise:

Problem:

(a) How much heat transfer is required to raise the temperature of a 0.750-kg aluminum pot containing 2.50 kg of water from $30.0\ ^\circ\mathrm{C}$ to the boiling point and then boil away 0.750 kg of water? (b) How long does this take if the rate of heat transfer is 500 W?

Exercise:

Problem:

Condensation on a glass of ice water causes the ice to melt faster than it would otherwise. If 8.00 g of vapor condense on a glass containing both water and 200 g of ice, how many grams of the ice will melt as a result? Assume no other heat transfer occurs. Use L_v for water at $37\ ^\circ\mathrm{C}$ as a better approximation than L_v for water at $100\ ^\circ\mathrm{C}$.)

Solution:

58.1 g

Exercise:

Problem:

On a trip, you notice that a 3.50-kg bag of ice lasts an average of one day in your cooler. What is the average power in watts entering the ice if it starts at $0\ ^\circ\mathrm{C}$ and completely melts to $0\ ^\circ\mathrm{C}$ water in exactly one day?

Exercise:

Problem:

On a certain dry sunny day, a swimming pool's temperature would rise by $1.50\ ^\circ\text{C}$ if not for evaporation. What fraction of the water must evaporate to carry away precisely enough energy to keep the temperature constant?

Solution:

Let *M* be the mass of pool water and *m* be the mass of pool water that evaporates.
$Mc\Delta T = mL_{\text{V}(37\ ^\circ\text{C})} \Rightarrow \frac{m}{M} = \frac{c\Delta T}{L_{\text{V}(37\ ^\circ\text{C})}} = \frac{(1.00\ \text{kcal/kg}\cdot{}^\circ\text{C})(1.50\ ^\circ\text{C})}{580\ \text{kcal/kg}} = 2.59 \times 10^{-3}$;
(Note that L_V for water at $37\ ^\circ\text{C}$ is used here as a better approximation than L_V for $100\ ^\circ\text{C}$ water.)

Exercise:

Problem:

(a) How much heat transfer is necessary to raise the temperature of a 0.200-kg piece of ice from $-20.0\ ^\circ\text{C}$ to $130.0\ ^\circ\text{C}$, including the energy needed for phase changes? (b) How much time is required for each stage, assuming a constant 20.0 kJ/s rate of heat transfer? (c) Make a graph of temperature versus time for this process.

Exercise:

Problem:

In 1986, an enormous iceberg broke away from the Ross Ice Shelf in Antarctica. It was an approximately rectangular prism 160 km long, 40.0 km wide, and 250 m thick. (a) What is the mass of this iceberg, given that the density of ice is $917\ \text{kg/m}^3$? (b) How much heat transfer (in joules) is needed to melt it? (c) How many years would it take sunlight alone to melt ice this thick, if the ice absorbs an average of $100\ \text{W/m}^2$, 12.00 h per day?

Solution:

a. 1.47×10^{15} kg; b. 4.90×10^{20} J; c. 48.5 y

Exercise:

Problem:

How many grams of coffee must evaporate from 350 g of coffee in a 100-g glass cup to cool the coffee and the cup from $95.0\ ^\circ\text{C}$ to $45.0\ ^\circ\text{C}$? Assume the coffee has the same thermal properties as water and that the average heat of vaporization is 2340 kJ/kg (560 kcal/g). Neglect heat losses through processes other than evaporation, as well as the change in mass of the coffee as it cools. Do the latter two assumptions cause your answer to be higher or lower than the true answer?

Exercise:

Problem:

(a) It is difficult to extinguish a fire on a crude oil tanker, because each liter of crude oil releases 2.80×10^7 J of energy when burned. To illustrate this difficulty, calculate the number of liters of water that must be expended to absorb the energy released by burning 1.00 L of crude oil, if the water's temperature rises from $20.0\ ^\circ\text{C}$ to $100\ ^\circ\text{C}$, it boils, and the resulting steam's temperature rises to $300\ ^\circ\text{C}$ at constant pressure. (b) Discuss additional complications caused by the fact that crude oil is less dense than water.

Solution:

a. 9.67 L; b. Crude oil is less dense than water, so it floats on top of the water, thereby exposing it to the oxygen in the air, which it uses to burn. Also, if the water is under the oil, it is less able to absorb the heat generated by the oil.

Exercise:

Problem:

The energy released from condensation in thunderstorms can be very large. Calculate the energy released into the atmosphere for a small storm of radius 1 km, assuming that 1.0 cm of rain is precipitated uniformly over this area.

Exercise:

Problem:

To help prevent frost damage, 4.00 kg of water at $0\ ^\circ\text{C}$ is sprayed onto a fruit tree. (a) How much heat transfer occurs as the water freezes? (b) How much would the temperature of the 200-kg tree decrease if this amount of heat transferred from the tree? Take the specific heat to be $3.35\ \text{kJ/kg}\cdot\ ^\circ\text{C}$, and assume that no phase change occurs in the tree.

Solution:

a. 319 kcal; b. $2.00\ ^\circ\text{C}$

Exercise:

Problem:

A 0.250-kg aluminum bowl holding 0.800 kg of soup at $25.0\ ^\circ\text{C}$ is placed in a freezer. What is the final temperature if 388 kJ of energy is transferred from the bowl and soup, assuming the soup's thermal properties are the same as that of water?

Exercise:

Problem:

A 0.0500-kg ice cube at $-30.0\ ^\circ\text{C}$ is placed in 0.400 kg of 35.0-$^\circ\text{C}$ water in a very well-insulated container. What is the final temperature?

Solution:

First bring the ice up to $0\ ^\circ\text{C}$ and melt it with heat Q_1 : 4.74 kcal. This lowers the temperature of water by ΔT_2 : $23.15\ ^\circ\text{C}$. Now, the heat lost by the hot water equals that gained by the cold water (T_f is the final temperature): $20.6\ ^\circ\text{C}$

Exercise:

Problem:

If you pour 0.0100 kg of $20.0\ ^\circ\text{C}$ water onto a 1.20-kg block of ice (which is initially at $-15.0\ ^\circ\text{C}$), what is the final temperature? You may assume that the water cools so rapidly that effects of the surroundings are negligible.

Exercise:

Problem:

Indigenous people sometimes cook in watertight baskets by placing hot rocks into water to bring it to a boil. What mass of 500-$^\circ\text{C}$ granite must be placed in 4.00 kg of 15.0-$^\circ\text{C}$ water to bring its temperature to $100\ ^\circ\text{C}$, if 0.0250 kg of water escapes as vapor from the initial sizzle? You may neglect the effects of the surroundings.

Solution:

Let the subscripts r, e, v, and w represent rock, equilibrium, vapor, and water, respectively.
$m_{\text{r}}c_{\text{r}}\left(T_1-T_{\text{e}}\right)=m_{\text{V}}L_{\text{V}}+m_{\text{W}}c_{\text{W}}\left(T_{\text{e}}-T_2\right);$

$$\begin{aligned}m_{\text{r}} &= \frac{m_{\text{V}}L_{\text{V}}+m_{\text{W}}c_{\text{W}}(T_{\text{e}}-T_2)}{c_{\text{r}}(T_1-T_{\text{e}})}\\ &= \frac{(0.0250\text{ kg})\left(2256\times10^3\text{ J/kg}\right)+(3.975\text{ kg})\left(4186\times10^3\text{ J/kg}\cdot{}^\circ\text{C}\right)(100\,^\circ\text{C}-15\,^\circ\text{C})}{(840\text{ J/kg}\cdot{}^\circ\text{C})(500\,^\circ\text{C}-100\,^\circ\text{C})}\\ &= 4.38\text{ kg}\end{aligned}$$

Exercise:

Problem:

What would the final temperature of the pan and water be in [link] if 0.260 kg of water were placed in the pan and 0.0100 kg of the water evaporated immediately, leaving the remainder to come to a common temperature with the pan?

Glossary

critical point
for a given substance, the combination of temperature and pressure above which the liquid and gas phases are indistinguishable

critical pressure
pressure at the critical point

critical temperature
temperature at the critical point

heat of fusion
energy per unit mass required to change a substance from the solid phase to the liquid phase, or released when the substance changes from liquid to solid

heat of sublimation
energy per unit mass required to change a substance from the solid phase to the vapor phase

heat of vaporization
energy per unit mass required to change a substance from the liquid phase to the vapor phase

latent heat coefficient
general term for the heats of fusion, vaporization, and sublimation

phase diagram
graph of pressure vs. temperature of a particular substance, showing at which pressures and temperatures the phases of the substance occur

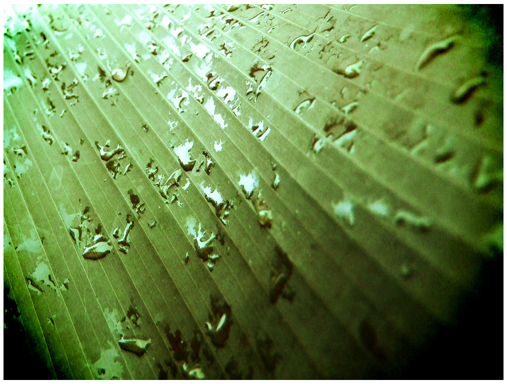
sublimation
phase change from solid to gas

vapor
gas at a temperature below the boiling temperature

vapor pressure
pressure at which a gas coexists with its solid or liquid phase

Humidity, Evaporation, and Boiling

- Explain the relationship between vapor pressure of water and the capacity of air to hold water vapor.
- Explain the relationship between relative humidity and partial pressure of water vapor in the air.
- Calculate vapor density using vapor pressure.
- Calculate humidity and dew point.

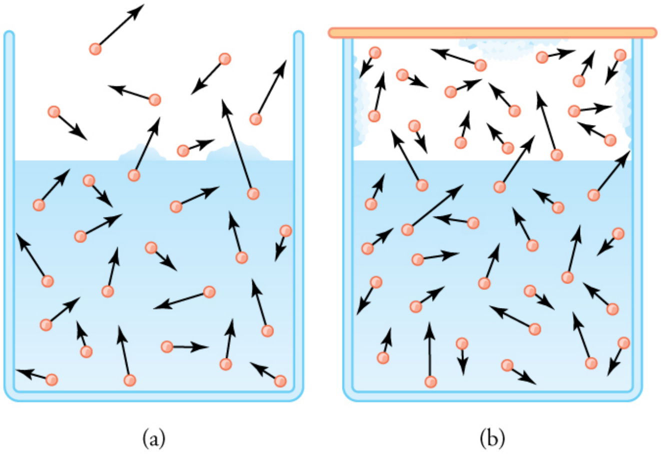


Dew drops like these, on a banana leaf photographed just after sunrise, form when the air temperature drops to or below the dew point. At the dew point, the rate at which water molecules join together is greater than the rate at which they separate, and some of the water condenses to form droplets. (credit: Aaron Escobar, Flickr)

The expression “it’s not the heat, it’s the humidity” makes a valid point. We keep cool in hot weather by evaporating sweat from our skin and water from our breathing passages. Because evaporation is inhibited by high humidity, we feel hotter at a given temperature when the humidity is high. Low humidity, on the other hand, can cause discomfort from excessive drying of mucous membranes and can lead to an increased risk of respiratory infections.

When we say humidity, we really mean **relative humidity**. Relative humidity tells us how much water vapor is in the air compared with the maximum possible. At its maximum, denoted as **saturation**, the relative humidity is 100%, and evaporation is inhibited. The amount of water vapor in the air depends on temperature. For example, relative humidity rises in the evening, as air temperature declines, sometimes reaching the **dew point**. At the dew point temperature, relative humidity is 100%, and fog may result from the condensation of water droplets if they are small enough to stay in suspension. Conversely, if you wish to dry something (perhaps your hair), it is more effective to blow hot air over it rather than cold air, because, among other things, the increase in temperature increases the energy of the molecules, so the rate of evaporation increases.

The amount of water vapor in the air depends on the vapor pressure of water. The liquid and solid phases are continuously giving off vapor because some of the molecules have high enough speeds to enter the gas phase; see [link](a). If a lid is placed over the container, as in [link](b), evaporation continues, increasing the pressure, until sufficient vapor has built up for condensation to balance evaporation. Then equilibrium has been achieved, and the vapor pressure is equal to the partial pressure of water in the container. Vapor pressure increases with temperature because molecular speeds are higher as temperature increases. [link] gives representative values of water vapor pressure over a range of temperatures.



(a) Because of the distribution of speeds and kinetic energies, some water molecules can break away to the vapor phase even at temperatures below the ordinary boiling point. (b) If the container is sealed, evaporation will continue until there is enough vapor density for the condensation rate to equal the evaporation rate. This vapor density and the partial pressure it creates are the saturation values. They increase with temperature and are independent of the presence of other gases, such as air. They depend only on the vapor pressure of water.

Relative humidity is related to the partial pressure of water vapor in the air. At 100% humidity, the partial pressure is equal to the vapor pressure, and no more water can enter the vapor phase. If the partial pressure is less than the vapor pressure, then evaporation will take place, as humidity is less than 100%. If the partial pressure is greater than the vapor pressure, condensation takes place. In everyday language, people sometimes refer to the capacity of air to “hold” water vapor, but this is not actually what happens. The water vapor is not held by the air. The amount of water in air is determined by the vapor pressure of water and has nothing to do with the properties of air.

Temperature (°C)	**Vapor pressure (Pa)**	**Saturation vapor density (g/m^3)**
−50	4.0	0.039
−20	1.04×10^2	0.89
−10	2.60×10^2	2.36
0	6.10×10^2	4.84
5	8.68×10^2	6.80
10	1.19×10^3	9.40

Temperature (ºC)	**Vapor pressure (Pa)**	**Saturation vapor density (g/m^3)**
15	1.69×10^3	12.8
20	2.33×10^3	17.2
25	3.17×10^3	23.0
30	4.24×10^3	30.4
37	6.31×10^3	44.0
40	7.34×10^3	51.1
50	1.23×10^4	82.4
60	1.99×10^4	130
70	3.12×10^4	197

Temperature (°C)	**Vapor pressure (Pa)**	**Saturation vapor density (g/m^3)**
80	4.73×10^4	294
90	7.01×10^4	418
95	8.59×10^4	505
100	$\mathbf{1.01 \times 10^5}$	**598**
120	1.99×10^5	1095
150	4.76×10^5	2430
200	1.55×10^6	7090
220	2.32×10^6	10,200

Saturation Vapor Density of Water

Note:

Percent Relative Humidity

We define **percent relative humidity** as the ratio of vapor density to saturation vapor density, or

Equation:

$$\text{percent relative humidity} = \frac{\text{vapor density}}{\text{saturation vapor density}} \times 100$$

We can use this and the data in [link] to do a variety of interesting calculations, keeping in mind that relative humidity is based on the comparison of the partial pressure of water vapor in air and ice.

Example:

Calculating Humidity and Dew Point

(a) Calculate the percent relative humidity on a day when the temperature is 25.0^{o}C and the air contains 9.40 g of water vapor per m^3. (b) At what temperature will this air reach 100% relative humidity (the saturation density)? This temperature is the dew point. (c) What is the humidity when the air temperature is 25.0^{o}C and the dew point is -10.0^{o}C?

Strategy and Solution

(a) Percent relative humidity is defined as the ratio of vapor density to saturation vapor density.

Equation:

$$\text{percent relative humidity} = \frac{\text{vapor density}}{\text{saturation vapor density}} \times 100$$

The first is given to be $9.40\ \text{g}/\text{m}^3$, and the second is found in [link] to be $23.0\ \text{g}/\text{m}^3$. Thus,

Equation:

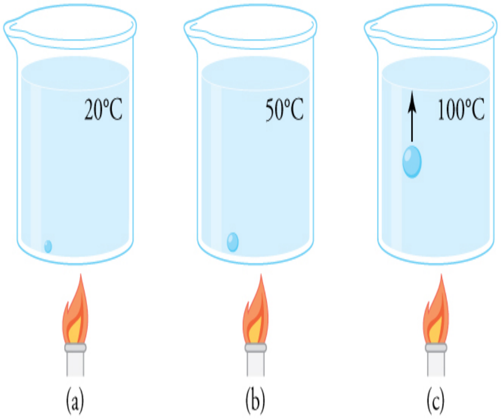
$$\text{percent relative humidity} = \frac{9.40\ \text{g/m}^3}{23.0\ \text{g/m}^3} \times 100 = 40.9.\%$$

(b) The air contains $9.40\ \text{g/m}^3$ of water vapor. The relative humidity will be 100% at a temperature where $9.40\ \text{g/m}^3$ is the saturation density. Inspection of [link] reveals this to be the case at 10.0ºC, where the relative humidity will be 100%. That temperature is called the dew point for air with this concentration of water vapor.
(c) Here, the dew point temperature is given to be -10.0ºC. Using [link], we see that the vapor density is $2.36\ \text{g/m}^3$, because this value is the saturation vapor density at -10.0ºC. The saturation vapor density at 25.0ºC is seen to be $23.0\ \text{g/m}^3$. Thus, the relative humidity at 25.0ºC is
Equation:

$$\text{percent relative humidity} = \frac{2.36\ \text{g/m}^3}{23.0\ \text{g/m}^3} \times 100 = 10.3\%.$$

Discussion
The importance of dew point is that air temperature cannot drop below 10.0ºC in part (b), or -10.0ºC in part (c), without water vapor condensing out of the air. If condensation occurs, considerable transfer of heat occurs (discussed in Heat and Heat Transfer Methods), which prevents the temperature from further dropping. When dew points are below 0ºC, freezing temperatures are a greater possibility, which explains why farmers keep track of the dew point. Low humidity in deserts means low dew-point temperatures. Thus condensation is unlikely. If the temperature drops, vapor does not condense in liquid drops. Because no heat is released into the air, the air temperature drops more rapidly compared to air with higher humidity. Likewise, at high temperatures, liquid droplets do not evaporate, so that no heat is removed from the gas to the liquid phase. This explains the large range of temperature in arid regions.

Why does water boil at 100ºC? You will note from [link] that the vapor pressure of water at 100ºC is $1.01 \times 10^5\ \text{Pa}$, or 1.00 atm. Thus, it can evaporate without limit at this temperature and pressure. But why does it form bubbles when it boils? This is because water ordinarily contains significant amounts of dissolved air and other impurities, which are observed as small bubbles of air in a glass of water. If a bubble starts out at the bottom of the container at 20ºC, it contains water vapor (about 2.30%). The pressure inside the bubble is fixed at 1.00 atm (we ignore the slight pressure exerted by the water around it). As the temperature rises, the amount of air in the bubble stays the same, but the water vapor increases; the bubble expands to keep the pressure at 1.00 atm. At 100ºC, water vapor enters the bubble continuously since the partial pressure of water is equal to 1.00 atm in equilibrium. It cannot reach this pressure, however, since the bubble also contains air and total pressure is 1.00 atm. The bubble grows in size and thereby increases the buoyant force. The bubble breaks away and rises rapidly to the surface—we call this boiling! (See [link].)



(a) An air bubble in water starts out saturated with water vapor at 20ºC. (b) As the temperature rises, water vapor enters the bubble because its vapor pressure increases. The bubble expands to keep its pressure at 1.00 atm. (c) At 100ºC, water vapor enters the bubble continuously because water’s vapor pressure exceeds its partial pressure in the bubble, which must be less than 1.00 atm. The bubble grows and rises to the surface.

Exercise:
Check Your Understanding

Problem:

Freeze drying is a process in which substances, such as foods, are dried by placing them in a vacuum chamber and lowering the atmospheric pressure around them. How does the lowered atmospheric pressure speed the drying process, and why does it cause the temperature of the food to drop?

Solution:

Decreased the atmospheric pressure results in decreased partial pressure of water, hence a lower humidity. So evaporation of water from food, for example, will be enhanced. The molecules of water most likely to break away from the food will be those with the greatest velocities. Those remaining thus have a lower average velocity and a lower temperature. This can (and does) result in the freezing and drying of the food; hence the process is aptly named freeze drying.

Note:
PhET Explorations: States of Matter
Watch different types of molecules form a solid, liquid, or gas. Add or remove heat and watch the phase change. Change the temperature or volume of a container and see a pressure-temperature diagram respond in real time. Relate the interaction potential to the forces between molecules.
https://phet.colorado.edu/sims/html/states-of-matter/latest/states-of-

matter_en.html

Section Summary

- Relative humidity is the fraction of water vapor in a gas compared to the saturation value.
- The saturation vapor density can be determined from the vapor pressure for a given temperature.
- Percent relative humidity is defined to be
  Equation:

$$\text{percent relative humidity} = \frac{\text{vapor density}}{\text{saturation vapor density}} \times 100.$$

- The dew point is the temperature at which air reaches 100% relative humidity.

Conceptual Questions

Exercise:

Problem:

Because humidity depends only on water’s vapor pressure and temperature, are the saturation vapor densities listed in [link] valid in an atmosphere of helium at a pressure of $1.01 \times 10^5\ \text{N/m}^2$, rather than air? Are those values affected by altitude on Earth?

Exercise:

Problem:

Why does a beaker of 40.0ºC water placed in a vacuum chamber start to boil as the chamber is evacuated (air is pumped out of the chamber)? At what pressure does the boiling begin? Would food cook any faster in such a beaker?

Exercise:

Problem:

Why does rubbing alcohol evaporate much more rapidly than water at STP (standard temperature and pressure)?

Problems & Exercises

Exercise:

Problem:

What is the relative humidity on a 25.0°C day when the air contains $18.0\ \text{g/m}^3$ of water vapor?

Solution:

78.3%

Exercise:

Problem:

What is the density of water vapor in g/m^3 on a hot dry day in the desert when the temperature is 40.0°C and the relative humidity is 6.00%?

Exercise:

Problem:

If the relative humidity is 90.0% on a muggy summer morning when the temperature is 20.0°C, what will it be later in the day when the temperature is 30.0°C, assuming the water vapor density remains constant?

Exercise:

Problem:

Late on an autumn day, the relative humidity is 45.0% and the temperature is 20.0°C. What will the relative humidity be that evening when the temperature has dropped to 10.0°C, assuming constant water vapor density?

Solution:

82.3%

Exercise:

Problem:

Atmospheric pressure atop Mt. Everest is $3.30\times10^{4}\ \text{N/m}^{2}$. (a) What is the partial pressure of oxygen there if it is 20.9% of the air? (b) What percent oxygen should a mountain climber breathe so that its partial pressure is the same as at sea level, where atmospheric pressure is $1.01\times10^{5}\ \text{N/m}^{2}$? (c) One of the most severe problems for those climbing very high mountains is the extreme drying of breathing passages. Why does this drying occur?

Exercise:

Problem:

What is the dew point (the temperature at which 100% relative humidity would occur) on a day when relative humidity is 39.0% at a temperature of 20.0°C?

Solution:

4.77°C

Exercise:

Problem:

On a certain day, the temperature is 25.0^{o}C and the relative humidity is 90.0%. How many grams of water must condense out of each cubic meter of air if the temperature falls to 15.0^{o}C? Such a drop in temperature can, thus, produce heavy dew or fog.

Exercise:

Problem: **Integrated Concepts**

(a) At what depth in fresh water is the critical pressure of water reached, given that the surface is at sea level? (b) At what temperature will this water boil? (c) Is a significantly higher temperature needed to boil water at a greater depth?

Exercise:

Problem: **Unreasonable Results**

(a) An automobile mechanic claims that an aluminum rod fits loosely into its hole on an aluminum engine block because the engine is hot and the rod is cold. If the hole is 10.0% bigger in diameter than the 22.0^{o}C rod, at what temperature will the rod be the same size as the hole? (b) What is unreasonable about this temperature? (c) Which premise is responsible?

Exercise:

Problem: **Unreasonable Results**

The temperature inside a supernova explosion is said to be 2.00×10^{13} K. (a) What would the average velocity $_{\text{rms}}$ of hydrogen atoms be? (b) What is unreasonable about this velocity? (c) Which premise or assumption is responsible?

Solution:

(a) $7.03 \times 10^8\ \text{m/s}$

(b) The velocity is too high—it’s greater than the speed of light.

(c) The assumption that hydrogen inside a supernova behaves as an idea gas is responsible, because of the great temperature and density in the core of a star. Furthermore, when a velocity greater than the speed of light is obtained, classical physics must be replaced by relativity, a subject not yet covered.

Exercise:

Problem: Unreasonable Results

Suppose the relative humidity is 80% on a day when the temperature is 30.0^{o}C. (a) What will the relative humidity be if the air cools to 25.0^{o}C and the vapor density remains constant? (b) What is unreasonable about this result? (c) Which premise is responsible?

Glossary

dew point	the temperature at which relative humidity is 100%; the temperature at which water starts to condense out of the air
saturation	the condition of 100% relative humidity
percent relative humidity	the ratio of vapor density to saturation vapor density
relative humidity	the amount of water in the air relative to the maximum amount the air can hold

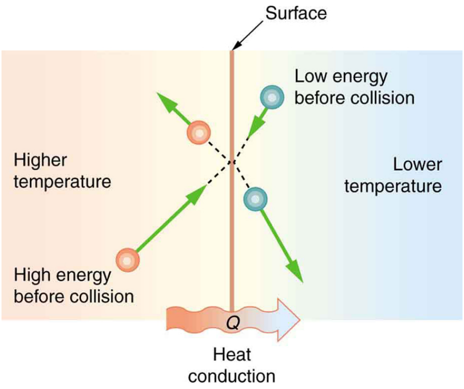
Conduction

- Calculate thermal conductivity.
- Observe conduction of heat in collisions.
- Study thermal conductivities of common substances.

Insulation is used to limit the conduction of heat from the inside to the outside (in winters) and from the outside to the inside (in summers). (credit: Giles Douglas)

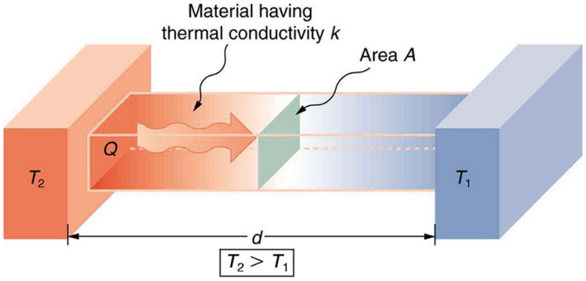
Your feet feel cold as you walk barefoot across the living room carpet in your cold house and then step onto the kitchen tile floor. This result is intriguing, since the carpet and tile floor are both at the same temperature. The different sensation you feel is explained by the different rates of heat transfer: the heat loss during the same time interval is greater for skin in contact with the tiles than with the carpet, so the temperature drop is greater on the tiles.

Some materials conduct thermal energy faster than others. In general, good conductors of electricity (metals like copper, aluminum, gold, and silver) are also good heat conductors, whereas insulators of electricity (wood, plastic, and rubber) are poor heat conductors. [link] shows molecules in two bodies at different temperatures. The (average) kinetic energy of a molecule in the hot body is higher than in the colder body. If two molecules collide, an energy transfer from the molecule with greater kinetic energy to the molecule with less kinetic energy occurs. The cumulative effect from all collisions results in a net flux of heat from the hot body to the colder body. The heat flux thus depends on the temperature difference $\Delta T = T_{\text{hot}} - T_{\text{cold}}$. Therefore, you will get a more severe burn from boiling water than from hot tap water. Conversely, if the temperatures are the same, the net heat transfer rate falls to zero, and equilibrium is achieved. Owing to the fact that the number of collisions increases with increasing area, heat conduction depends on the cross-sectional area. If you touch a cold wall with your palm, your hand cools faster than if you just touch it with your fingertip.



The molecules in two bodies at different temperatures have different average kinetic energies. Collisions occurring at the contact surface tend to transfer energy from high-temperature regions to low-temperature regions. In this illustration, a molecule in the lower temperature region (right side) has low energy before collision, but its energy increases after colliding with the contact surface. In contrast, a molecule in the higher temperature region (left side) has high energy before collision, but its energy decreases after colliding with the contact surface.

A third factor in the mechanism of conduction is the thickness of the material through which heat transfers. The figure below shows a slab of material with different temperatures on either side. Suppose that T_2 is greater than T_1, so that heat is transferred from left to right. Heat transfer from the left side to the right side is accomplished by a series of molecular collisions. The thicker the material, the more time it takes to transfer the same amount of heat. This model explains why thick clothing is warmer than thin clothing in winters, and why Arctic mammals protect themselves with thick blubber.



Heat conduction occurs through any material, represented here by a rectangular bar, whether window glass or walrus blubber. The temperature of the material is T_2 on the left and T_1 on the right, where T_2 is greater than T_1.

The rate of heat transfer by conduction is directly proportional to the surface area A, the temperature difference $T_2 - T_1$, and the substance’s conductivity k. The rate of heat transfer is inversely proportional to the thickness d.

Lastly, the heat transfer rate depends on the material properties described by the coefficient of thermal conductivity. All four factors are included in a simple equation that was deduced from and is confirmed by experiments. The **rate of conductive heat transfer** through a slab of material, such as the one in [link], is given by

Equation:

$$\frac{Q}{t} = \frac{\mathrm{kA}(T_2 - T_1)}{d},$$

where Q/t is the rate of heat transfer in watts or kilocalories per second, k is the **thermal conductivity** of the material, A and d are its surface area and thickness, as shown in [link], and $(T_2 - T_1)$ is the temperature difference across the slab. [link] gives representative values of thermal conductivity.

Example:

Calculating Heat Transfer Through Conduction: Conduction Rate Through an Ice Box

A Styrofoam ice box has a total area of $0.950\ \mathrm{m}^2$ and walls with an average thickness of 2.50 cm. The box contains ice, water, and canned beverages at $0^\mathrm{o}\mathrm{C}$. The inside of the box is kept cold by melting ice. How much ice melts in one day if the ice box is kept in the trunk of a car at $35.0^\mathrm{o}\mathrm{C}$?

Strategy

This question involves both heat for a phase change (melting of ice) and the transfer of heat by conduction. To find the amount of ice melted, we must find the net heat transferred. This value can be obtained by calculating the rate of heat transfer by conduction and multiplying by time.

Solution

1. Identify the knowns.
   Equation:
   $$A = 0.950\ \mathrm{m}^2;\ d = 2.50\ \mathrm{cm} = 0.0250\ \mathrm{m};\ T_1 = 0^\mathrm{o}\mathrm{C};\ T_2 = 35.0^\mathrm{o}\mathrm{C},\ t = 1\ \mathrm{day} = 24\ \mathrm{hours} = 86{,}400\ \mathrm{s}.$$
2. Identify the unknowns. We need to solve for the mass of the ice, m. We will also need to solve for the net heat transferred to melt the ice, Q.
3. Determine which equations to use. The rate of heat transfer by conduction is given by
   Equation:
   $$\frac{Q}{t} = \frac{\mathrm{kA}(T_2 - T_1)}{d}.$$
4. The heat is used to melt the ice: $Q = \mathrm{mL}_\mathrm{f}$.
5. Insert the known values:
   Equation:
   $$\frac{Q}{t} = \frac{\left(0.010\ \mathrm{J/s \cdot m \cdot {}^o\, C}\right)\left(0.950\ \ \mathrm{m}^2\right)\left(35.0^\mathrm{o}\mathrm{C} - 0^\mathrm{o}\mathrm{C}\right)}{0.0250\ \mathrm{m}} = 13.3\ \mathrm{J/s}.$$

6. Multiply the rate of heat transfer by the time (1 day $=$ 86,400 s):
   Equation:

$$Q = (Q/t)t = (13.3\text{ J/s})(86{,}400\text{ s}) = 1.15 \times 10^6\text{ J}.$$

7. Set this equal to the heat transferred to melt the ice: $Q = \mathrm{mL}_\mathrm{f}$. Solve for the mass m:
   Equation:

$$m = \frac{Q}{L_\mathrm{f}} = \frac{1.15 \times 10^6\text{ J}}{334\ \times 10^3\text{ J/kg}} = 3.44\text{kg}.$$

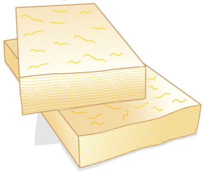
Discussion
The result of 3.44 kg, or about 7.6 lbs, seems about right, based on experience. You might expect to use about a 4 kg (7–10 lb) bag of ice per day. A little extra ice is required if you add any warm food or beverages. Inspecting the conductivities in [link] shows that Styrofoam is a very poor conductor and thus a good insulator. Other good insulators include fiberglass, wool, and goose-down feathers. Like Styrofoam, these all incorporate many small pockets of air, taking advantage of air's poor thermal conductivity.

Substance	Thermal conductivity k $(\mathrm{J/s{\cdot}m{\cdot}^\circ C})$
Silver	420
Copper	390
Gold	318
Aluminum	220
Steel iron	80
Steel (stainless)	14
Ice	2.2
Glass (average)	0.84
Concrete brick	0.84
Water	0.6
Fatty tissue (without blood)	0.2
Asbestos	0.16
Plasterboard	0.16
Wood	0.08–0.16

Substance	**Thermal conductivity k $(\mathbf{J/s \cdot m \cdot {}^{\circ}C})$**
Snow (dry)	0.10
Cork	0.042
Glass wool	0.042
Wool	0.04
Down feathers	0.025
Air	0.023
Styrofoam	0.010

Thermal Conductivities of Common Substances[footnote]
At temperatures near 0ºC.

A combination of material and thickness is often manipulated to develop good insulators—the smaller the conductivity k and the larger the thickness d, the better. The ratio of d/k will thus be large for a good insulator. The ratio d/k is called the R **factor**. The rate of conductive heat transfer is inversely proportional to R. The larger the value of R, the better the insulation. R factors are most commonly quoted for household insulation, refrigerators, and the like—unfortunately, it is still in non-metric units of $\text{ft}^2\cdot$°F·h/Btu, although the unit usually goes unstated (1 British thermal unit [Btu] is the amount of energy needed to change the temperature of 1.0 lb of water by 1.0 °F). A couple of representative values are an R factor of 11 for 3.5-in-thick fiberglass batts (pieces) of insulation and an R factor of 19 for 6.5-in-thick fiberglass batts. Walls are usually insulated with 3.5-in batts, while ceilings are usually insulated with 6.5-in batts. In cold climates, thicker batts may be used in ceilings and walls.



The fiberglass batt is used for insulation of walls and ceilings to prevent heat transfer between the inside of the building and the outside environment.

Note that in [link], the best thermal conductors—silver, copper, gold, and aluminum—are also the best electrical conductors, again related to the density of free electrons in them. Cooking utensils are typically made from good conductors.

Example:

Calculating the Temperature Difference Maintained by a Heat Transfer: Conduction Through an Aluminum Pan

Water is boiling in an aluminum pan placed on an electrical element on a stovetop. The sauce pan has a bottom that is 0.800 cm thick and 14.0 cm in diameter. The boiling water is evaporating at the rate of 1.00 g/s. What is the temperature difference across (through) the bottom of the pan?

Strategy

Conduction through the aluminum is the primary method of heat transfer here, and so we use the equation for the rate of heat transfer and solve for the temperature difference.

Equation:

$$T_2 - T_1 = \frac{Q}{t}\left(\frac{d}{\mathrm{kA}}\right).$$

Solution

1. Identify the knowns and convert them to the SI units.

   The thickness of the pan, $d = 0.800\text{ cm} = 8.0 \times 10^{-3}\text{ m}$, the area of the pan, $A = \pi(0.14/2)^2\text{ m}^2 = 1.54 \times 10^{-2}\text{ m}^2$, and the thermal conductivity, $k = 220\text{ J/s}\cdot\text{m}\cdot{}^\circ\text{C}$.
2. Calculate the necessary heat of vaporization of 1 g of water:
   Equation:

   $$Q = \mathrm{mL_v} = \left(1.00 \times 10^{-3}\text{ kg}\right)\left(2256 \times 10^3\text{ J/kg}\right) = 2256\text{ J}.$$
3. Calculate the rate of heat transfer given that 1 g of water melts in one second:
   Equation:

   $$Q/t = 2256\text{ J/s or }2.26\text{ kW}.$$
4. Insert the knowns into the equation and solve for the temperature difference:
   Equation:

   $$T_2 - T_1 = \frac{Q}{t}\left(\frac{d}{\mathrm{kA}}\right) = (2256\text{ J/s})\frac{8.00 \times 10^{-3}\text{m}}{(220\text{ J/s}\cdot\text{m}\cdot{}^\text{o}\text{ C})\left(1.54 \times 10^{-2}\text{ m}^2\right)} = 5.33^\text{o}\text{C}.$$

Discussion

The value for the heat transfer $Q/t = 2.26\text{kW}$ or 2256 J/s is typical for an electric stove. This value gives a remarkably small temperature difference between the stove and the pan. Consider that the stove burner is red hot while the inside of the pan is nearly 100ºC because of its contact with boiling water. This contact effectively cools the bottom of the pan in spite of its proximity to the very hot stove burner. Aluminum is such a good conductor that it only takes this small temperature difference to produce a heat transfer of 2.26 kW into the pan.

Conduction is caused by the random motion of atoms and molecules. As such, it is an ineffective mechanism for heat transport over macroscopic distances and short time distances. Take, for example, the temperature on the Earth, which would be unbearably cold during the night and extremely hot during the day if heat transport in the atmosphere was to be only through conduction. In another example, car engines would overheat unless there was a more efficient way to remove excess heat from the pistons.

Exercise:
Check Your Understanding

Problem:

How does the rate of heat transfer by conduction change when all spatial dimensions are doubled?

Solution:

Because area is the product of two spatial dimensions, it increases by a factor of four when each dimension is doubled $\left(A_{\text{final}} = (2d)^2 = 4d^2 = 4A_{\text{initial}}\right)$. The distance, however, simply doubles. Because the temperature difference and the coefficient of thermal conductivity are independent of the spatial dimensions, the rate of heat transfer by conduction increases by a factor of four divided by two, or two:

Equation:

$$\left(\frac{Q}{t}\right)_{\text{final}} = \frac{\mathrm{kA}_{\text{final}}(T_2 - T_1)}{d_{\text{final}}} = \frac{k(4\mathrm{A}_{\text{initial}})(T_2 - T_1)}{2\mathrm{d}_{\text{initial}}} = 2\frac{\mathrm{kA}_{\text{initial}}(T_2 - T_1)}{d_{\text{initial}}} = 2\left(\frac{Q}{t}\right)_{\text{initial}}.$$

Summary

- Heat conduction is the transfer of heat between two objects in direct contact with each other.
- The rate of heat transfer Q/t (energy per unit time) is proportional to the temperature difference $T_2 - T_1$ and the contact area A and inversely proportional to the distance d between the objects:
  Equation:

  $$\frac{Q}{t} = \frac{\mathrm{kA}(T_2 - T_1)}{d}.$$

Conceptual Questions

Exercise:

Problem:

Some electric stoves have a flat ceramic surface with heating elements hidden beneath. A pot placed over a heating element will be heated, while it is safe to touch the surface only a few centimeters away. Why is ceramic, with a conductivity less than that of a metal but greater than that of a good insulator, an ideal choice for the stove top?

Exercise:

Problem:

Loose-fitting white clothing covering most of the body is ideal for desert dwellers, both in the hot Sun and during cold evenings. Explain how such clothing is advantageous during both day and night.

A jellabiya is worn by many men in Egypt. (credit: Zerida)

Problems & Exercises

Exercise:

Problem:

(a) Calculate the rate of heat conduction through house walls that are 13.0 cm thick and that have an average thermal conductivity twice that of glass wool. Assume there are no windows or doors. The surface area of the walls is $120\ \text{m}^2$ and their inside surface is at 18.0°C, while their outside surface is at 5.00°C. (b) How many 1-kW room heaters would be needed to balance the heat transfer due to conduction?

Solution:

(a) 1.01×10^3 W

(b) One

Exercise:

Problem:

The rate of heat conduction out of a window on a winter day is rapid enough to chill the air next to it. To see just how rapidly the windows transfer heat by conduction, calculate the rate of conduction in watts through a 3.00-m^2 window that is $0.635\ \text{cm}$ thick (1/4 in) if the temperatures of the inner and outer surfaces are 5.00°C and -10.0°C, respectively. This rapid rate will not be maintained—the inner surface will cool, and even result in frost formation.

Exercise:

Problem:

Calculate the rate of heat conduction out of the human body, assuming that the core internal temperature is 37.0°C, the skin temperature is 34.0°C, the thickness of the tissues between averages $1.00\ \text{cm}$, and the surface area is $1.40\ \text{m}^2$.

Solution:

84.0 W

Exercise:

Problem:

Suppose you stand with one foot on ceramic flooring and one foot on a wool carpet, making contact over an area of $80.0\ \text{cm}^2$ with each foot. Both the ceramic and the carpet are 2.00 cm thick and are 10.0^oC on their bottom sides. At what rate must heat transfer occur from each foot to keep the top of the ceramic and carpet at 33.0^oC?

Exercise:

Problem:

A man consumes 3000 kcal of food in one day, converting most of it to maintain body temperature. If he loses half this energy by evaporating water (through breathing and sweating), how many kilograms of water evaporate?

Solution:

2.59 kg

Exercise:

Problem:

(a) A firewalker runs across a bed of hot coals without sustaining burns. Calculate the heat transferred by conduction into the sole of one foot of a firewalker given that the bottom of the foot is a 3.00-mm-thick callus with a conductivity at the low end of the range for wood and its density is $300\ \text{kg/m}^3$. The area of contact is $25.0\ \text{cm}^2$, the temperature of the coals is 700^oC, and the time in contact is 1.00 s.

(b) What temperature increase is produced in the $25.0\ \text{cm}^3$ of tissue affected?

(c) What effect do you think this will have on the tissue, keeping in mind that a callus is made of dead cells?

Exercise:

Problem:

(a) What is the rate of heat conduction through the 3.00-cm-thick fur of a large animal having a 1.40-m^2 surface area? Assume that the animal’s skin temperature is 32.0^oC, that the air temperature is -5.00^oC, and that fur has the same thermal conductivity as air. (b) What food intake will the animal need in one day to replace this heat transfer?

Solution:

(a) 39.7 W

(b) 820 kcal

Exercise:

Problem:

A walrus transfers energy by conduction through its blubber at the rate of 150 W when immersed in -1.00°C water. The walrus's internal core temperature is 37.0°C, and it has a surface area of $2.00\ \text{m}^2$. What is the average thickness of its blubber, which has the conductivity of fatty tissues without blood?

Walrus on ice. (credit: Captain Budd Christman, NOAA Corps)

Exercise:

Problem:

Compare the rate of heat conduction through a 13.0-cm-thick wall that has an area of $10.0\ \text{m}^2$ and a thermal conductivity twice that of glass wool with the rate of heat conduction through a window that is 0.750 cm thick and that has an area of $2.00\ \text{m}^2$, assuming the same temperature difference across each.

Solution:

35 to 1, window to wall

Exercise:

Problem:

Suppose a person is covered head to foot by wool clothing with average thickness of 2.00 cm and is transferring energy by conduction through the clothing at the rate of 50.0 W. What is the temperature difference across the clothing, given the surface area is $1.40\ \text{m}^2$?

Exercise:

Problem:

Some stove tops are smooth ceramic for easy cleaning. If the ceramic is 0.600 cm thick and heat conduction occurs through the same area and at the same rate as computed in [link], what is the temperature difference across it? Ceramic has the same thermal conductivity as glass and brick.

Solution:

$1.05 \times 10^3\ \text{K}$

Exercise:

Problem:

One easy way to reduce heating (and cooling) costs is to add extra insulation in the attic of a house. Suppose the house already had 15 cm of fiberglass insulation in the attic and in all the exterior surfaces. If you added an extra 8.0 cm of fiberglass to the attic, then by what percentage would the heating cost of the house drop? Take the single story house to be of dimensions 10 m by 15 m by 3.0 m. Ignore air infiltration and heat loss through windows and doors.

Exercise:

Problem:

(a) Calculate the rate of heat conduction through a double-paned window that has a 1.50-m^2 area and is made of two panes of 0.800-cm-thick glass separated by a 1.00-cm air gap. The inside surface temperature is 15.0°C, while that on the outside is -10.0°C. (Hint: There are identical temperature drops across the two glass panes. First find these and then the temperature drop across the air gap. This problem ignores the increased heat transfer in the air gap due to convection.)

(b) Calculate the rate of heat conduction through a 1.60-cm-thick window of the same area and with the same temperatures. Compare your answer with that for part (a).

Solution:

(a) 83 W

(b) 24 times that of a double pane window.

Exercise:

Problem:

Many decisions are made on the basis of the payback period: the time it will take through savings to equal the capital cost of an investment. Acceptable payback times depend upon the business or philosophy one has. (For some industries, a payback period is as small as two years.) Suppose you wish to install the extra insulation in [link]. If energy cost $1.00 per million joules and the insulation was $4.00 per square meter, then calculate the simple payback time. Take the average ΔT for the 120 day heating season to be 15.0°C.

Exercise:

Problem:

For the human body, what is the rate of heat transfer by conduction through the body's tissue with the following conditions: the tissue thickness is 3.00 cm, the change in temperature is 2.00°C, and the skin area is $1.50\ \text{m}^2$. How does this compare with the average heat transfer rate to the body resulting from an energy intake of about 2400 kcal per day? (No exercise is included.)

Solution:

20.0 W, 17.2% of 2400 kcal per day

Glossary

R factor	the ratio of thickness to the conductivity of a material
rate of conductive heat transfer	rate of heat transfer from one material to another

thermal conductivity

the property of a material’s ability to conduct heat

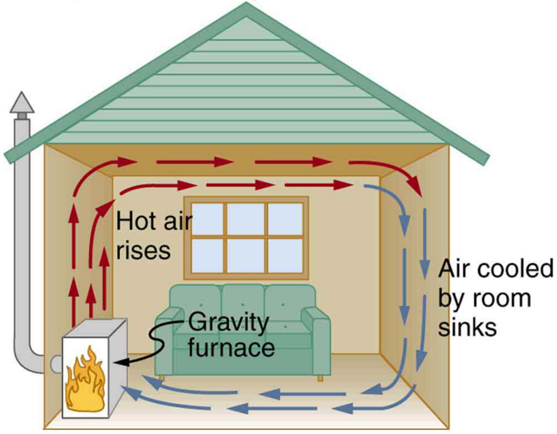
Convection

- Discuss the method of heat transfer by convection.

Convection is driven by large-scale flow of matter. In the case of Earth, the atmospheric circulation is caused by the flow of hot air from the tropics to the poles, and the flow of cold air from the poles toward the tropics. (Note that Earth's rotation causes the observed easterly flow of air in the northern hemisphere). Car engines are kept cool by the flow of water in the cooling system, with the water pump maintaining a flow of cool water to the pistons. The circulatory system is used the body: when the body overheats, the blood vessels in the skin expand (dilate), which increases the blood flow to the skin where it can be cooled by sweating. These vessels become smaller when it is cold outside and larger when it is hot (so more fluid flows, and more energy is transferred).

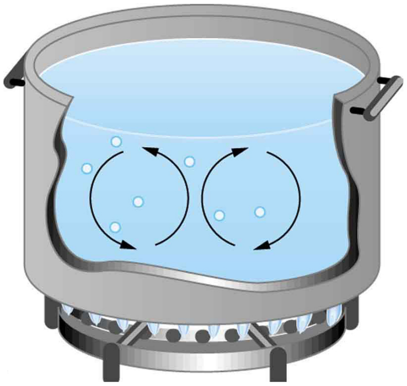
The body also loses a significant fraction of its heat through the breathing process.

While convection is usually more complicated than conduction, we can describe convection and do some straightforward, realistic calculations of its effects. Natural convection is driven by buoyant forces: hot air rises because density decreases as temperature increases. The house in [link] is kept warm in this manner, as is the pot of water on the stove in [link]. Ocean currents and large-scale atmospheric circulation transfer energy from one part of the globe to another. Both are examples of natural convection.



Air heated by the so-called

gravity furnace expands and rises, forming a convective loop that transfers energy to other parts of the room. As the air is cooled at the ceiling and outside walls, it contracts, eventually becoming denser than room air and sinking to the floor. A properly designed heating system using natural convection, like this one, can be quite efficient in uniformly heating a home.



Convection plays an important role in heat transfer inside this pot of water. Once conducted to the inside, heat transfer to other parts of the pot is mostly by convection. The hotter water expands, decreases in density, and rises to transfer heat to other regions of the water, while colder water sinks to the bottom. This process keeps repeating.

Note:

Take-Home Experiment: Convection Rolls in a Heated Pan

Take two small pots of water and use an eye dropper to place a drop of food coloring near the bottom of each. Leave one on a bench top and heat the other over a stovetop. Watch how the color spreads and how long it takes the color to reach the top. Watch how convective loops form.

Example:

Calculating Heat Transfer by Convection: Convection of Air Through the Walls of a House

Most houses are not airtight: air goes in and out around doors and windows, through cracks and crevices, following wiring to switches and outlets, and so on. The air in a typical house is completely replaced in less than an hour. Suppose that a moderately-sized house has inside dimensions $12.0\text{m} \times 18.0\text{m} \times 3.00\text{m}$ high, and that all air is replaced in 30.0 min. Calculate the heat transfer per unit time in watts needed to warm the incoming cold air by 10.0ºC, thus replacing the heat transferred by convection alone.

Strategy

Heat is used to raise the temperature of air so that $Q = \text{mc}\Delta T$. The rate of heat transfer is then Q/t, where t is the time for air turnover. We are given that ΔT is 10.0ºC, but we must still find values for the mass of air and its specific heat before we can calculate Q. The specific heat of air is a weighted average of the specific heats of nitrogen and oxygen, which gives $c = c_{\text{p}} \cong 1000\ \text{J/kg} \cdot^{\text{o}}\ \text{C}$ from [link] (note that the specific heat at constant pressure must be used for this process).

Solution

1. Determine the mass of air from its density and the given volume of the house. The density is given from the density ρ and the volume
   Equation:

$$m = \text{ρV} = \left(1.29\ \text{kg/m}^3\right)(12.0\ \text{m} \times 18.0\ \text{m} \times 3.00\ \text{m}) = 836\ \text{kg}.$$

2. Calculate the heat transferred from the change in air temperature: $Q = \text{mc}\Delta T$ so that
   Equation:

$$Q = (836\ \text{kg})(1000\ \text{J/kg} \cdot^{\text{o}}\ \text{C})(10.0^{\text{o}}\text{C}) = 8.36 \times 10^6\ \text{J}.$$

3. Calculate the heat transfer from the heat Q and the turnover time t. Since air is turned over in $t = 0.500\ \text{h} = 1800\ \text{s}$, the heat transferred per unit time is
   Equation:

$$\frac{Q}{t} = \frac{8.36 \times 10^6\ \text{J}}{1800\ \text{s}} = 4.64\ \text{kW}.$$

Discussion

This rate of heat transfer is equal to the power consumed by about forty-six 100-W light bulbs. Newly constructed homes are designed for a turnover time of 2 hours or more, rather than 30 minutes for the house of this example. Weather stripping, caulking, and improved window seals are commonly employed. More extreme measures are sometimes taken in very cold (or hot) climates to achieve a tight standard of more than 6 hours for one air turnover. Still longer turnover times are unhealthy, because a minimum amount of fresh air is necessary to supply oxygen for breathing and to dilute household pollutants. The term used for the process by which outside air leaks into the house from cracks around windows, doors, and the foundation is called “air infiltration.”

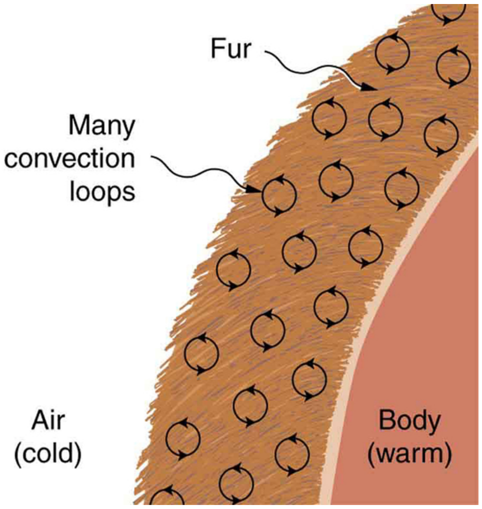
A cold wind is much more chilling than still cold air, because convection combines with conduction in the body to increase the rate at which energy is transferred away from the body. The table below gives approximate wind-chill factors, which are the temperatures of still air that produce the same rate of cooling as air of a given temperature and speed. Wind-chill factors are a dramatic reminder of convection’s ability to transfer heat faster than conduction. For example, a 15.0 m/s wind at 0ºC has the chilling equivalent of still air at about −18ºC.

Moving air temperature	**Wind speed (m/s)**				
(ºC)	**2**	**5**	**10**	**15**	**20**
5	3	−1	−8	−10	−12
2	0	−7	−12	−16	−18
0	−2	−9	−15	−18	−20

Moving air temperature	**Wind speed (m/s)**				
−5	−7	−15	−22	−26	−29
−10	−12	−21	−29	−34	−36
−20	−23	−34	−44	−50	−52
−40	−44	−59	−73	−82	−84

Wind-Chill Factors

Although air can transfer heat rapidly by convection, it is a poor conductor and thus a good insulator. The amount of available space for airflow determines whether air acts as an insulator or conductor. The space between the inside and outside walls of a house, for example, is about 9 cm (3.5 in)—large enough for convection to work effectively. The addition of wall insulation prevents airflow, so heat loss (or gain) is decreased. Similarly, the gap between the two panes of a double-paned window is about 1 cm, which prevents convection and takes advantage of air’s low conductivity to prevent greater loss. Fur, fiber, and fiberglass also take advantage of the low conductivity of air by trapping it in spaces too small to support convection, as shown in the figure. Fur and feathers are lightweight and thus ideal for the protection of animals.



Fur is filled with air, breaking it up into many small pockets. Convection is very slow here, because the loops are so small. The low conductivity of air makes fur a very good lightweight insulator.

Some interesting phenomena happen *when convection is accompanied by a phase change*. It allows us to cool off by sweating, even if the temperature of the surrounding air exceeds body temperature. Heat from the skin is required for sweat to evaporate from the skin, but without air flow, the air becomes saturated and evaporation stops. Air flow caused by convection replaces the saturated air by dry air and evaporation continues.

Example:
Calculate the Flow of Mass during Convection: Sweat-Heat Transfer away from the Body

The average person produces heat at the rate of about 120 W when at rest. At what rate must water evaporate from the body to get rid of all this energy? (This evaporation might occur when a person is sitting in the shade and surrounding temperatures are the same as skin temperature, eliminating heat transfer by other methods.)

Strategy

Energy is needed for a phase change ($Q = \mathrm{mL_v}$). Thus, the energy loss per unit time is

Equation:

$$\frac{Q}{t} = \frac{\mathrm{mL_v}}{t} = 120\ \ \mathrm{W} = 120\ \mathrm{J/s}.$$

We divide both sides of the equation by L_v to find that the mass evaporated per unit time is

Equation:

$$\frac{m}{t} = \frac{120\ \mathrm{J/s}}{L_\mathrm{v}}.$$

Solution

(1) Insert the value of the latent heat from [link], $L_\mathrm{v} = 2430\ \mathrm{kJ/kg} = 2430\ \mathrm{J/g}$. This yields

Equation:

$$\frac{m}{t} = \frac{120\ \mathrm{J/s}}{2430\ \mathrm{J/g}} = 0.0494\ \mathrm{g/s} = 2.96\ \mathrm{g/min}.$$

Discussion

Evaporating about 3 g/min seems reasonable. This would be about 180 g (about 7 oz) per hour. If the air is very dry, the sweat may evaporate without even being noticed. A significant amount of evaporation also takes place in the lungs and breathing passages.

Another important example of the combination of phase change and convection occurs when water evaporates from the oceans. Heat is removed from the ocean when water evaporates. If the water vapor condenses in liquid droplets as clouds form, heat is released in the atmosphere. Thus, there is an overall transfer of heat from the ocean to the atmosphere. This process is the driving power behind thunderheads, those great cumulus clouds that rise as much as 20.0 km into the stratosphere. Water vapor carried in by convection condenses, releasing tremendous amounts of energy. This energy causes the air to expand and rise, where it is colder. More condensation occurs in these colder regions, which in turn drives the cloud even higher. Such a mechanism is called positive feedback, since the process reinforces and accelerates itself. These systems sometimes produce violent storms, with lightning and hail, and constitute the mechanism driving hurricanes.

Cumulus clouds are caused by water vapor that rises because of convection. The rise of clouds is driven by a positive feedback mechanism. (credit: Mike Love)

Convection accompanied by a phase change releases the energy needed to drive this thunderhead into the stratosphere. (credit: Gerardo García Moretti)

The phase change that occurs when this iceberg melts involves tremendous heat

transfer. (credit:
Dominic Alves)

The movement of icebergs is another example of convection accompanied by a phase change. Suppose an iceberg drifts from Greenland into warmer Atlantic waters. Heat is removed from the warm ocean water when the ice melts and heat is released to the land mass when the iceberg forms on Greenland.

Exercise:

Check Your Understanding

Problem: Explain why using a fan in the summer feels refreshing!

Solution:

Using a fan increases the flow of air: warm air near your body is replaced by cooler air from elsewhere. Convection increases the rate of heat transfer so that moving air "feels" cooler than still air.

Summary

- Convection is heat transfer by the macroscopic movement of mass. Convection can be natural or forced and generally transfers thermal energy faster than conduction. [link] gives wind-chill factors, indicating that moving air has the same chilling effect of much colder stationary air. *Convection that occurs along with a phase change* can transfer energy from cold regions to warm ones.

Conceptual Questions

Exercise:

Problem:

One way to make a fireplace more energy efficient is to have an external air supply for the combustion of its fuel. Another is to have room air circulate around the outside of the fire box and back into the room. Detail the methods of heat transfer involved in each.

Exercise:

Problem:

On cold, clear nights horses will sleep under the cover of large trees. How does this help them keep warm?

Problems & Exercises

Exercise:

Problem:

At what wind speed does -10°C air cause the same chill factor as still air at -29°C?

Solution:

10 m/s

Exercise:

Problem:

At what temperature does still air cause the same chill factor as -5°C air moving at 15 m/s?

Exercise:

Problem:

The “steam” above a freshly made cup of instant coffee is really water vapor droplets condensing after evaporating from the hot coffee. What is the final temperature of 250 g of hot coffee initially at 90.0°C if 2.00 g evaporates from it? The coffee is in a Styrofoam cup, so other methods of heat transfer can be neglected.

Solution:

85.7°C

Exercise:

Problem:

(a) How many kilograms of water must evaporate from a 60.0-kg woman to lower her body temperature by 0.750°C?

(b) Is this a reasonable amount of water to evaporate in the form of perspiration, assuming the relative humidity of the surrounding air is low?

Exercise:

Problem:

On a hot dry day, evaporation from a lake has just enough heat transfer to balance the $1.00\ \text{kW/m}^2$ of incoming heat from the Sun. What mass of water evaporates in 1.00 h from each square meter? Explicitly show how you follow the steps in the Problem-Solving Strategies for the Effects of Heat Transfer.

Solution:

1.48 kg

Exercise:

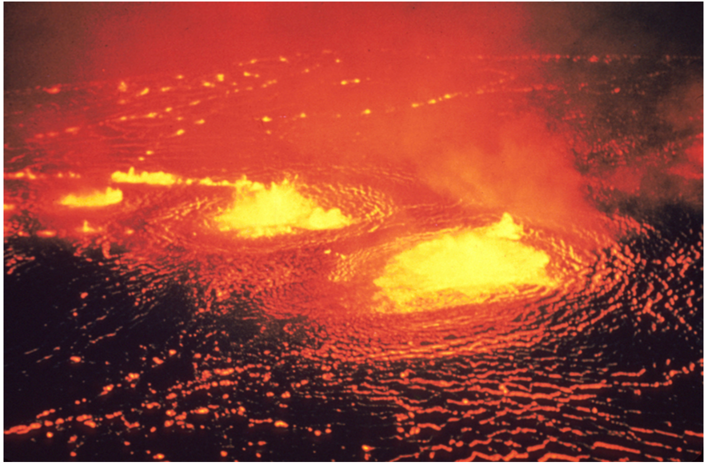
Problem:

One winter day, the climate control system of a large university classroom building malfunctions. As a result, $500\ \text{m}^3$ of excess cold air is brought in each minute. At what rate in kilowatts must heat transfer occur to warm this air by 10.0°C (that is, to bring the air to room temperature)?

Exercise:

Problem:

The Kilauea volcano in Hawaii is the world’s most active, disgorging about $5 \times 10^5\ \text{m}^3$ of 1200°C lava per day. What is the rate of heat transfer out of Earth by convection if this lava has a density of $2700\ \text{kg/m}^3$ and eventually cools to 30°C? Assume that the specific heat of lava is the same as that of granite.



Lava flow on Kilauea volcano in Hawaii. (credit: J. P. Eaton, U.S. Geological Survey)

Solution:

$2 \times 10^4\ \text{MW}$

Exercise:

Problem:

During heavy exercise, the body pumps 2.00 L of blood per minute to the surface, where it is cooled by 2.00°C. What is the rate of heat transfer from this forced convection alone, assuming blood has the same specific heat as water and its density is $1050\ \text{kg/m}^3$?

Exercise:

Problem:

A person inhales and exhales 2.00 L of 37.0°C air, evaporating 4.00×10^{-2} g of water from the lungs and breathing passages with each breath.

(a) How much heat transfer occurs due to evaporation in each breath?

(b) What is the rate of heat transfer in watts if the person is breathing at a moderate rate of 18.0 breaths per minute?

(c) If the inhaled air had a temperature of 20.0°C, what is the rate of heat transfer for warming the air?

(d) Discuss the total rate of heat transfer as it relates to typical metabolic rates. Will this breathing be a major form of heat transfer for this person?

Solution:

(a) 97.2 J

(b) 29.2 W

(c) 9.49 W

(d) The total rate of heat loss would be $29.2\ \text{W} + 9.49\ \text{W} = 38.7\ \text{W}$. While sleeping, our body consumes 83 W of power, while sitting it consumes 120 to 210 W. Therefore, the total rate of heat loss from breathing will not be a major form of heat loss for this person.

Exercise:

Problem:

A glass coffee pot has a circular bottom with a 9.00-cm diameter in contact with a heating element that keeps the coffee warm with a continuous heat transfer rate of 50.0 W

(a) What is the temperature of the bottom of the pot, if it is 3.00 mm thick and the inside temperature is 60.0°C?

(b) If the temperature of the coffee remains constant and all of the heat transfer is removed by evaporation, how many grams per minute evaporate? Take the heat of vaporization to be 2340 kJ/kg.

Radiation

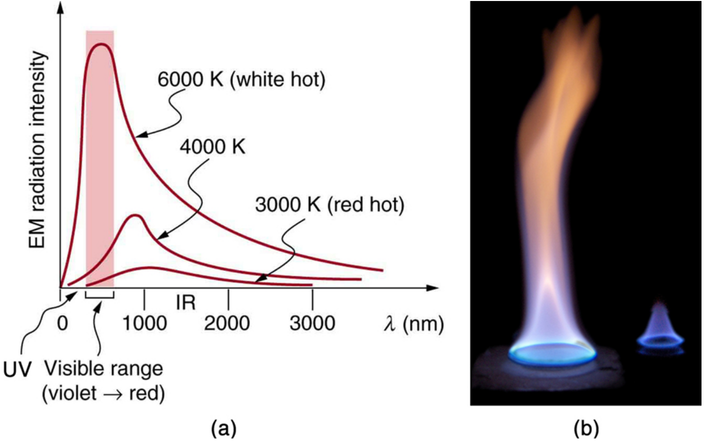
- Discuss heat transfer by radiation.
- Explain the power of different materials.

You can feel the heat transfer from a fire and from the Sun. Similarly, you can sometimes tell that the oven is hot without touching its door or looking inside—it may just warm you as you walk by. The space between the Earth and the Sun is largely empty, without any possibility of heat transfer by convection or conduction. In these examples, heat is transferred by radiation. That is, the hot body emits electromagnetic waves that are absorbed by our skin: no medium is required for electromagnetic waves to propagate. Different names are used for electromagnetic waves of different wavelengths: radio waves, microwaves, infrared **radiation**, visible light, ultraviolet radiation, X-rays, and gamma rays.

Most of the heat transfer from this fire to the observers is through infrared radiation. The visible light, although dramatic, transfers relatively little thermal energy. Convection transfers energy away from the observers as hot air rises, while conduction is negligibly slow here. Skin is very sensitive to infrared radiation, so that you can sense the presence of a fire without looking at it directly. (credit: Daniel X. O’Neil)

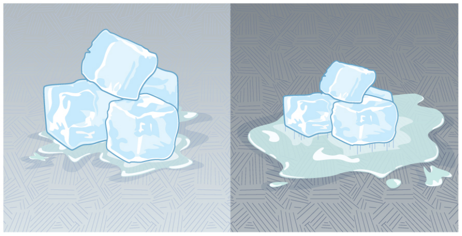
The energy of electromagnetic radiation depends on the wavelength (color) and varies over a wide range: a smaller wavelength (or higher frequency) corresponds to a higher energy. Because more heat is radiated at higher temperatures, a temperature change is accompanied by a color change. Take, for example, an electrical element on a stove, which glows from red to orange, while the higher-temperature steel in a blast furnace glows from yellow to white. The radiation you feel is mostly infrared, which corresponds to a lower temperature than that of the electrical element and the steel. The radiated energy depends on its intensity, which is represented in the figure below by the height of the distribution.

Electromagnetic Waves explains more about the electromagnetic spectrum and Introduction to Quantum Physics discusses how the decrease in wavelength corresponds to an increase in energy.



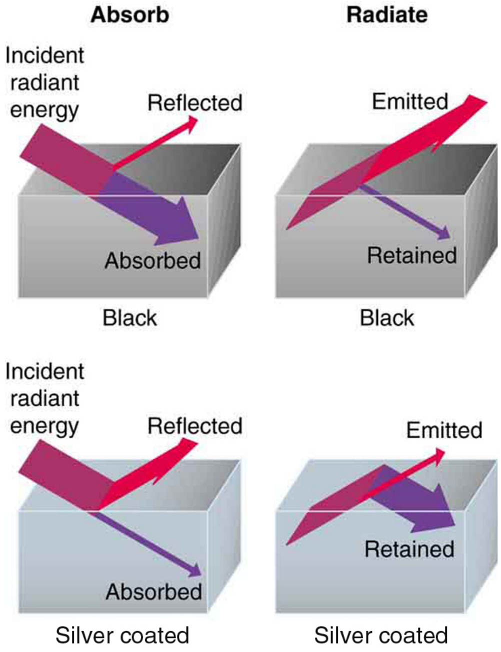
(a) A graph of the spectra of electromagnetic waves emitted from an ideal radiator at three different temperatures. The intensity or rate of radiation emission increases dramatically with temperature, and the spectrum shifts toward the visible and ultraviolet parts of the spectrum. The shaded portion denotes the visible part of the spectrum. It is apparent that the shift toward the ultraviolet with temperature makes the visible appearance shift from red to white to blue as temperature increases. (b) Note the variations in color corresponding to variations in flame temperature. (credit: Tuohirulla)

All objects absorb and emit electromagnetic radiation. The rate of heat transfer by radiation is largely determined by the color of the object. Black is the most effective, and white is the least effective. People living in hot climates generally avoid wearing black clothing, for instance (see [link]). Similarly, black asphalt in a parking lot will be hotter than adjacent gray sidewalk on a summer day, because black absorbs better than gray. The reverse is also true—black radiates better than gray. Thus, on a clear summer night, the asphalt will be colder than the gray sidewalk, because black radiates the energy more rapidly than gray. An *ideal radiator* is the same color as an *ideal absorber*, and captures all the radiation that falls on it. In contrast, white is a poor absorber and is also a poor radiator. A white object reflects all radiation, like a mirror. (A perfect, polished white surface is mirror-like in appearance, and a crushed mirror looks white.)



This illustration shows that the darker pavement is hotter than the lighter pavement (much more of the ice on the right has melted), although both have been in the sunlight for the same time. The thermal conductivities of the pavements are the same.

Gray objects have a uniform ability to absorb all parts of the electromagnetic spectrum. Colored objects behave in similar but more complex ways, which gives them a particular color in the visible range and may make them special in other ranges of the nonvisible spectrum. Take, for example, the strong absorption of infrared radiation by the skin, which allows us to be very sensitive to it.



A black object is a good absorber and a good radiator, while a white (or silver) object is a poor absorber and a poor radiator. It is as if radiation from the inside is reflected back into the silver object, whereas radiation from the inside of the black object is “absorbed” when it hits the surface and finds itself on the outside and is strongly emitted.

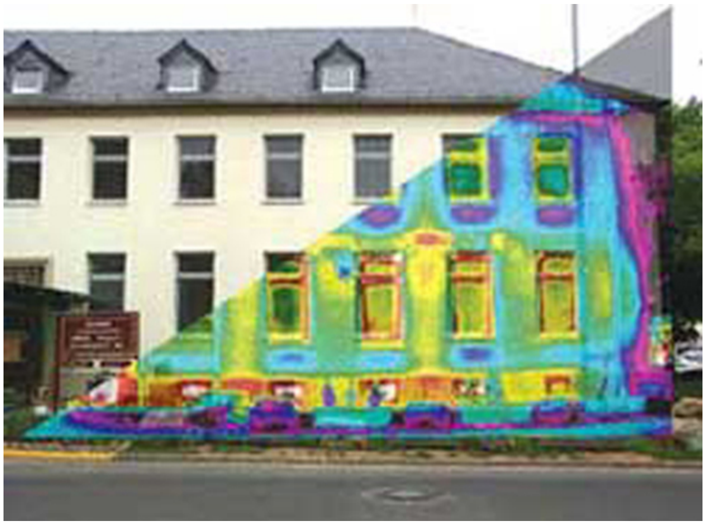
The rate of heat transfer by emitted radiation is determined by the **Stefan-Boltzmann law of radiation**:

Equation:

$$\frac{Q}{t}=\sigma eAT^4,$$

where $\sigma = 5.67 \times 10^{-8}\ \mathrm{J/s \cdot m^2 \cdot K^4}$ is the Stefan-Boltzmann constant, A is the surface area of the object, and T is its absolute temperature in kelvin. The symbol e stands for the **emissivity** of the object, which is a measure of how well it radiates. An ideal jet-black (or black body) radiator has $e = 1$, whereas a perfect reflector has $e = 0$. Real objects fall between these two values. Take, for example, tungsten light bulb filaments which have an e of about 0.5, and carbon black (a material used in printer toner), which has the (greatest known) emissivity of about 0.99.

The radiation rate is directly proportional to the *fourth power* of the absolute temperature—a remarkably strong temperature dependence. Furthermore, the radiated heat is proportional to the surface area of the object. If you knock apart the coals of a fire, there is a noticeable increase in radiation due to an increase in radiating surface area.



A thermograph of part of a building shows temperature variations, indicating where heat transfer to the outside is most severe. Windows are a major region of heat transfer to the outside of homes. (credit: U.S. Army)

Skin is a remarkably good absorber and emitter of infrared radiation, having an emissivity of 0.97 in the infrared spectrum. Thus, we are all nearly (jet) black in the infrared, in spite of the obvious variations in skin color. This high infrared emissivity is why we can so easily feel radiation on our skin. It is also the basis for the use of night scopes used by law enforcement and the military to detect human beings. Even small temperature variations can be detected because of the T^4 dependence. Images, called *thermographs*, can be used medically to detect regions of abnormally high temperature in the body, perhaps indicative of disease. Similar techniques can be used to detect heat leaks in homes [link], optimize performance of blast furnaces, improve comfort levels in work environments, and even remotely map the Earth’s temperature profile.

All objects emit and absorb radiation. The *net* rate of heat transfer by radiation (absorption minus emission) is related to both the temperature of the object and the temperature of its surroundings. Assuming that an object with a temperature T_1 is surrounded by an environment with uniform temperature T_2, the **net rate of heat transfer by radiation** is

Equation:

$$\frac{Q_{\text{net}}}{t} = \sigma e A \left(T_2^4 - T_1^4\right),$$

where e is the emissivity of the object alone. In other words, it does not matter whether the surroundings are white, gray, or black; the balance of radiation into and out of the object depends on how well it emits and absorbs radiation. When $T_2 > T_1$, the quantity Q_{net}/t is positive; that is, the net heat transfer is from hot to cold.

Note:

Take-Home Experiment: Temperature in the Sun

Place a thermometer out in the sunshine and shield it from direct sunlight using an aluminum foil. What is the reading? Now remove the shield, and note what the thermometer reads. Take a handkerchief soaked in nail polish remover, wrap it around the thermometer and place it in the sunshine. What does the thermometer read?

Example:

Calculate the Net Heat Transfer of a Person: Heat Transfer by Radiation

What is the rate of heat transfer by radiation, with an unclothed person standing in a dark room whose ambient temperature is 22.0°C. The person has a normal skin temperature of 33.0°C and a surface area of $1.50\ \text{m}^2$. The emissivity of skin is 0.97 in the infrared, where the radiation takes place.

Strategy

We can solve this by using the equation for the rate of radiative heat transfer.

Solution

Insert the temperatures values $T_2 = 295\ \text{K}$ and $T_1 = 306\ \text{K}$, so that

Equation:

$$\frac{Q}{t} = \sigma e A\left(T_2^4 - T_1^4\right)$$

Equation:

$$= \left(5.67 \times 10^{-8}\ \text{J/s} \cdot\ \text{m}^2 \cdot\ \text{K}^{\,4}\right)(0.97)\left(1.50\ \text{m}^2\right)\left[(295\ \text{K})^4 - (306\ \text{K})^4\right]$$

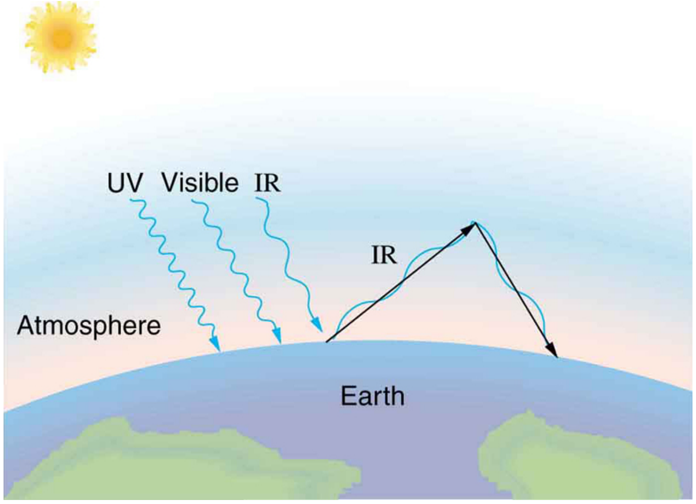
Equation:

$$= -99\ \text{J/s} = -99\ \text{W}.$$

Discussion

This value is a significant rate of heat transfer to the environment (note the minus sign), considering that a person at rest may produce energy at the rate of 125 W and that conduction and convection will also be transferring energy to the environment. Indeed, we would probably expect this person to feel cold. Clothing significantly reduces heat transfer to the environment by many methods, because clothing slows down both conduction and convection, and has a lower emissivity (especially if it is white) than skin.

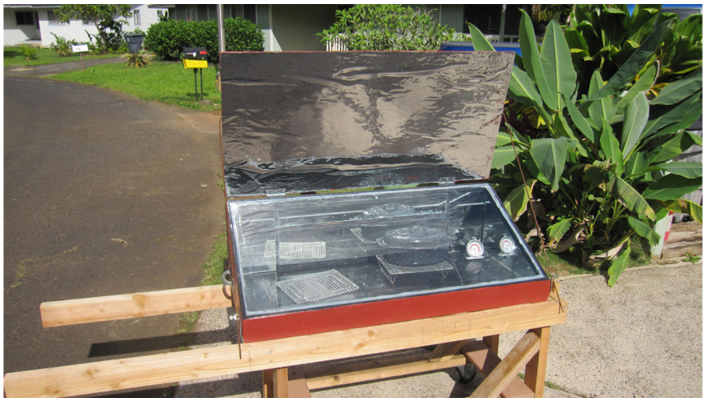
The Earth receives almost all its energy from radiation of the Sun and reflects some of it back into outer space. Because the Sun is hotter than the Earth, the net energy flux is from the Sun to the Earth. However, the rate of energy transfer is less than the equation for the radiative heat transfer would predict because the Sun does not fill the sky. The average emissivity (e) of the Earth is about 0.65, but the calculation of this value is complicated by the fact that the highly reflective cloud coverage varies greatly from day to day. There is a negative feedback (one in which a change produces an effect that opposes that change) between clouds and heat transfer; greater temperatures evaporate more water to form more clouds, which reflect more radiation back into space, reducing the temperature. The often mentioned **greenhouse effect** is directly related to the variation of the Earth's emissivity with radiation type (see the figure given below). The greenhouse effect is a natural phenomenon responsible for providing temperatures suitable for life on Earth. The Earth’s relatively constant temperature is a result of the energy balance between the incoming solar radiation and the energy radiated from the Earth. Most of the infrared radiation emitted from the Earth is absorbed by carbon dioxide (CO_2) and water (H_2O) in the atmosphere and then re-radiated back to the Earth or into outer space. Re-radiation back to the Earth maintains its surface temperature about $40^{\circ}C$ higher than it would be if there was no atmosphere, similar to the way glass increases temperatures in a greenhouse.



The greenhouse effect is a name given to the trapping of energy in the Earth’s atmosphere by a process similar to that used in greenhouses. The atmosphere, like window glass, is transparent to incoming visible radiation and most of the Sun’s infrared. These wavelengths are absorbed by the Earth and re-emitted as infrared. Since Earth’s temperature is much lower than that of the Sun, the infrared radiated by the Earth has a much longer wavelength. The atmosphere, like glass, traps these longer infrared rays, keeping the Earth warmer than it would otherwise be. The amount of trapping depends on concentrations of trace gases like carbon dioxide, and a change in the concentration of these gases is believed to affect the Earth's surface temperature.

The greenhouse effect is also central to the discussion of global warming due to emission of carbon dioxide and methane (and other so-called greenhouse gases) into the Earth's atmosphere from industrial production and farming. Changes in global climate could lead to more intense storms, precipitation changes (affecting agriculture), reduction in rain forest biodiversity, and rising sea levels.

Heating and cooling are often significant contributors to energy use in individual homes. Current research efforts into developing environmentally friendly homes quite often focus on reducing conventional heating and cooling through better building materials, strategically positioning windows to optimize radiation gain from the Sun, and opening spaces to allow convection. It is possible to build a zero-energy house that allows for comfortable living in most parts of the United States with hot and humid summers and cold winters.



This simple but effective solar cooker uses the greenhouse effect and reflective material to trap and retain solar energy. Made of inexpensive, durable materials, it saves money and labor, and is of particular economic value in energy-poor developing countries. (credit: E.B. Kauai)

Conversely, dark space is very cold, about $3\text{K}\ (-\ 454^{\text{o}}\text{F})$, so that the Earth radiates energy into the dark sky. Owing to the fact that clouds have lower emissivity than either oceans or land masses, they reflect some of the radiation back to the surface, greatly reducing heat transfer into dark space, just as they greatly reduce heat transfer into the atmosphere during the day. The rate of heat transfer from soil and grasses can be so rapid that frost may occur on clear summer evenings, even in warm latitudes.

Exercise:

Check Your Understanding

Problem:

What is the change in the rate of the radiated heat by a body at the temperature $T_1 = 20^{\text{o}}\text{C}$ compared to when the body is at the temperature $T_2 = 40^{\text{o}}\text{C}$?

Solution:

The radiated heat is proportional to the fourth power of the *absolute temperature*. Because $T_1 = 293\text{ K}$ and $T_2 = 313\text{ K}$, the rate of heat transfer increases by about 30 percent of the original rate.

Note:

Career Connection: Energy Conservation Consultation

The cost of energy is generally believed to remain very high for the foreseeable future. Thus, passive control of heat loss in both commercial and domestic housing will become increasingly important. Energy consultants measure and analyze the flow of energy into and out of houses and ensure that a healthy exchange of air is maintained inside the house. The job prospects for an energy consultant are strong.

Note:

Problem-Solving Strategies for the Methods of Heat Transfer

1. *Examine the situation to determine what type of heat transfer is involved.*
2. *Identify the type(s) of heat transfer—conduction, convection, or radiation.*
3. *Identify exactly what needs to be determined in the problem (identify the unknowns). A written list is very useful.*
4. *Make a list of what is given or can be inferred from the problem as stated (identify the knowns).*
5. *Solve the appropriate equation for the quantity to be determined (the unknown).*
6. For conduction, equation $\frac{Q}{t}=\frac{\text{kA}(T_2-T_1)}{d}$ is appropriate. [link] lists thermal conductivities. For convection, determine the amount of matter moved and use equation $Q=\text{mc}\Delta T$, to calculate the heat transfer involved in the temperature change of the fluid. If a phase change accompanies convection, equation $Q=\text{mL}_\text{f}$ or $Q=\text{mL}_\text{v}$ is appropriate to find the heat transfer involved in the phase change. [link] lists information relevant to phase change. For radiation, equation $\frac{Q_\text{net}}{t}=\sigma eA\left(T_2^4-T_1^4\right)$ gives the net heat transfer rate.
7. *Insert the knowns along with their units into the appropriate equation and obtain numerical solutions complete with units.*
8. *Check the answer to see if it is reasonable. Does it make sense?*

Summary

- Radiation is the rate of heat transfer through the emission or absorption of electromagnetic waves.

- The rate of heat transfer depends on the surface area and the fourth power of the absolute temperature:
  Equation:

  $$\frac{Q}{t} = \sigma e A T^4,$$

  where $\sigma = 5.67 \times 10^{-8}\ \mathrm{J/s \cdot m^2 \cdot K^4}$ is the Stefan-Boltzmann constant and e is the emissivity of the body. For a black body, $e = 1$ whereas a shiny white or perfect reflector has $e = 0$, with real objects having values of e between 1 and 0. The net rate of heat transfer by radiation is
  Equation:

  $$\frac{Q_{\text{net}}}{t} = \sigma e A \left(T_2^4 - T_1^4\right)$$

  where T_1 is the temperature of an object surrounded by an environment with uniform temperature T_2 and e is the emissivity of the *object*.

Conceptual Questions

Exercise:

Problem:

When watching a daytime circus in a large, dark-colored tent, you sense significant heat transfer from the tent. Explain why this occurs.

Exercise:

Problem:

Satellites designed to observe the radiation from cold (3 K) dark space have sensors that are shaded from the Sun, Earth, and Moon and that are cooled to very low temperatures. Why must the sensors be at low temperature?

Exercise:

Problem: Why are cloudy nights generally warmer than clear ones?

Exercise:

Problem:

Why are thermometers that are used in weather stations shielded from the sunshine? What does a thermometer measure if it is shielded from the sunshine and also if it is not?

Exercise:

Problem:

On average, would Earth be warmer or cooler without the atmosphere? Explain your answer.

Problems & Exercises

Exercise:

Problem:

At what net rate does heat radiate from a 275-m^2 black roof on a night when the roof's temperature is 30.0°C and the surrounding temperature is 15.0°C? The emissivity of the roof is 0.900.

Solution:

$-21.7\ \text{kW}$
Note that the negative answer implies heat loss to the surroundings.

Exercise:

Problem:

(a) Cherry-red embers in a fireplace are at 850°C and have an exposed area of $0.200\ \text{m}^2$ and an emissivity of 0.980. The surrounding room has a temperature of 18.0°C. If 50% of the radiant energy enters the room, what is the net rate of radiant heat transfer in kilowatts? (b) Does your answer support the contention that most of the heat transfer into a room by a fireplace comes from infrared radiation?

Exercise:

Problem:

Radiation makes it impossible to stand close to a hot lava flow. Calculate the rate of heat transfer by radiation from $1.00\ \text{m}^2$ of 1200°C fresh lava into 30.0°C surroundings, assuming lava's emissivity is 1.00.

Solution:

$-266\ \text{kW}$

Exercise:

Problem:

(a) Calculate the rate of heat transfer by radiation from a car radiator at 110°C into a 50.0°C environment, if the radiator has an emissivity of 0.750 and a 1.20-m^2 surface area. (b) Is this a significant fraction of the heat transfer by an automobile engine? To answer this, assume a horsepower of $200\ \text{hp}\ (1.5\ \text{kW})$ and the efficiency of automobile engines as 25%.

Exercise:

Problem:

Find the net rate of heat transfer by radiation from a skier standing in the shade, given the following. She is completely clothed in white (head to foot, including a ski mask), the clothes have an emissivity of 0.200 and a surface temperature of 10.0°C, the surroundings are at -15.0°C, and her surface area is $1.60\ \text{m}^2$.

Solution:

$-36.0\ \text{W}$

Exercise:

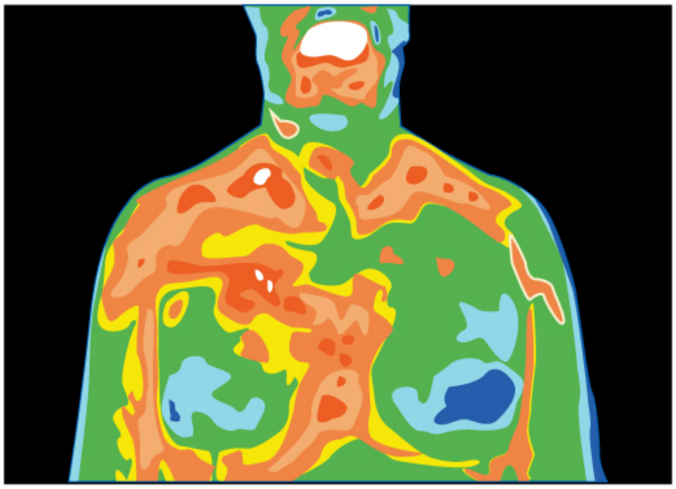
Problem:

Suppose you walk into a sauna that has an ambient temperature of 50.0°C. (a) Calculate the rate of heat transfer to you by radiation given your skin temperature is 37.0°C, the emissivity of skin is 0.98, and the surface area of your body is $1.50\ \text{m}^2$. (b) If all other forms of heat transfer are balanced (the net heat transfer is zero), at what rate will your body temperature increase if your mass is 75.0 kg?

Exercise:

Problem:

Thermography is a technique for measuring radiant heat and detecting variations in surface temperatures that may be medically, environmentally, or militarily meaningful.(a) What is the percent increase in the rate of heat transfer by radiation from a given area at a temperature of 34.0°C compared with that at 33.0°C, such as on a person’s skin? (b) What is the percent increase in the rate of heat transfer by radiation from a given area at a temperature of 34.0°C compared with that at 20.0°C, such as for warm and cool automobile hoods?



Artist's rendition of a thermograph of a patient's upper body, showing the distribution of heat represented by different colors.

Solution:

(a) 1.31%

(b) 20.5%

Exercise:

Problem:

The Sun radiates like a perfect black body with an emissivity of exactly 1. (a) Calculate the surface temperature of the Sun, given that it is a sphere with a $7.00 \times 10^8\text{-m}$ radius that radiates 3.80×10^{26} W into 3-K space. (b) How much power does the Sun radiate per square meter of its surface? (c) How much power in watts per square meter is that value at the distance of Earth, 1.50×10^{11} m away? (This number is called the solar constant.)

Exercise:

Problem:

A large body of lava from a volcano has stopped flowing and is slowly cooling. The interior of the lava is at 1200^{o}C, its surface is at 450^{o}C, and the surroundings are at 27.0^{o}C. (a) Calculate the rate at which energy is transferred by radiation from $1.00\ \text{m}^2$ of surface lava into the surroundings, assuming the emissivity is 1.00. (b) Suppose heat conduction to the surface occurs at the same rate. What is the thickness of the lava between the 450^{o}C surface and the 1200^{o}C interior, assuming that the lava's conductivity is the same as that of brick?

Solution:

(a) -15.0 kW

(b) 4.2 cm

Exercise:

Problem:

Calculate the temperature the entire sky would have to be in order to transfer energy by radiation at 1000 W/m^2 —about the rate at which the Sun radiates when it is directly overhead on a clear day. This value is the effective temperature of the sky, a kind of average that takes account of the fact that the Sun occupies only a small part of the sky but is much hotter than the rest. Assume that the body receiving the energy has a temperature of 27.0^{o}C.

Exercise:

Problem:

(a) A shirtless rider under a circus tent feels the heat radiating from the sunlit portion of the tent. Calculate the temperature of the tent canvas based on the following information: The shirtless rider's skin temperature is $34.0^{\mathrm{o}}\mathrm{C}$ and has an emissivity of 0.970. The exposed area of skin is $0.400\ \mathrm{m}^2$. He receives radiation at the rate of 20.0 W—half what you would calculate if the entire region behind him was hot. The rest of the surroundings are at $34.0^{\mathrm{o}}\mathrm{C}$. (b) Discuss how this situation would change if the sunlit side of the tent was nearly pure white and if the rider was covered by a white tunic.

Solution:

(a) $48.5^{\mathrm{o}}\mathrm{C}$

(b) A pure white object reflects more of the radiant energy that hits it, so a white tent would prevent more of the sunlight from heating up the inside of the tent, and the white tunic would prevent that heat which entered the tent from heating the rider. Therefore, with a white tent, the temperature would be lower than $48.5^{\mathrm{o}}\mathrm{C}$, and the rate of radiant heat transferred to the rider would be less than 20.0 W.

Exercise:

Problem: Integrated Concepts

One $30.0^{\mathrm{o}}\mathrm{C}$ day the relative humidity is 75.0%, and that evening the temperature drops to $20.0^{\mathrm{o}}\mathrm{C}$, well below the dew point. (a) How many grams of water condense from each cubic meter of air? (b) How much heat transfer occurs by this condensation? (c) What temperature increase could this cause in dry air?

Exercise:

Problem: Integrated Concepts

Large meteors sometimes strike the Earth, converting most of their kinetic energy into thermal energy. (a) What is the kinetic energy of a 10^9 kg meteor moving at 25.0 km/s? (b) If this meteor lands in a deep ocean and 80% of its kinetic energy goes into heating water, how many kilograms of water could it raise by 5.0°C? (c) Discuss how the energy of the meteor is more likely to be deposited in the ocean and the likely effects of that energy.

Solution:

(a) $3\times 10^{17}\ \text{J}$

(b) $1\times 10^{13}\ \text{kg}$

(c) When a large meteor hits the ocean, it causes great tidal waves, dissipating large amount of its energy in the form of kinetic energy of the water.

Exercise:

Problem: Integrated Concepts

Frozen waste from airplane toilets has sometimes been accidentally ejected at high altitude. Ordinarily it breaks up and disperses over a large area, but sometimes it holds together and strikes the ground. Calculate the mass of 0°C ice that can be melted by the conversion of kinetic and gravitational potential energy when a $20.0\ \text{kg}$ piece of frozen waste is released at 12.0 km altitude while moving at 250 m/s and strikes the ground at 100 m/s (since less than 20.0 kg melts, a significant mess results).

Exercise:

Problem: Integrated Concepts

(a) A large electrical power facility produces 1600 MW of “waste heat,” which is dissipated to the environment in cooling towers by warming air flowing through the towers by 5.00°C. What is the necessary flow rate of air in m^3/s? (b) Is your result consistent with the large cooling towers used by many large electrical power plants?

Solution:

(a) $3.44 \times 10^5\ \text{m}^3/\text{s}$

(b) This is equivalent to 12 million cubic feet of air per second. That is tremendous. This is too large to be dissipated by heating the air by only 5^oC. Many of these cooling towers use the circulation of cooler air over warmer water to increase the rate of evaporation. This would allow much smaller amounts of air necessary to remove such a large amount of heat because evaporation removes larger quantities of heat than was considered in part (a).

Exercise:

Problem: Integrated Concepts

(a) Suppose you start a workout on a Stairmaster, producing power at the same rate as climbing 116 stairs per minute. Assuming your mass is 76.0 kg and your efficiency is 20.0%, how long will it take for your body temperature to rise 1.00^oC if all other forms of heat transfer in and out of your body are balanced? (b) Is this consistent with your experience in getting warm while exercising?

Exercise:

Problem: Integrated Concepts

A 76.0-kg person suffering from hypothermia comes indoors and shivers vigorously. How long does it take the heat transfer to increase the person’s body temperature by 2.00^oC if all other forms of heat transfer are balanced?

Solution:

20.9 min

Exercise:

Problem: **Integrated Concepts**

In certain large geographic regions, the underlying rock is hot. Wells can be drilled and water circulated through the rock for heat transfer for the generation of electricity. (a) Calculate the heat transfer that can be extracted by cooling $1.00\ \text{km}^3$ of granite by 100°C. (b) How long will this take if heat is transferred at a rate of 300 MW, assuming no heat transfers back into the 1.00 km of rock by its surroundings?

Exercise:

Problem: **Integrated Concepts**

Heat transfers from your lungs and breathing passages by evaporating water. (a) Calculate the maximum number of grams of water that can be evaporated when you inhale 1.50 L of 37°C air with an original relative humidity of 40.0%. (Assume that body temperature is also 37°C.) (b) How many joules of energy are required to evaporate this amount? (c) What is the rate of heat transfer in watts from this method, if you breathe at a normal resting rate of 10.0 breaths per minute?

Solution:

(a) 3.96×10^{-2} g

(b) 96.2 J

(c) 16.0 W

Exercise:

Problem: **Integrated Concepts**

(a) What is the temperature increase of water falling 55.0 m over Niagara Falls? (b) What fraction must evaporate to keep the temperature constant?

Exercise:

Problem: **Integrated Concepts**

Hot air rises because it has expanded. It then displaces a greater volume of cold air, which increases the buoyant force on it. (a) Calculate the ratio of the buoyant force to the weight of 50.0°C air surrounded by 20.0°C air. (b) What energy is needed to cause 1.00 m^3 of air to go from 20.0°C to 50.0°C? (c) What gravitational potential energy is gained by this volume of air if it rises 1.00 m? Will this cause a significant cooling of the air?

Solution:

(a) 1.102

(b) $2.79 \times 10^4\text{ J}$

(c) 12.6 J. This will not cause a significant cooling of the air because it is much less than the energy found in part (b), which is the energy required to warm the air from 20.0°C to 50.0°C.

Exercise:

Problem: **Unreasonable Results**

(a) What is the temperature increase of an 80.0 kg person who consumes 2500 kcal of food in one day with 95.0% of the energy transferred as heat to the body? (b) What is unreasonable about this result? (c) Which premise or assumption is responsible?

Solution:

(a) 36°C

(b) Any temperature increase greater than about 3°C would be unreasonably large. In this case the final temperature of the person would rise to $73^{\circ}\text{C}\ (163^{\circ}\text{F})$.

(c) The assumption of 95% heat retention is unreasonable.

Exercise:

Problem: Unreasonable Results

A slightly deranged Arctic inventor surrounded by ice thinks it would be much less mechanically complex to cool a car engine by melting ice on it than by having a water-cooled system with a radiator, water pump, antifreeze, and so on. (a) If 80.0% of the energy in 1.00 gal of gasoline is converted into “waste heat” in a car engine, how many kilograms of 0°C ice could it melt? (b) Is this a reasonable amount of ice to carry around to cool the engine for 1.00 gal of gasoline consumption? (c) What premises or assumptions are unreasonable?

Exercise:

Problem: Unreasonable Results

(a) Calculate the rate of heat transfer by conduction through a window with an area of $1.00\ \text{m}^2$ that is 0.750 cm thick, if its inner surface is at 22.0°C and its outer surface is at 35.0°C. (b) What is unreasonable about this result? (c) Which premise or assumption is responsible?

Solution:

(a) 1.46 kW

(b) Very high power loss through a window. An electric heater of this power can keep an entire room warm.

(c) The surface temperatures of the window do not differ by as great an amount as assumed. The inner surface will be warmer, and the outer surface will be cooler.

Exercise:

Problem: Unreasonable Results

A meteorite 1.20 cm in diameter is so hot immediately after penetrating the atmosphere that it radiates 20.0 kW of power. (a) What is its temperature, if the surroundings are at 20.0ºC and it has an emissivity of 0.800? (b) What is unreasonable about this result? (c) Which premise or assumption is responsible?

Exercise:

Problem: Construct Your Own Problem

Consider a new model of commercial airplane having its brakes tested as a part of the initial flight permission procedure. The airplane is brought to takeoff speed and then stopped with the brakes alone. Construct a problem in which you calculate the temperature increase of the brakes during this process. You may assume most of the kinetic energy of the airplane is converted to thermal energy in the brakes and surrounding materials, and that little escapes. Note that the brakes are expected to become so hot in this procedure that they ignite and, in order to pass the test, the airplane must be able to withstand the fire for some time without a general conflagration.

Exercise:

Problem: Construct Your Own Problem

Consider a person outdoors on a cold night. Construct a problem in which you calculate the rate of heat transfer from the person by all three heat transfer methods. Make the initial circumstances such that at rest the person will have a net heat transfer and then decide how much physical activity of a chosen type is necessary to balance the rate of heat transfer. Among the things to consider are the size of the person, type of clothing, initial metabolic rate, sky conditions, amount of water evaporated, and volume of air breathed. Of course, there are many other factors to consider and your instructor may wish to guide you in the assumptions made as well as the detail of analysis and method of presenting your results.

Glossary

emissivity

measure of how well an object radiates

greenhouse effect

warming of the Earth that is due to gases such as carbon dioxide and methane that absorb infrared radiation from the Earth’s surface and reradiate it in all directions, thus sending a fraction of it back toward the surface of the Earth

net rate of heat transfer by radiation

is $\frac{Q_{\text{net}}}{t} = \sigma e A \left(T_2^4 - T_1^4\right)$

radiation

energy transferred by electromagnetic waves directly as a result of a temperature difference

Stefan-Boltzmann law of radiation

$\frac{Q}{t} = \sigma e A T^4$, where σ is the Stefan-Boltzmann constant, A is the surface area of the object, T is the absolute temperature, and e is the emissivity

Introduction to Fluid Statics class="introduction"

The fluid essential to all life has a beauty of its own. It also helps support the weight of this swimmer. (credit: Terren, Wikimedia Commons)

Much of what we value in life is fluid: a breath of fresh winter air; the hot blue flame in our gas cooker; the water we drink, swim in, and bathe in; the

blood in our veins. What exactly is a fluid? Can we understand fluids with the laws already presented, or will new laws emerge from their study? The physical characteristics of static or stationary fluids and some of the laws that govern their behavior are the topics of this chapter. Fluid Dynamics and Its Biological and Medical Applications explores aspects of fluid flow.
https://www.youtube.com/embed/iShUultAD9M

Pressure

- Define pressure.
- Explain the relationship between pressure and force.
- Calculate force given pressure and area.

You have no doubt heard the word **pressure** being used in relation to blood (high or low blood pressure) and in relation to the weather (high- and low-pressure weather systems). These are only two of many examples of pressures in fluids. Pressure P is defined as

Equation:

$$P=\frac{F}{A}$$

where F is a force applied to an area A that is perpendicular to the force.

Note:

Pressure

Pressure is defined as the force divided by the area perpendicular to the force over which the force is applied, or

Equation:

$$P=\frac{F}{A}.$$

A given force can have a significantly different effect depending on the area over which the force is exerted, as shown in [link]. The SI unit for pressure is the *pascal*, where

Equation:

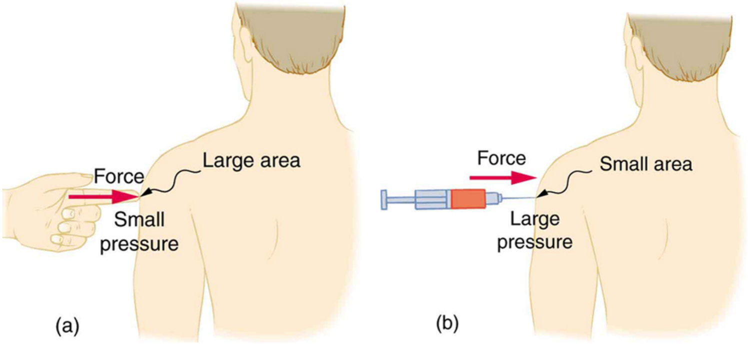
$$1\ \text{Pa}=1\ \text{N/m}^2.$$

In addition to the pascal, there are many other units for pressure that are in common use. In meteorology, atmospheric pressure is often described in units of millibar (mb), where

Equation:

$$100\text{ mb}=1\times10^{4}\text{ Pa}\enspace.$$

Pounds per square inch $\left(\text{lb/in}^{2}\text{ or psi}\right)$ is still sometimes used as a measure of tire pressure, and millimeters of mercury (mm Hg) is still often used in the measurement of blood pressure. Pressure is defined for all states of matter but is particularly important when discussing fluids.



(a) While the person being poked with the finger might be irritated, the force has little lasting effect. (b) In contrast, the same force applied to an area the size of the sharp end of a needle is great enough to break the skin.

Example:

Calculating Force Exerted by the Air: What Force Does a Pressure Exert?

An astronaut is working outside the International Space Station where the atmospheric pressure is essentially zero. The pressure gauge on her air tank reads 6.90×10^6 Pa. What force does the air inside the tank exert on the flat end of the cylindrical tank, a disk 0.150 m in diameter?

Strategy

We can find the force exerted from the definition of pressure given in $P = \frac{F}{A}$, provided we can find the area A acted upon.

Solution

By rearranging the definition of pressure to solve for force, we see that

Equation:

$$F = \text{PA}.$$

Here, the pressure P is given, as is the area of the end of the cylinder A, given by $A = \pi r^2$. Thus,

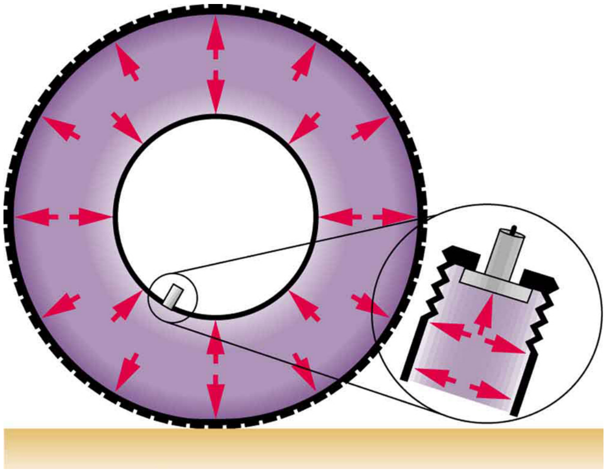
Equation:

$$\begin{aligned} F &= \left(6.90 \times 10^6\ \text{N/m}^2\right)(3.14)(0.0750\ \text{m})^2 \\ &= 1.22 \times 10^5\ \text{N}. \end{aligned}$$

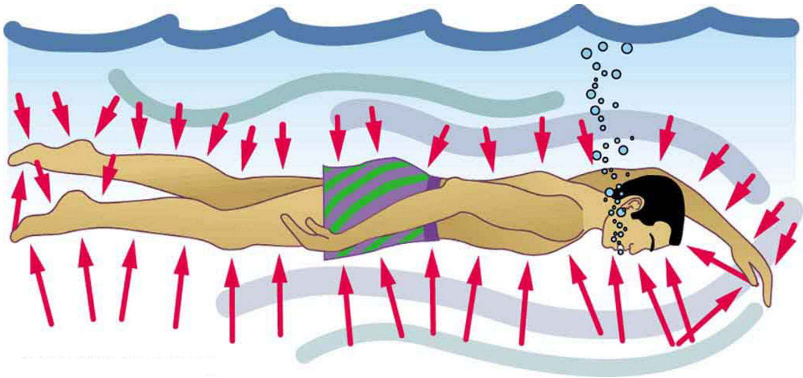
Discussion

Wow! No wonder the tank must be strong. Since we found $F = \text{PA}$, we see that the force exerted by a pressure is directly proportional to the area acted upon as well as the pressure itself.

The force exerted on the end of the tank is perpendicular to its inside surface. This direction is because the force is exerted by a static or stationary fluid. We have already seen that fluids cannot *withstand* shearing (sideways) forces; they cannot *exert* shearing forces, either. Fluid pressure has no direction, being a scalar quantity. The forces due to pressure have well-defined directions: they are always exerted perpendicular to any surface. (See the tire in [link], for example.) Finally, note that pressure is exerted on all surfaces. Swimmers, as well as the tire, feel pressure on all sides. (See [link].)



Pressure inside this tire exerts forces perpendicular to all surfaces it contacts. The arrows give representative directions and magnitudes of the forces exerted at various points. Note that static fluids do not exert shearing forces.



Pressure is exerted on all sides of this swimmer, since the water would flow into the space he occupies if he were not there. The arrows represent the directions and magnitudes of the forces exerted at various points on the swimmer. Note that the forces are larger underneath, due to greater depth, giving a net upward or buoyant force that is balanced by the weight of the swimmer.

Note:
PhET Explorations: Gas Properties
Pump gas molecules to a box and see what happens as you change the volume, add or remove heat, change gravity, and more. Measure the temperature and pressure, and discover how the properties of the gas vary in relation to each other. Click to open media in new browser.

Section Summary

- Pressure is the force per unit perpendicular area over which the force is applied. In equation form, pressure is defined as
  Equation:
  $$P = \frac{F}{A}.$$
- The SI unit of pressure is pascal and $1\ \text{Pa} = 1\ \text{N/m}^2$.

Conceptual Questions

Exercise:

Problem:

How is pressure related to the sharpness of a knife and its ability to cut?

Exercise:

Problem:

Why does a dull hypodermic needle hurt more than a sharp one?

Exercise:

Problem:

The outward force on one end of an air tank was calculated in [link]. How is this force balanced? (The tank does not accelerate, so the force must be balanced.)

Exercise:

Problem:

Why is force exerted by static fluids always perpendicular to a surface?

Exercise:

Problem:

In a remote location near the North Pole, an iceberg floats in a lake. Next to the lake (assume it is not frozen) sits a comparably sized glacier sitting on land. If both chunks of ice should melt due to rising global temperatures (and the melted ice all goes into the lake), which ice chunk would give the greatest increase in the level of the lake water, if any?

Exercise:

Problem:

How do jogging on soft ground and wearing padded shoes reduce the pressures to which the feet and legs are subjected?

Exercise:

Problem:

Toe dancing (as in ballet) is much harder on toes than normal dancing or walking. Explain in terms of pressure.

Exercise:

Problem:

How do you convert pressure units like millimeters of mercury, centimeters of water, and inches of mercury into units like newtons per meter squared without resorting to a table of pressure conversion factors?

Problems & Exercises

Exercise:

Problem:

As a woman walks, her entire weight is momentarily placed on one heel of her high-heeled shoes. Calculate the pressure exerted on the floor by the heel if it has an area of $1.50\ \text{cm}^2$ and the woman's mass is 55.0 kg. Express the pressure in Pa. (In the early days of commercial flight, women were not allowed to wear high-heeled shoes because aircraft floors were too thin to withstand such large pressures.)

Solution:

$3.59 \times 10^6\ \text{Pa}$; or $521\ \text{lb/in}^2$

Exercise:

Problem:

The pressure exerted by a phonograph needle on a record is surprisingly large. If the equivalent of 1.00 g is supported by a needle, the tip of which is a circle 0.200 mm in radius, what pressure is exerted on the record in N/m^2?

Exercise:

Problem:

Nail tips exert tremendous pressures when they are hit by hammers because they exert a large force over a small area. What force must be exerted on a nail with a circular tip of 1.00 mm diameter to create a pressure of $3.00 \times 10^9 \text{ N/m}^2$?(This high pressure is possible because the hammer striking the nail is brought to rest in such a short distance.)

Solution:

$2.36 \times 10^3 \text{ N}$

Glossary

pressure
: the force per unit area perpendicular to the force, over which the force acts

Pascal’s Principle

LEARNING OBJECTIVES

By the end of this section, you will be able to:

- Define pressure.
- State Pascal's principle.
- Understand applications of Pascal's principle.
- Derive relationships between forces in a hydraulic system.

Pressure is defined as force per unit area. Can pressure be increased in a fluid by pushing directly on the fluid? Yes, but it is much easier if the fluid is enclosed. The heart, for example, increases blood pressure by pushing directly on the blood in an enclosed system (valves closed in a chamber). If you try to push on a fluid in an open system, such as a river, the fluid flows away. An enclosed fluid cannot flow away, and so pressure is more easily increased by an applied force.

What happens to a pressure in an enclosed fluid? Since atoms in a fluid are free to move about, they transmit the pressure to all parts of the fluid and to the walls of the container. Remarkably, the pressure is transmitted *undiminished*. This phenomenon is called **Pascal's principle**, because it was first clearly stated by the French philosopher and scientist Blaise Pascal (1623–1662): A change in pressure applied to an enclosed fluid is transmitted undiminished to all portions of the fluid and to the walls of its container.

Note:

Pascal's Principle

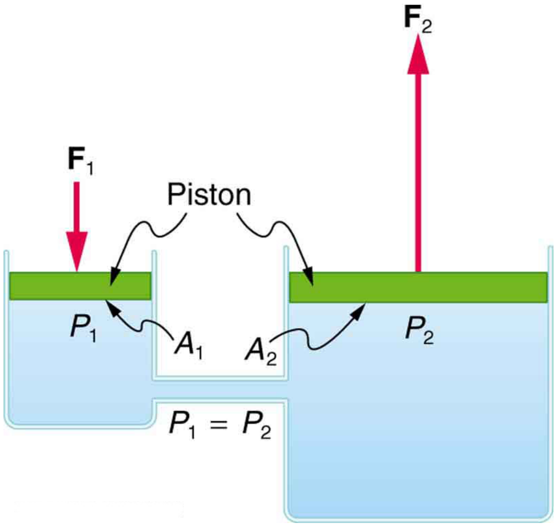
A change in pressure applied to an enclosed fluid is transmitted undiminished to all portions of the fluid and to the walls of its container.

Pascal's principle, an experimentally verified fact, is what makes pressure so important in fluids. Since a change in pressure is transmitted undiminished in an enclosed fluid, we often know more about pressure than other physical quantities in fluids. Moreover, Pascal's principle implies that *the total pressure in a fluid is the sum of the pressures from different sources*. We shall find this fact—that pressures add—very useful.

Blaise Pascal had an interesting life in that he was home-schooled by his father who removed all of the mathematics textbooks from his house and forbade him to study mathematics until the age of 15. This, of course, raised the boy's curiosity, and by the age of 12, he started to teach himself geometry. Despite this early deprivation, Pascal went on to make major contributions in the mathematical fields of probability theory, number theory, and geometry. He is also well known for being the inventor of the first mechanical digital calculator, in addition to his contributions in the field of fluid statics.

Application of Pascal's Principle

One of the most important technological applications of Pascal's principle is found in a *hydraulic system*, which is an enclosed fluid system used to exert forces. The most common hydraulic systems are those that operate car brakes. Let us first consider the simple hydraulic system shown in [link].



A typical hydraulic system with two fluid-filled cylinders, capped with pistons and connected by a tube called a hydraulic line. A downward force $\mathbf{F}_1$ on the left piston creates a pressure that is transmitted undiminished to all parts of the enclosed fluid. This results in an upward force $\mathbf{F}_2$ on the right piston that is larger than $\mathbf{F}_1$ because the right piston has a larger area.

Relationship Between Forces in a Hydraulic System

We can derive a relationship between the forces in the simple hydraulic system shown in [link] by applying Pascal's principle. Note first that the two pistons in the system are at the same height, and so there will be no difference in pressure due to a difference in depth. Now the pressure due to F_1 acting on area A_1 is simply $P_1 = \frac{F_1}{A_1}$, as defined by $P = \frac{F}{A}$. According to Pascal's principle, this pressure is transmitted undiminished throughout the fluid and to all walls of the container. Thus, a pressure P_2 is felt at the other piston that is equal to P_1. That is $P_1 = P_2$.

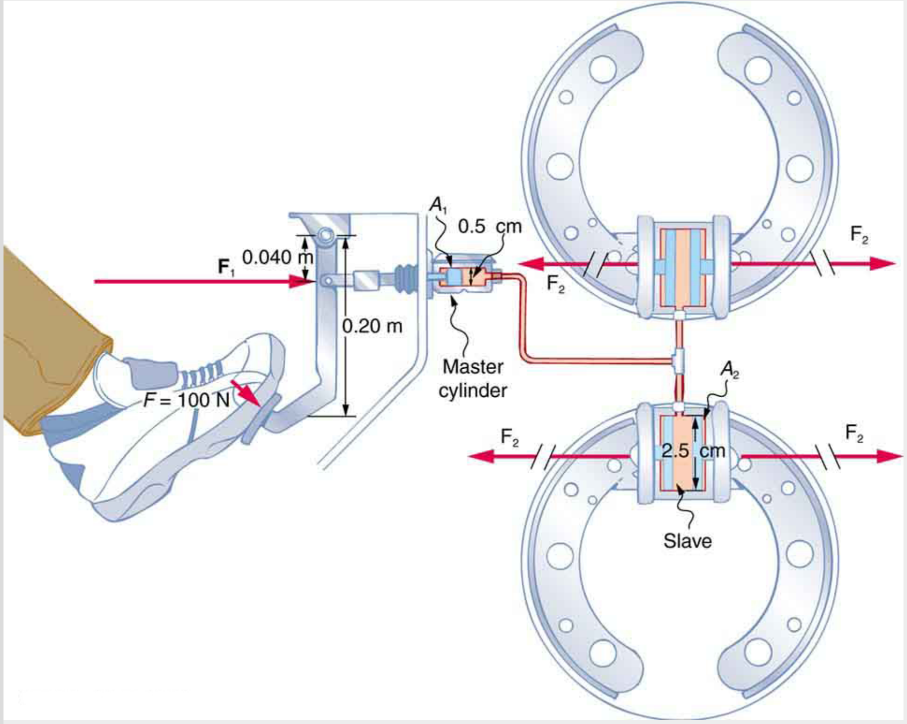
But since $P_2 = \frac{F_2}{A_2}$, we see that $\frac{F_1}{A_1} = \frac{F_2}{A_2}$.

This equation relates the ratios of force to area in any hydraulic system, providing the pistons are at the same vertical height and that friction in the system is negligible. Hydraulic systems can increase or decrease the force applied to them. To make the force larger, the pressure is applied to a larger area. For example, if a 100-N force is applied to the left cylinder in [link] and the right one has an area five times greater, then the force out is 500 N. Hydraulic systems are analogous to simple levers, but they have the advantage that pressure can be sent through tortuously curved lines to several places at once.

Example:

Calculating Force of Slave Cylinders: Pascal Puts on the Brakes

Consider the automobile hydraulic system shown in [link].



Hydraulic brakes use Pascal's principle. The driver exerts a force of 100 N on the brake pedal. This force is increased by the simple lever and again by the hydraulic system. Each of the identical slave cylinders receives the same pressure and, therefore, creates the same force output F_2. The circular cross-sectional areas of the master and slave cylinders are represented by A_1 and A_2, respectively

A force of 100 N is applied to the brake pedal, which acts on the cylinder—called the master—through a lever. A force of 500 N is exerted on the master cylinder. (The reader can verify that the force is 500 N using techniques of statics from Applications of Statics, Including Problem-Solving Strategies.) Pressure created in the master cylinder is transmitted to four so-called slave cylinders. The master cylinder has a diameter of 0.500 cm, and each slave cylinder has a diameter of 2.50 cm. Calculate the force F_2 created at each of the slave cylinders.

Strategy

We are given the force F_1 that is applied to the master cylinder. The cross-sectional areas A_1 and A_2 can be calculated from their given diameters. Then $\frac{F_1}{A_1} = \frac{F_2}{A_2}$ can be used to find the force F_2. Manipulate this algebraically to get F_2 on one side and substitute known values:

Solution

Pascal's principle applied to hydraulic systems is given by $\frac{F_1}{A_1} = \frac{F_2}{A_2}$:

Equation:

$$F_2 = \frac{A_2}{A_1}F_1 = \frac{\pi r_2^2}{\pi r_1^2}F_1 = \frac{(1.25\text{ cm})^2}{(0.250\text{ cm})^2} \times 500\text{ N} = 1.25 \times 10^4\text{ N}.$$

Discussion

This value is the force exerted by each of the four slave cylinders. Note that we can add as many slave cylinders as we wish. If each has a 2.50-cm diameter, each will exert 1.25×10^4 N.

A simple hydraulic system, such as a simple machine, can increase force but cannot do more work than done on it. Work is force times distance moved, and the slave cylinder moves through a smaller distance than the master cylinder. Furthermore, the more slaves added, the smaller the distance each moves. Many hydraulic systems—such as power brakes and those in bulldozers—have a motorized pump that actually does most of the work in the system. The movement of the legs of a spider is achieved partly by hydraulics. Using hydraulics, a jumping spider can create a force that makes it capable of jumping 25 times its length!

Note:
Making Connections: Conservation of Energy
Conservation of energy applied to a hydraulic system tells us that the system cannot do more work than is done on it. Work transfers energy, and so the work output cannot exceed the work input. Power brakes and other similar hydraulic systems use pumps to supply extra energy when needed.

Section Summary

- Pressure is force per unit area.
- A change in pressure applied to an enclosed fluid is transmitted undiminished to all portions of the fluid and to the walls of its container.
- A hydraulic system is an enclosed fluid system used to exert forces.

Conceptual Questions

Exercise:

Problem:

Suppose the master cylinder in a hydraulic system is at a greater height than the slave cylinder. Explain how this will affect the force produced at the slave cylinder.

Problems & Exercises

Exercise:

Problem:

How much pressure is transmitted in the hydraulic system considered in [link]? Express your answer in pascals and in atmospheres.

Solution:

$2.55 \times 10^7\ \text{Pa}$; or 251 atm

Exercise:

Problem:

What force must be exerted on the master cylinder of a hydraulic lift to support the weight of a 2000-kg car (a large car) resting on the slave cylinder? The master cylinder has a 2.00-cm diameter and the slave has a 24.0-cm diameter.

Exercise:

Problem:

A crass host pours the remnants of several bottles of wine into a jug after a party. He then inserts a cork with a 2.00-cm diameter into the bottle, placing it in direct contact with the wine. He is amazed when he pounds the cork into place and the bottom of the jug (with a 14.0-cm diameter) breaks away. Calculate the extra force exerted against the bottom if he pounded the cork with a 120-N force.

Solution:

5.76×10^3 N extra force

Exercise:

Problem:

A certain hydraulic system is designed to exert a force 100 times as large as the one put into it. (a) What must be the ratio of the area of the slave cylinder to the area of the master cylinder? (b) What must be the ratio of their diameters? (c) By what factor is the distance through which the output force moves reduced relative to the distance through which the input force moves? Assume no losses to friction.

Exercise:

Problem:

(a) Verify that work input equals work output for a hydraulic system assuming no losses to friction. Do this by showing that the distance the output force moves is reduced by the same factor that the output force is increased. Assume the volume of the fluid is constant. (b) What effect would friction within the fluid and between components in the system have on the output force? How would this depend on whether or not the fluid is moving?

Solution:

(a) $V = d_{\text{i}}A_{\text{i}} = d_{\text{o}}A_{\text{o}} \Rightarrow d_{\text{o}} = d_{\text{i}}\left(\frac{A_{\text{i}}}{A_{\text{o}}}\right).$

Now, using equation:

$$\frac{F_1}{A_1} = \frac{F_2}{A_2} \Rightarrow F_{\text{o}} = F_{\text{i}}\left(\frac{A_{\text{o}}}{A_{\text{i}}}\right).$$

Finally,

$$W_{\text{o}} = F_{\text{o}}d_{\text{o}} = \left(\frac{F_{\text{i}}A_{\text{o}}}{A_{\text{i}}}\right)\left(\frac{d_{\text{i}}A_{\text{i}}}{A_{\text{o}}}\right) = F_{\text{i}}d_{\text{i}} = W_{\text{i}}.$$

In other words, the work output equals the work input.

(b) If the system is not moving, friction would not play a role. With friction, we know there are losses, so that $W_{\text{out}} = W_{\text{in}} - W_{\text{f}}$;

therefore, the work output is less than the work input. In other words, with friction, you need to push harder on the input piston than was calculated for the nonfriction case.

Glossary

Pascal's Principle
: a change in pressure applied to an enclosed fluid is transmitted undiminished to all portions of the fluid and to the walls of its container

Variation of Pressure with Depth in a Fluid

- Define pressure in terms of weight.
- Explain the variation of pressure with depth in a fluid.
- Calculate density given pressure and altitude.

If your ears have ever popped on a plane flight or ached during a deep dive in a swimming pool, you have experienced the effect of depth on pressure in a fluid. At the Earth's surface, the air pressure exerted on you is a result of the weight of air above you. This pressure is reduced as you climb up in altitude and the weight of air above you decreases. Under water, the pressure exerted on you increases with increasing depth. In this case, the pressure being exerted upon you is a result of both the weight of water above you *and* that of the atmosphere above you. You may notice an air pressure change on an elevator ride that transports you many stories, but you need only dive a meter or so below the surface of a pool to feel a pressure increase. The difference is that water is much denser than air, about 775 times as dense.

Consider the container in [link]. Its bottom supports the weight of the fluid in it. Let us calculate the pressure exerted on the bottom by the weight of the fluid. That **pressure** is the weight of the fluid mg divided by the area A supporting it (the area of the bottom of the container):
Equation:

$$P=\frac{\text{mg}}{A}.$$

We can find the mass of the fluid from its volume and density:
Equation:

$$m=\rho V.$$

The volume of the fluid V is related to the dimensions of the container. It is
Equation:

$$V=\text{Ah},$$

where A is the cross-sectional area and h is the depth. Combining the last two equations gives

Equation:

$$m=\rho \text{Ah}.$$

If we enter this into the expression for pressure, we obtain

Equation:

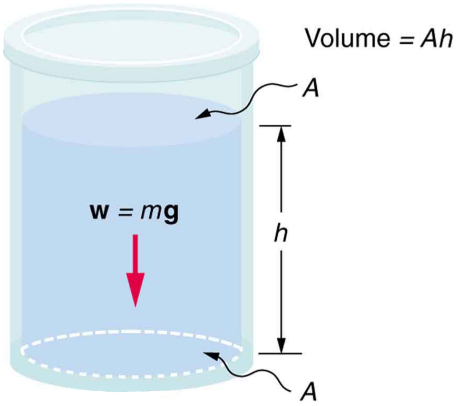
$$P=\frac{(\rho \text{Ah})g}{A}.$$

The area cancels, and rearranging the variables yields

Equation:

$$P=h\rho g.$$

This value is the *pressure due to the weight of a fluid*. The equation has general validity beyond the special conditions under which it is derived here. Even if the container were not there, the surrounding fluid would still exert this pressure, keeping the fluid static. Thus the equation $P=h\rho g$ represents the pressure due to the weight of any fluid of *average density* ρ at any depth h below its surface. For liquids, which are nearly incompressible, this equation holds to great depths. For gases, which are quite compressible, one can apply this equation as long as the density changes are small over the depth considered. [link] illustrates this situation.



The bottom of this container supports the entire weight of the fluid in it. The vertical sides cannot exert an upward force on the fluid (since it cannot withstand a shearing force), and so the bottom must support it all.

Example:

Calculating the Average Pressure and Force Exerted: What Force Must a Dam Withstand?

In [link], we calculated the mass of water in a large reservoir. We will now consider the pressure and force acting on the dam retaining water. (See [link].) The dam is 500 m wide, and the water is 80.0 m deep at the dam. (a) What is the average pressure on the dam due to the water? (b) Calculate the force exerted against the dam and compare it with the weight of water in the dam (previously found to be 1.96×10^{13} N).

Strategy for (a)

The average pressure P due to the weight of the water is the pressure at the average depth h of 40.0 m, since pressure increases linearly with depth.

Solution for (a)

The average pressure due to the weight of a fluid is

Equation:

$$P = h\rho g.$$

Entering the density of water from [link] and taking h to be the average depth of 40.0 m, we obtain

Equation:

$$\begin{aligned} P &= (40.0\ \text{m})\left(10^3\ \frac{\text{kg}}{\text{m}^3}\right)\left(9.80\ \frac{\text{m}}{\text{s}^2}\right) \\ &= 3.92 \times 10^5\ \frac{\text{N}}{\text{m}^2} = 392\ \text{kPa}. \end{aligned}$$

Strategy for (b)

The force exerted on the dam by the water is the average pressure times the area of contact:

Equation:

$$F = PA.$$

Solution for (b)

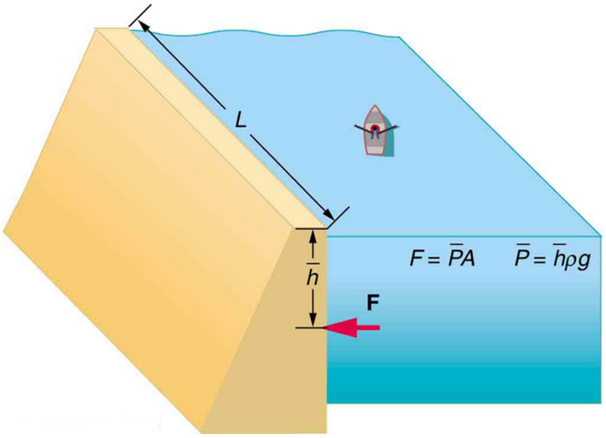
We have already found the value for P. The area of the dam is $A = 80.0\ \text{m} \times 500\ \text{m} = 4.00 \times 10^4\ \text{m}^2$, so that

Equation:

$$\begin{aligned} F &= (3.92 \times 10^5\ \text{N/m}^2)(4.00 \times 10^4\ \text{m}^2) \\ &= 1.57 \times 10^{10}\ \text{N}. \end{aligned}$$

Discussion

Although this force seems large, it is small compared with the 1.96×10^{13} N weight of the water in the reservoir—in fact, it is only 0.0800% of the weight. Note that the pressure found in part (a) is completely independent of the width and length of the lake—it depends only on its average depth at the dam. Thus the force depends only on the water’s average depth and the dimensions of the dam, *not* on the horizontal extent of the reservoir. In the diagram, the thickness of the dam increases with depth to balance the increasing force due to the increasing pressure.epth to balance the increasing force due to the increasing pressure.



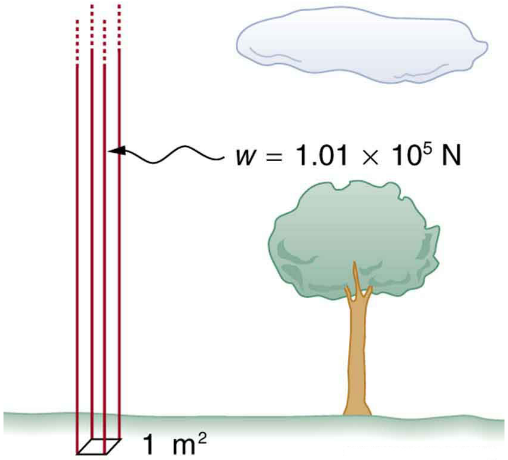
The dam must withstand the force exerted against it by the water it retains. This force is small compared with the weight of the water behind the dam.

Atmospheric pressure is another example of pressure due to the weight of a fluid, in this case due to the weight of *air* above a given height. The atmospheric pressure at the Earth’s surface varies a little due to the large-scale flow of the atmosphere induced by the Earth’s rotation (this creates weather “highs” and “lows”). However, the average pressure at sea level is given by the *standard atmospheric pressure* P_{atm}, measured to be

Equation:

$$1 \text{ atmosphere (atm)} = P_{\text{atm}} = 1.01 \times 10^5 \text{ N/m}^2 = 101 \text{ kPa}.$$

This relationship means that, on average, at sea level, a column of air above $1.00\ \text{m}^2$ of the Earth's surface has a weight of $1.01 \times 10^5\ \text{N}$, equivalent to $1\ \text{atm}$. (See [link].)



Atmospheric pressure at sea level averages $1.01 \times 10^5\ \text{Pa}$ (equivalent to 1 atm), since the column of air over this $1\ \text{m}^2$, extending to the top of the atmosphere, weighs $1.01 \times 10^5\ \text{N}$.

Example:

Calculating Average Density: How Dense Is the Air?

Calculate the average density of the atmosphere, given that it extends to an altitude of 120 km. Compare this density with that of air listed in [link].

Strategy

If we solve $P = h\rho g$ for density, we see that

Equation:

$$\rho = \frac{P}{\text{hg}}.$$

We then take P to be atmospheric pressure, h is given, and g is known, and so we can use this to calculate ρ.

Solution

Entering known values into the expression for ρ yields

Equation:

$$\rho = \frac{1.01 \times 10^5\ \text{N/m}^2}{(120 \times 10^3\ \text{m})(9.80\ \text{m/s}^2)} = 8.59 \times 10^{-2}\ \text{kg/m}^3.$$

Discussion

This result is the average density of air between the Earth’s surface and the top of the Earth’s atmosphere, which essentially ends at 120 km. The density of air at sea level is given in [link] as $1.29\ \text{kg/m}^3$ —about 15 times its average value. Because air is so compressible, its density has its highest value near the Earth’s surface and declines rapidly with altitude.

Example:

Calculating Depth Below the Surface of Water: What Depth of Water Creates the Same Pressure as the Entire Atmosphere?

Calculate the depth below the surface of water at which the pressure due to the weight of the water equals 1.00 atm.

Strategy

We begin by solving the equation $P = h\rho g$ for depth h:

Equation:

$$h = \frac{P}{\rho g}.$$

Then we take P to be 1.00 atm and ρ to be the density of the water that creates the pressure.

Solution

Entering the known values into the expression for h gives

Equation:

$$h = \frac{1.01 \times 10^5\ \text{N/m}^2}{(1.00 \times 10^3\ \text{kg/m}^3)(9.80\ \text{m/s}^2)} = 10.3\ \text{m}.$$

Discussion

Just 10.3 m of water creates the same pressure as 120 km of air. Since water is nearly incompressible, we can neglect any change in its density over this depth.

What do you suppose is the *total* pressure at a depth of 10.3 m in a swimming pool? Does the atmospheric pressure on the water's surface affect the pressure below? The answer is yes. This seems only logical, since both the water's weight and the atmosphere's weight must be supported. So the *total* pressure at a depth of 10.3 m is 2 atm—half from the water above and half from the air above. We shall see in Pascal's Principle that fluid pressures always add in this way.

Section Summary

- Pressure is the weight of the fluid mg divided by the area A supporting it (the area of the bottom of the container):
  Equation:

  $$P = \frac{\text{mg}}{A}.$$

- Pressure due to the weight of a liquid is given by
  Equation:

  $$P = h\rho g,$$

  where P is the pressure, h is the height of the liquid, ρ is the density of the liquid, and g is the acceleration due to gravity.

Conceptual Questions

Exercise:

Problem:

Atmospheric pressure exerts a large force (equal to the weight of the atmosphere above your body—about 10 tons) on the top of your body when you are lying on the beach sunbathing. Why are you able to get up?

Exercise:

Problem:

Why does atmospheric pressure decrease more rapidly than linearly with altitude?

Exercise:

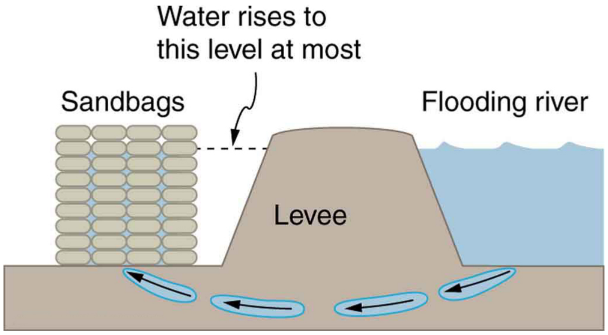
Problem:

What are two reasons why mercury rather than water is used in barometers?

Exercise:

Problem:

[link] shows how sandbags placed around a leak outside a river levee can effectively stop the flow of water under the levee. Explain how the small amount of water inside the column formed by the sandbags is able to balance the much larger body of water behind the levee.



Because the river level is very high, it has started to leak under the levee. Sandbags are placed around the leak, and the water held by them rises until it is the same level as the river, at which point the water there stops rising.

Exercise:

Problem:

Why is it difficult to swim under water in the Great Salt Lake?

Exercise:

Problem:

Is there a net force on a dam due to atmospheric pressure? Explain your answer.

Exercise:

Problem:

Does atmospheric pressure add to the gas pressure in a rigid tank? In a toy balloon? When, in general, does atmospheric pressure *not* affect the total pressure in a fluid?

Exercise:

Problem:

You can break a strong wine bottle by pounding a cork into it with your fist, but the cork must press directly against the liquid filling the bottle—there can be no air between the cork and liquid. Explain why the bottle breaks, and why it will not if there is air between the cork and liquid.

Problems & Exercises

Exercise:

Problem: What depth of mercury creates a pressure of 1.00 atm?

Solution:

0.760 m

Exercise:

Problem:

The greatest ocean depths on the Earth are found in the Marianas Trench near the Philippines. Calculate the pressure due to the ocean at the bottom of this trench, given its depth is 11.0 km and assuming the density of seawater is constant all the way down.

Exercise:

Problem: Verify that the SI unit of $h\rho g$ is N/m^2.

Solution:
Equation:

$$\begin{aligned}
(h\rho g)_{\text{units}} &= (\text{m})\left(\text{kg/m}^3\right)\left(\text{m/s}^2\right) = \left(\text{kg}\cdot\text{m}^2\right)/\left(\text{m}^3\cdot\text{s}^2\right)\\
&= \left(\text{kg}\cdot\text{m/s}^2\right)\left(1/\text{m}^2\right)\\
&= \text{N/m}^2
\end{aligned}$$

Exercise:

Problem:

Water towers store water above the level of consumers for times of heavy use, eliminating the need for high-speed pumps. How high above a user must the water level be to create a gauge pressure of $3.00 \times 10^5 \text{ N/m}^2$?

Exercise:

Problem:

The aqueous humor in a person’s eye is exerting a force of 0.300 N on the 1.10-cm^2 area of the cornea. (a) What pressure is this in mm Hg? (b) Is this value within the normal range for pressures in the eye?

Solution:

(a) 20.5 mm Hg

(b) The range of pressures in the eye is 12–24 mm Hg, so the result in part (a) is within that range

Exercise:

Problem:

How much force is exerted on one side of an 8.50 cm by 11.0 cm sheet of paper by the atmosphere? How can the paper withstand such a force?

Exercise:

Problem:

What pressure is exerted on the bottom of a 0.500-m-wide by 0.900-m-long gas tank that can hold 50.0 kg of gasoline by the weight of the gasoline in it when it is full?

Solution:

$1.09 \times 10^3 \text{ N/m}^2$

Exercise:

Problem:

Calculate the average pressure exerted on the palm of a shot-putter's hand by the shot if the area of contact is 50.0 cm^2 and he exerts a force of 800 N on it. Express the pressure in N/m^2 and compare it with the $1.00 \times 10^6 \text{ Pa}$ pressures sometimes encountered in the skeletal system.

Exercise:

Problem:

The left side of the heart creates a pressure of 120 mm Hg by exerting a force directly on the blood over an effective area of 15.0 cm^2. What force does it exert to accomplish this?

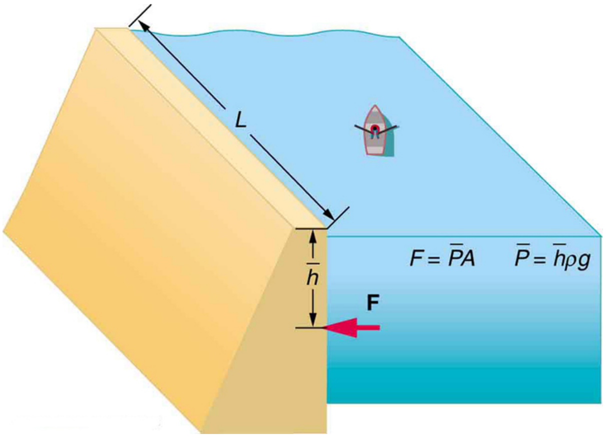
Solution:

24.0 N

Exercise:

Problem:

Show that the total force on a rectangular dam due to the water behind it increases with the *square* of the water depth. In particular, show that this force is given by $F = \rho gh^2 L/2$, where ρ is the density of water, h is its depth at the dam, and L is the length of the dam. You may assume the face of the dam is vertical. (Hint: Calculate the average pressure exerted and multiply this by the area in contact with the water. (See [link].)



Glossary

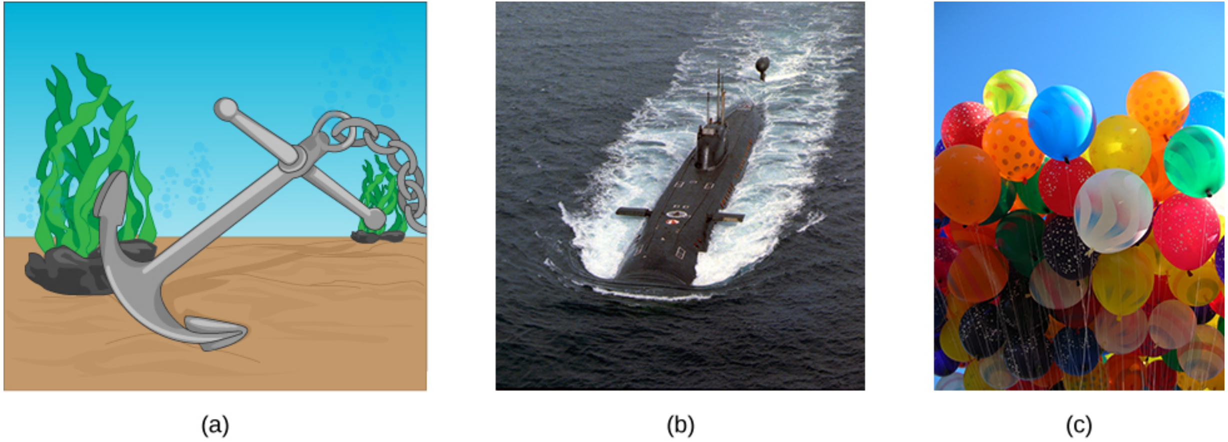
pressure
: the weight of the fluid divided by the area supporting it

Archimedes’ Principle and Buoyancy

By the end of this section, you will be able to:

- Define buoyant force
- State Archimedes’ principle
- Describe the relationship between density and Archimedes’ principle

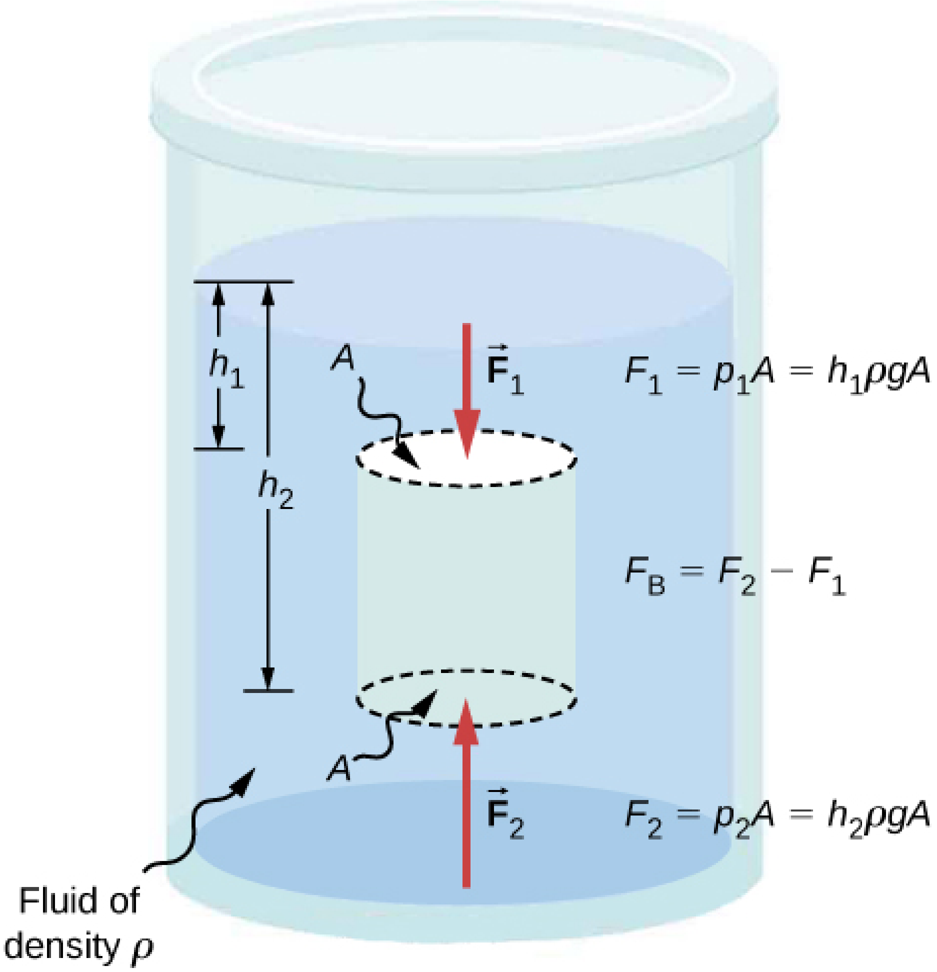
When placed in a fluid, some objects float due to a buoyant force. Where does this buoyant force come from? Why is it that some things float and others do not? Do objects that sink get any support at all from the fluid? Is your body buoyed by the atmosphere, or are only helium balloons affected ([link])?



(a) Even objects that sink, like this anchor, are partly supported by water when submerged. (b) Submarines have adjustable density (ballast tanks) so that they may float or sink as desired. (c) Helium-filled balloons tug upward on their strings, demonstrating air’s buoyant effect. (credit b: modification of work by Allied Navy; credit c: modification of work by “Crystl”/Flickr)

Answers to all these questions, and many others, are based on the fact that pressure increases with depth in a fluid. This means that the upward force on the bottom of an object in a fluid is greater than the downward force on top of the object. There is an upward force, or **buoyant force**, on any object in any fluid ([link]). If the buoyant force is greater than the object’s weight, the object rises to the surface and floats. If the buoyant force is less than the object’s weight, the object sinks. If the buoyant force equals the object’s weight, the object can remain suspended at its present depth. The buoyant force is always present, whether the object floats, sinks, or is suspended in a fluid.

Note:
Buoyant Force
The buoyant force is the upward force on any object in any fluid.



Pressure due to the weight of a fluid increases with depth because $p = hpg$. This change in pressure and associated upward force on the bottom of the cylinder are greater than the downward force on the top of the cylinder. The differences in the force results in the buoyant force F_{B}. (Horizontal forces cancel.)

Archimedes’ Principle

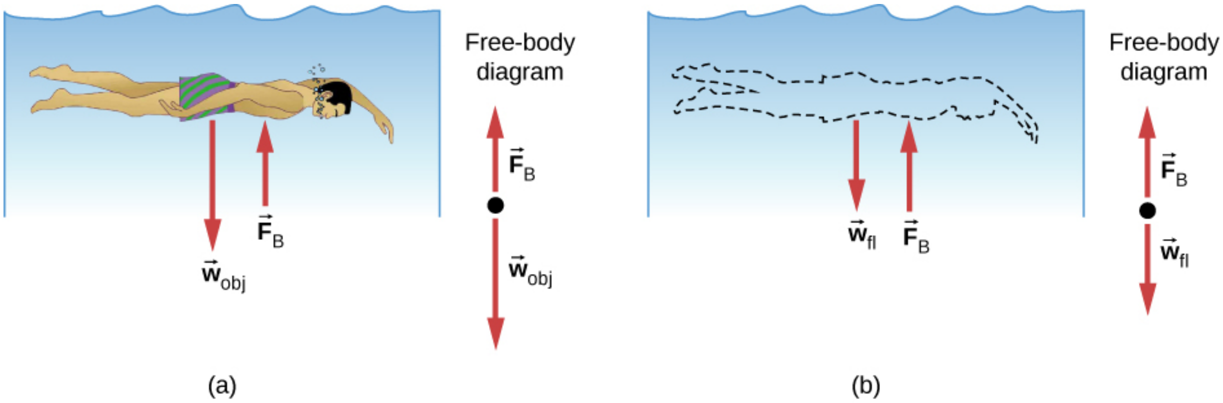
Just how large a force is buoyant force? To answer this question, think about what happens when a submerged object is removed from a fluid, as in [link]. If the object were not in the fluid, the space the object occupied would be filled by fluid having a weight w_{fl}. This weight is supported by the surrounding fluid, so the buoyant force must equal w_{fl}, the weight of the fluid displaced by the object.

Note:
Archimedes’ Principle
The buoyant force on an object equals the weight of the fluid it displaces. In equation form, **Archimedes’ principle** is
Equation:

$$F_{\text{B}} = w_{\text{fl}},$$

where F_{B} is the buoyant force and w_{fl} is the weight of the fluid displaced by the object.

This principle is named after the Greek mathematician and inventor Archimedes (ca. 287–212 BCE), who stated this principle long before concepts of force were well established.



(a) An object submerged in a fluid experiences a buoyant force F_B. If F_B is greater than the weight of the object, the object rises. If F_B is less than the weight of the object, the object sinks. (b) If the object is removed, it is replaced by fluid having weight w_{fl}. Since this weight is supported by surrounding fluid, the buoyant force must equal the weight of the fluid displaced.

Archimedes' principle refers to the force of buoyancy that results when a body is submerged in a fluid, whether partially or wholly. The force that provides the pressure of a fluid acts on a body perpendicular to the surface of the body. In other words, the force due to the pressure at the bottom is pointed up, while at the top, the force due to the pressure is pointed down; the forces due to the pressures at the sides are pointing into the body.

Since the bottom of the body is at a greater depth than the top of the body, the pressure at the lower part of the body is higher than the pressure at the upper part, as shown in [link]. Therefore a net upward force acts on the body. This upward force is the force of buoyancy, or simply *buoyancy*.

Note:
The exclamation "Eureka" (meaning "I found it") has often been credited to Archimedes as he made the discovery that would lead to Archimedes' principle. Some say it all started in a bathtub. To hear this story, watch this video or explore Scientific American to learn more.

Density and Archimedes' Principle

If you drop a lump of clay in water, it will sink. But if you mold the same lump of clay into the shape of a boat, it will float. Because of its shape, the clay boat displaces more water than the lump and experiences a greater buoyant force, even though its mass is the same. The same is true of steel ships.

The average density of an object is what ultimately determines whether it floats. If an object's average density is less than that of the surrounding fluid, it will float. The reason is that the fluid, having a higher density, contains more mass and hence more weight in the same volume. The buoyant force, which equals the weight of the fluid displaced, is thus greater than the weight of the object. Likewise, an object denser than the fluid will sink.

The extent to which a floating object is submerged depends on how the object's density compares to the density of the fluid. In [link], for example, the unloaded ship has a lower density and less of it is submerged compared with the same ship when loaded. We can derive a quantitative expression for the fraction submerged by considering density. The fraction submerged is the ratio of the volume submerged to the volume of the object, or

Equation:

$$\text{fraction submerged} = \frac{V_{\text{sub}}}{V_{\text{obj}}} = \frac{V_{\text{fl}}}{V_{\text{obj}}}.$$

The volume submerged equals the volume of fluid displaced, which we call V_{fl}. Now we can obtain the relationship between the densities by substituting $\rho = \frac{m}{V}$ into the expression. This gives

Equation:

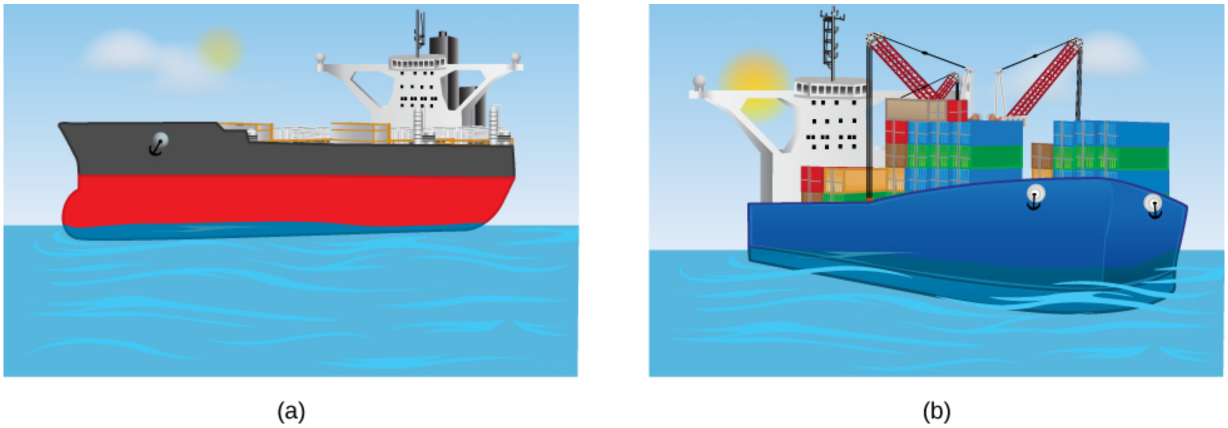
$$\frac{V_{\text{fl}}}{V_{\text{obj}}} = \frac{m_{\text{fl}}/\rho_{\text{fl}}}{m_{\text{obj}}/\rho_{\text{obj}}},$$

where ρ_{obj} is the average density of the object and ρ_{fl} is the density of the fluid. Since the object floats, its mass and that of the displaced fluid are equal, so they cancel from the equation, leaving

Equation:

$$\text{fraction submerged} = \frac{\rho_{\text{obj}}}{\rho_{\text{fl}}}.$$

We can use this relationship to measure densities.


(a)
(b)

An unloaded ship (a) floats higher in the water than a loaded ship (b).

Example:

Calculating Average Density

Suppose a 60.0-kg woman floats in fresh water with 97.0% of her volume submerged when her lungs are full of air. What is her average density?

Strategy

We can find the woman's density by solving the equation

Equation:

$$\text{fraction submerged} = \frac{\rho_{\text{obj}}}{\rho_{\text{fl}}}$$

for the density of the object. This yields

Equation:

$$\rho_{\text{obj}} = \rho_{\text{person}} = (\text{fraction submerged}) \cdot \rho_{\text{fl}}.$$

We know both the fraction submerged and the density of water, so we can calculate the woman’s density.

Solution

Entering the known values into the expression for her density, we obtain

Equation:

$$\rho_{\text{person}} = 0.970 \cdot \left(10^3 \ \frac{\text{kg}}{\text{m}^3}\right) = 970 \frac{\text{kg}}{\text{m}^3}.$$

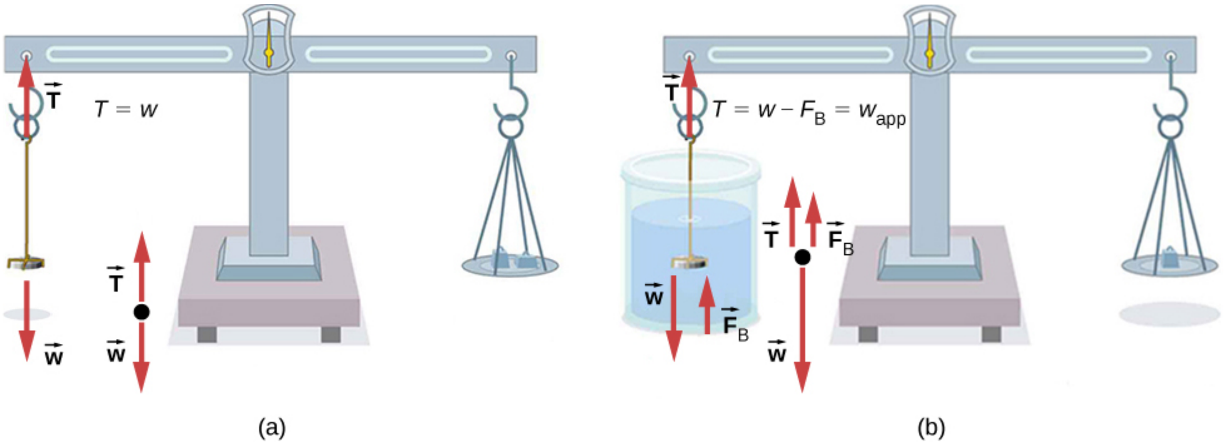
Significance

The woman’s density is less than the fluid density. We expect this because she floats.

Numerous lower-density objects or substances float in higher-density fluids: oil on water, a hot-air balloon in the atmosphere, a bit of cork in wine, an iceberg in salt water, and hot wax in a “lava lamp,” to name a few. A less obvious example is mountain ranges floating on the higher-density crust and mantle beneath them. Even seemingly solid Earth has fluid characteristics.

Measuring Density

One of the most common techniques for determining density is shown in [link].



(a) A coin is weighed in air. (b) The apparent weight of the coin is determined while it is completely submerged in a fluid of known density. These two measurements are used to calculate the density of the coin.

An object, here a coin, is weighed in air and then weighed again while submerged in a liquid. The density of the coin, an indication of its authenticity, can be calculated if the fluid density is known. We can use this same technique to determine the density of the fluid if the density of the coin is known.

All of these calculations are based on Archimedes’ principle, which states that the buoyant force on the object equals the weight of the fluid displaced. This, in turn, means that the object appears to weigh less when submerged; we call this measurement the object’s apparent weight. The object suffers an apparent weight loss equal to the weight of the fluid displaced. Alternatively, on balances that measure mass, the object suffers an apparent mass loss equal to the mass of fluid displaced. That is, apparent weight loss equals weight of fluid displaced, or apparent mass loss equals mass of fluid displaced.

Summary

- Buoyant force is the net upward force on any object in any fluid. If the buoyant force is greater than the object’s weight, the object will rise to

the surface and float. If the buoyant force is less than the object’s weight, the object will sink. If the buoyant force equals the object’s weight, the object can remain suspended at its present depth. The buoyant force is always present and acting on any object immersed either partially or entirely in a fluid.
- Archimedes’ principle states that the buoyant force on an object equals the weight of the fluid it displaces.

Conceptual Questions

Exercise:

Problem:

More force is required to pull the plug in a full bathtub than when it is empty. Does this contradict Archimedes’ principle? Explain your answer.

Solution:

Not at all. Pascal’s principle says that the change in the pressure is exerted through the fluid. The reason that the full tub requires more force to pull the plug is because of the weight of the water above the plug.

Exercise:

Problem:

Do fluids exert buoyant forces in a “weightless” environment, such as in the space shuttle? Explain your answer.

Exercise:

Problem:

Will the same ship float higher in salt water than in freshwater? Explain your answer.

Solution:

The buoyant force is equal to the weight of the fluid displaced. The greater the density of the fluid, the less fluid that is needed to be displaced to have the weight of the object be supported and to float. Since the density of salt water is higher than that of fresh water, less salt water will be displaced, and the ship will float higher.

Exercise:

Problem:

Marbles dropped into a partially filled bathtub sink to the bottom. Part of their weight is supported by buoyant force, yet the downward force on the bottom of the tub increases by exactly the weight of the marbles. Explain why.

Problems

Exercise:

Problem:

What fraction of ice is submerged when it floats in freshwater, given the density of water at $0\ ^{\circ}\text{C}$ is very close to $1000\ \text{kg/m}^3$?

Exercise:

Problem:

If a person’s body has a density of $995\ \text{kg/m}^3$, what fraction of the body will be submerged when floating gently in (a) freshwater? (b) In salt water with a density of $1027\ \text{kg/m}^3$?

Solution:

a. 99.5% submerged; b. 96.9% submerged

Exercise:

Problem:

A rock with a mass of 540 g in air is found to have an apparent mass of 342 g when submerged in water. (a) What mass of water is displaced? (b) What is the volume of the rock? (c) What is its average density? Is this consistent with the value for granite?

Exercise:

Problem:

Archimedes' principle can be used to calculate the density of a fluid as well as that of a solid. Suppose a chunk of iron with a mass of 390.0 g in air is found to have an apparent mass of 350.5 g when completely submerged in an unknown liquid. (a) What mass of fluid does the iron displace? (b) What is the volume of iron, using its density as given in [link]? (c) Calculate the fluid's density and identify it.

Solution:

a. 39.5 g; b. $50\ \text{cm}^3$; c. $0.79\ \text{g}/\text{cm}^3$; ethyl alcohol

Exercise:

Problem:

Calculate the buoyant force on a 2.00-L helium balloon. (b) Given the mass of the rubber in the balloon is 1.50 g, what is the net vertical force on the balloon if it is let go? Neglect the volume of the rubber.

Exercise:

Problem:

What is the density of a woman who floats in fresh water with 4.00% of her volume above the surface? (This could be measured by placing her in a tank with marks on the side to measure how much water she displaces when floating and when held under water.) (b) What percent of her volume is above the surface when she floats in seawater?

Solution:

a. 960 kg/m^3; b. 6.34%; She floats higher in seawater.

Exercise:

Problem:

A man has a mass of 80 kg and a density of 955kg/m^3 (excluding the air in his lungs). (a) Calculate his volume. (b) Find the buoyant force air exerts on him. (c) What is the ratio of the buoyant force to his weight?

Exercise:

Problem:

A simple compass can be made by placing a small bar magnet on a cork floating in water. (a) What fraction of a plain cork will be submerged when floating in water? (b) If the cork has a mass of 10.0 g and a 20.0-g magnet is placed on it, what fraction of the cork will be submerged? (c) Will the bar magnet and cork float in ethyl alcohol?

Solution:

a. 0.24; b. 0.72; c. Yes, the cork will float in ethyl alcohol.

Exercise:

Problem:

What percentage of an iron anchor's weight will be supported by buoyant force when submerged in salt water?

Exercise:

Problem:

Referring to [link], prove that the buoyant force on the cylinder is equal to the weight of the fluid displaced (Archimedes' principle). You may assume that the buoyant force is $F_2 - F_1$ and that the ends of the cylinder have equal areas A. Note that the volume of the cylinder (and that of the fluid it displaces) equals $(h_2 - h_1)A$.

Solution:

$$\begin{aligned}\text{net } F &= F_2 - F_1 = p_2 A - p_1 A = (p_2 - p_1)A = (h_2 \rho_{\text{fl}} g - h_1 \rho_{\text{fl}} g)A \\ &= (h_2 - h_1)\rho_{\text{fl}} g A, \text{ where } \rho_{\text{fl}} = \text{density of fluid.} \\ \text{net } F &= (h_2 - h_1) A \rho_{\text{fl}} g = V_{\text{fl}} \rho_{\text{fl}} g = m_{\text{fl}} g = w_{\text{fl}}\end{aligned}$$

Exercise:

Problem:

A 75.0-kg man floats in freshwater with 3.00% of his volume above water when his lungs are empty, and 5.00% of his volume above water when his lungs are full. Calculate the volume of air he inhales—called his lung capacity—in liters. (b) Does this lung volume seem reasonable?

Glossary

Archimedes’ principle
: buoyant force on an object equals the weight of the fluid it displaces

buoyant force
: net upward force on any object in any fluid due to the pressure difference at different depths

Gauge Pressure, Absolute Pressure, and Pressure Measurement

- Define gauge pressure and absolute pressure.
- Understand the working of aneroid and open-tube barometers.

If you limp into a gas station with a nearly flat tire, you will notice the tire gauge on the airline reads nearly zero when you begin to fill it. In fact, if there were a gaping hole in your tire, the gauge would read zero, even though atmospheric pressure exists in the tire. Why does the gauge read zero? There is no mystery here. Tire gauges are simply designed to read zero at atmospheric pressure and positive when pressure is greater than atmospheric.

Similarly, atmospheric pressure adds to blood pressure in every part of the circulatory system. (As noted in Pascal’s Principle, the total pressure in a fluid is the sum of the pressures from different sources—here, the heart and the atmosphere.) But atmospheric pressure has no net effect on blood flow since it adds to the pressure coming out of the heart and going back into it, too. What is important is how much *greater* blood pressure is than atmospheric pressure. Blood pressure measurements, like tire pressures, are thus made relative to atmospheric pressure.

In brief, it is very common for pressure gauges to ignore atmospheric pressure—that is, to read zero at atmospheric pressure. We therefore define **gauge pressure** to be the pressure relative to atmospheric pressure. Gauge pressure is positive for pressures above atmospheric pressure, and negative for pressures below it.

Note:
Gauge Pressure
Gauge pressure is the pressure relative to atmospheric pressure. Gauge pressure is positive for pressures above atmospheric pressure, and negative for pressures below it.

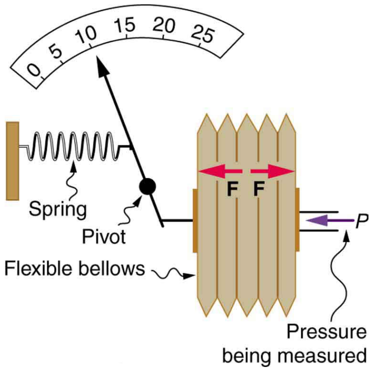
In fact, atmospheric pressure does add to the pressure in any fluid not enclosed in a rigid container. This happens because of Pascal’s principle. The total pressure, or **absolute pressure**, is thus the sum of gauge pressure and atmospheric pressure: $P_{\text{abs}} = P_{\text{g}} + P_{\text{atm}}$ where P_{abs} is absolute pressure, P_{g} is gauge pressure, and P_{atm} is atmospheric pressure. For example, if your tire gauge reads 34 psi (pounds per square inch), then the absolute pressure is 34 psi plus 14.7 psi (P_{atm} in psi), or 48.7 psi (equivalent to 336 kPa).

Note:
Absolute Pressure
Absolute pressure is the sum of gauge pressure and atmospheric pressure.

For reasons we will explore later, in most cases the absolute pressure in fluids cannot be negative. Fluids push rather than pull, so the smallest absolute pressure is zero. (A negative absolute pressure is a pull.) Thus the smallest possible gauge pressure is $P_{\text{g}} = -P_{\text{atm}}$ (this makes P_{abs} zero). There is no theoretical limit to how large a gauge pressure can be.

There are a host of devices for measuring pressure, ranging from tire gauges to blood pressure cuffs. Pascal's principle is of major importance in these devices. The undiminished transmission of pressure through a fluid allows precise remote sensing of pressures. Remote sensing is often more convenient than putting a measuring device into a system, such as a person's artery.

[link] shows one of the many types of mechanical pressure gauges in use today. In all mechanical pressure gauges, pressure results in a force that is converted (or transduced) into some type of readout.

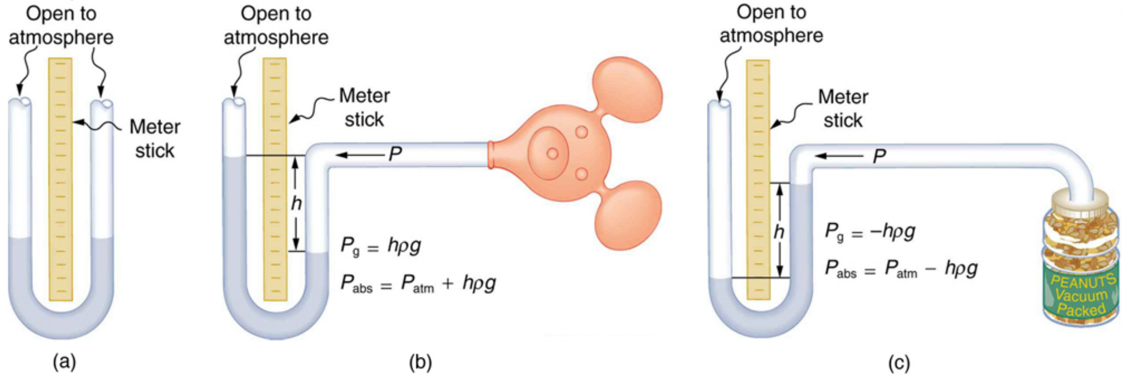


This aneroid gauge utilizes flexible bellows connected to a mechanical

indicator to
measure pressure.

An entire class of gauges uses the property that pressure due to the weight of a fluid is given by $P = h\rho g$. Consider the U-shaped tube shown in [link], for example. This simple tube is called a *manometer*. In [link](a), both sides of the tube are open to the atmosphere. Atmospheric pressure therefore pushes down on each side equally so its effect cancels. If the fluid is deeper on one side, there is a greater pressure on the deeper side, and the fluid flows away from that side until the depths are equal.

Let us examine how a manometer is used to measure pressure. Suppose one side of the U-tube is connected to some source of pressure P_{abs} such as the toy balloon in [link](b) or the vacuum-packed peanut jar shown in [link](c). Pressure is transmitted undiminished to the manometer, and the fluid levels are no longer equal. In [link](b), P_{abs} is greater than atmospheric pressure, whereas in [link](c), P_{abs} is less than atmospheric pressure. In both cases, P_{abs} differs from atmospheric pressure by an amount $h\rho g$, where ρ is the density of the fluid in the manometer. In [link](b), P_{abs} can support a column of fluid of height h, and so it must exert a pressure $h\rho g$ greater than atmospheric pressure (the gauge pressure P_{g} is positive). In [link](c), atmospheric pressure can support a column of fluid of height h, and so P_{abs} is less than atmospheric pressure by an amount $h\rho g$ (the gauge pressure P_{g} is negative). A manometer with one side open to the atmosphere is an ideal device for measuring gauge pressures. The gauge pressure is $P_{\text{g}} = h\rho g$ and is found by measuring h.



An open-tube manometer has one side open to the atmosphere. (a) Fluid depth must be the same on both sides, or the pressure each side exerts at the bottom will be unequal and there will be flow from the deeper side. (b) A positive gauge pressure $P_{\text{g}} = h\rho g$ transmitted to one side of the manometer can support a column of fluid of height h. (c) Similarly, atmospheric pressure is greater than a negative gauge pressure P_{g} by an amount $h\rho g$. The jar's rigidity prevents atmospheric pressure from being transmitted to the peanuts.

Mercury manometers are often used to measure arterial blood pressure. An inflatable cuff is placed on the upper arm as shown in [link]. By squeezing the bulb, the person making the measurement exerts pressure, which is transmitted undiminished to both the main artery in the arm and the manometer. When this applied pressure exceeds blood pressure, blood flow below the cuff is cut off. The person making the measurement then slowly lowers the applied pressure and listens for blood flow to resume. Blood pressure pulsates because of the pumping action of the heart, reaching a maximum, called **systolic pressure**, and a minimum, called **diastolic pressure**, with each heartbeat. Systolic pressure is measured by noting the value of h when blood flow first begins as cuff pressure is lowered. Diastolic pressure is measured by noting h when blood flows without interruption. The typical blood pressure of a young adult raises the mercury to a height of 120 mm at systolic and 80 mm at diastolic. This is commonly quoted as 120 over 80, or 120/80. The first pressure is representative of the maximum output of the heart; the second is due to the elasticity of the arteries in maintaining the pressure between beats. The density of the mercury fluid in the manometer is 13.6 times greater than water, so the height of the fluid will be 1/13.6 of that in a water manometer. This reduced height can make measurements difficult, so mercury manometers are used to measure larger pressures, such as blood pressure. The density of mercury is such that $1.0\text{ mm Hg} = 133\text{ Pa}$.

Note:
Systolic Pressure
Systolic pressure is the maximum blood pressure.

Note:
Diastolic Pressure
Diastolic pressure is the minimum blood pressure.

In routine blood pressure measurements, an inflatable cuff is placed on the upper arm at the same level as the heart. Blood flow is detected just below the cuff, and corresponding pressures are transmitted to a mercury-filled manometer. (credit: U.S. Army photo by Spc. Micah E. Clare\4TH BCT)

Example:

Calculating Height of IV Bag: Blood Pressure and Intravenous Infusions

Intravenous infusions are usually made with the help of the gravitational force. Assuming that the density of the fluid being administered is 1.00 g/ml, at what height should the IV bag be placed above the entry point so that the fluid just enters the vein if the blood pressure in the vein is 18 mm Hg above atmospheric pressure? Assume that the IV bag is collapsible.

Strategy for (a)

For the fluid to just enter the vein, its pressure at entry must exceed the blood pressure in the vein (18 mm Hg above atmospheric pressure). We therefore need to find the height of fluid that corresponds to this gauge pressure.

Solution

We first need to convert the pressure into SI units. Since $1.0\text{ mm Hg} = 133\text{ Pa}$,

Equation:

$$P = 18\text{ mm Hg} \times \frac{133\text{ Pa}}{1.0\text{ mm Hg}} = 2400\text{ Pa}.$$

Rearranging $P_{\text{g}} = h\rho g$ for h gives $h = \frac{P_{\text{g}}}{\rho g}$. Substituting known values into this equation gives

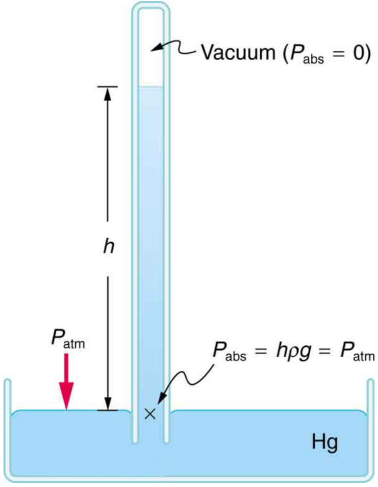
Equation:

$$\begin{aligned} h &= \frac{2400\text{ N/m}^2}{\left(1.0\times10^3\text{ kg/m}^3\right)\left(9.80\text{ m/s}^2\right)} \\ &= 0.24\text{ m.} \end{aligned}$$

Discussion

The IV bag must be placed at 0.24 m above the entry point into the arm for the fluid to just enter the arm. Generally, IV bags are placed higher than this. You may have noticed that the bags used for blood collection are placed below the donor to allow blood to flow easily from the arm to the bag, which is the opposite direction of flow than required in the example presented here.

A *barometer* is a device that measures atmospheric pressure. A mercury barometer is shown in [link]. This device measures atmospheric pressure, rather than gauge pressure, because there is a nearly pure vacuum above the mercury in the tube. The height of the mercury is such that $h\rho g = P_{\text{atm}}$. When atmospheric pressure varies, the mercury rises or falls, giving important clues to weather forecasters. The barometer can also be used as an altimeter, since average atmospheric pressure varies with altitude. Mercury barometers and manometers are so common that units of mm Hg are often quoted for atmospheric pressure and blood pressures. [link] gives conversion factors for some of the more commonly used units of pressure.



A mercury barometer measures atmospheric pressure. The pressure due to the mercury’s weight, $h\rho g$, equals atmospheric pressure. The atmosphere is able to force mercury in the tube to a height h because the pressure above the mercury is zero.

Conversion to N/m^2 (Pa)	Conversion from atm
$1.0 \text{ atm} = 1.013 \times 10^5 \text{ N/m}^2$	$1.0 \text{ atm} = 1.013 \times 10^5 \text{ N/m}^2$
$1.0 \text{ dyne/cm}^2 = 0.10 \text{ N/m}^2$	$1.0 \text{ atm} = 1.013 \times 10^6 \text{ dyne/cm}^2$
$1.0 \text{ kg/cm}^2 = 9.8 \times 10^4 \text{ N/m}^2$	$1.0 \text{ atm} = 1.013 \text{ kg/cm}^2$
$1.0 \text{ lb/in.}^2 = 6.90 \times 10^3 \text{ N/m}^2$	$1.0 \text{ atm} = 14.7 \text{ lb/in.}^2$
$1.0 \text{ mm Hg} = 133 \text{ N/m}^2$	$1.0 \text{ atm} = 760 \text{ mm Hg}$
$1.0 \text{ cm Hg} = 1.33 \times 10^3 \text{ N/m}^2$	$1.0 \text{ atm} = 76.0 \text{ cm Hg}$
$1.0 \text{ cm water} = 98.1 \text{ N/m}^2$	$1.0 \text{ atm} = 1.03 \times 10^3 \text{ cm water}$
$1.0 \text{ bar} = 1.000 \times 10^5 \text{ N/m}^2$	$1.0 \text{ atm} = 1.013 \text{ bar}$
$1.0 \text{ millibar} = 1.000 \times 10^2 \text{ N/m}^2$	$1.0 \text{ atm} = 1013 \text{ millibar}$

Conversion Factors for Various Pressure Units

Section Summary

- Gauge pressure is the pressure relative to atmospheric pressure.
- Absolute pressure is the sum of gauge pressure and atmospheric pressure.
- Aneroid gauge measures pressure using a bellows-and-spring arrangement connected to the pointer of a calibrated scale.
- Open-tube manometers have U-shaped tubes and one end is always open. It is used to measure pressure.
- A mercury barometer is a device that measures atmospheric pressure.

Conceptual Questions

Exercise:

Problem:

Explain why the fluid reaches equal levels on either side of a manometer if both sides are open to the atmosphere, even if the tubes are of different diameters.

Exercise:

Problem:

[link] shows how a common measurement of arterial blood pressure is made. Is there any effect on the measured pressure if the manometer is lowered? What is the effect of raising the arm above the shoulder? What is the effect of placing the cuff on the upper leg with the person standing? Explain your answers in terms of pressure created by the weight of a fluid.

Exercise:

Problem:

Considering the magnitude of typical arterial blood pressures, why are mercury rather than water manometers used for these measurements?

Problems & Exercises

Exercise:

Problem:

Find the gauge and absolute pressures in the balloon and peanut jar shown in [link], assuming the manometer connected to the balloon uses water whereas the manometer connected to the jar contains mercury. Express in units of centimeters of water for the balloon and millimeters of mercury for the jar, taking $h = 0.0500\text{ m}$ for each.

Solution:

Balloon:

$$\begin{aligned} P_{\text{g}} &= 5.00\text{ cm H}_2\text{O}, \\ P_{\text{abs}} &= 1.035 \times 10^3\text{ cm H}_2\text{O}. \end{aligned}$$

Jar:

$$\begin{aligned} P_{\text{g}} &= -50.0\text{ mm Hg}, \\ P_{\text{abs}} &= 710\text{ mm Hg}. \end{aligned}$$

Exercise:

Problem:

(a) Convert normal blood pressure readings of 120 over 80 mm Hg to newtons per meter squared using the relationship for pressure due to the weight of a fluid $(P = h\rho g)$ rather than a conversion factor. (b) Discuss why blood pressures for an infant could be smaller than those for an adult. Specifically, consider the smaller height to which blood must be pumped.

Exercise:

Problem:

How tall must a water-filled manometer be to measure blood pressures as high as 300 mm Hg?

Solution:

4.08 m

Exercise:

Problem:

Pressure cookers have been around for more than 300 years, although their use has strongly declined in recent years (early models had a nasty habit of exploding). How much force must the latches holding the lid onto a pressure cooker be able to withstand if the circular lid is 25.0 cm in diameter and the gauge pressure inside is 300 atm? Neglect the weight of the lid.

Exercise:

Problem:

Suppose you measure a standing person’s blood pressure by placing the cuff on his leg 0.500 m below the heart. Calculate the pressure you would observe (in units of mm Hg) if the pressure at the heart were 120 over 80 mm Hg. Assume that there is no loss of pressure due to resistance in the circulatory system (a reasonable assumption, since major arteries are large).

Solution:

$$\Delta P = 38.7\text{ mm Hg},$$

$$\text{Leg blood pressure} = \frac{159}{119}.$$

Exercise:

Problem:

A submarine is stranded on the bottom of the ocean with its hatch 25.0 m below the surface. Calculate the force needed to open the hatch from the inside, given it is circular and 0.450 m in diameter. Air pressure inside the submarine is 1.00 atm.

Exercise:

Problem:

Assuming bicycle tires are perfectly flexible and support the weight of bicycle and rider by pressure alone, calculate the total area of the tires in contact with the ground. The bicycle plus rider has a mass of 80.0 kg, and the gauge pressure in the tires is $3.50 \times 10^{5}\text{ Pa}$.

Solution:

$22.4\ \text{cm}^2$

Glossary

absolute pressure
the sum of gauge pressure and atmospheric pressure

diastolic pressure
the minimum blood pressure in the artery

gauge pressure
the pressure relative to atmospheric pressure

systolic pressure
the maximum blood pressure in the artery

Flow Rate and Its Relation to Velocity

- Calculate flow rate.
- Define units of volume.
- Describe incompressible fluids.
- Explain the consequences of the equation of continuity.

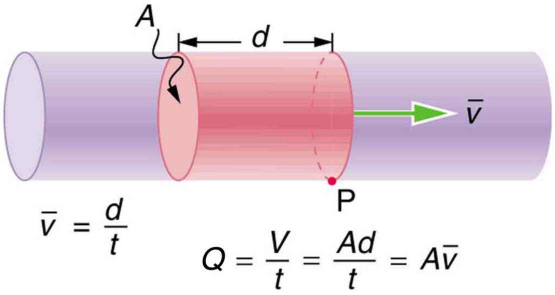
Flow rate Q is defined to be the volume of fluid passing by some location through an area during a period of time, as seen in [link]. In symbols, this can be written as

Equation:

$$Q=\frac{V}{t},$$

where V is the volume and t is the elapsed time.

The SI unit for flow rate is m^3/s, but a number of other units for Q are in common use. For example, the heart of a resting adult pumps blood at a rate of 5.00 liters per minute (L/min). Note that a **liter** (L) is 1/1000 of a cubic meter or 1000 cubic centimeters ($10^{-3}\ \text{m}^3$ or $10^3\ \text{cm}^3$). In this text we shall use whatever metric units are most convenient for a given situation.



Flow rate is the volume of fluid per unit time flowing past a point through the area A. Here the shaded cylinder of fluid flows past point P in a uniform pipe in time t. The volume of the cylinder is Ad

and the average velocity is
$v = d/t$ so that the flow rate
is $Q = \mathrm{Ad}/t = Av$.

Example:
Calculating Volume from Flow Rate: The Heart Pumps a Lot of Blood in a Lifetime

How many cubic meters of blood does the heart pump in a 75-year lifetime, assuming the average flow rate is 5.00 L/min?

Strategy

Time and flow rate Q are given, and so the volume V can be calculated from the definition of flow rate.

Solution

Solving $Q = V/t$ for volume gives

Equation:

$$V = \mathrm{Qt}.$$

Substituting known values yields

Equation:

$$\begin{aligned} V &= \left(\frac{5.00\text{ L}}{1\text{ min}}\right)(75\text{ y})\left(\frac{1\text{ m}^3}{10^3\text{ L}}\right)\left(5.26 \times 10^5\,\frac{\text{min}}{\text{y}}\right) \\ &= 2.0 \times 10^5\text{ m}^3. \end{aligned}$$

Discussion

This amount is about 200,000 tons of blood. For comparison, this value is equivalent to about 200 times the volume of water contained in a 6-lane 50-m lap pool.

Flow rate and velocity are related, but quite different, physical quantities. To make the distinction clear, think about the flow rate of a river. The

greater the velocity of the water, the greater the flow rate of the river. But flow rate also depends on the size of the river. A rapid mountain stream carries far less water than the Amazon River in Brazil, for example. The precise relationship between flow rate Q and velocity v is
Equation:

$$Q = Av,$$

where A is the cross-sectional area and v is the average velocity. This equation seems logical enough. The relationship tells us that flow rate is directly proportional to both the magnitude of the average velocity (hereafter referred to as the speed) and the size of a river, pipe, or other conduit. The larger the conduit, the greater its cross-sectional area. [link] illustrates how this relationship is obtained. The shaded cylinder has a volume
Equation:

$$V = \text{Ad},$$

which flows past the point P in a time t. Dividing both sides of this relationship by t gives
Equation:

$$\frac{V}{t} = \frac{\text{Ad}}{t}.$$

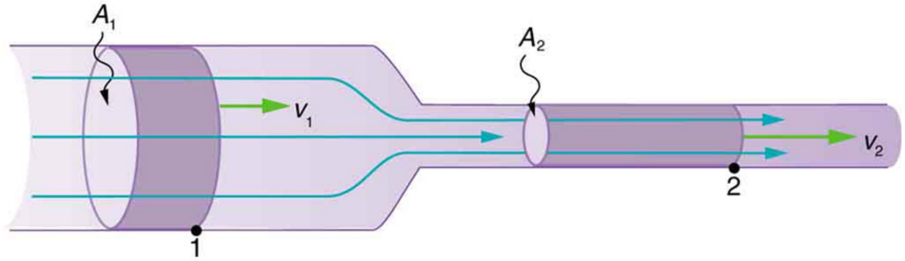
We note that $Q = V/t$ and the average speed is $v = d/t$. Thus the equation becomes $Q = Av$.

[link] shows an incompressible fluid flowing along a pipe of decreasing radius. Because the fluid is incompressible, the same amount of fluid must flow past any point in the tube in a given time to ensure continuity of flow. In this case, because the cross-sectional area of the pipe decreases, the velocity must necessarily increase. This logic can be extended to say that the flow rate must be the same at all points along the pipe. In particular, for points 1 and 2,

Equation:

$$\left.\begin{aligned} Q_1 &= Q_2 \\ A_1 v_1 &= A_2 v_2 \end{aligned}\right\}.$$

This is called the equation of continuity and is valid for any incompressible fluid. The consequences of the equation of continuity can be observed when water flows from a hose into a narrow spray nozzle: it emerges with a large speed—that is the purpose of the nozzle. Conversely, when a river empties into one end of a reservoir, the water slows considerably, perhaps picking up speed again when it leaves the other end of the reservoir. In other words, speed increases when cross-sectional area decreases, and speed decreases when cross-sectional area increases.



When a tube narrows, the same volume occupies a greater length. For the same volume to pass points 1 and 2 in a given time, the speed must be greater at point 2. The process is exactly reversible. If the fluid flows in the opposite direction, its speed will decrease when the tube widens. (Note that the relative volumes of the two cylinders and the corresponding velocity vector arrows are not drawn to scale.)

Since liquids are essentially incompressible, the equation of continuity is valid for all liquids. However, gases are compressible, and so the equation must be applied with caution to gases if they are subjected to compression or expansion.

Example:

Calculating Fluid Speed: Speed Increases When a Tube Narrows

A nozzle with a radius of 0.250 cm is attached to a garden hose with a radius of 0.900 cm. The flow rate through hose and nozzle is 0.500 L/s. Calculate the speed of the water (a) in the hose and (b) in the nozzle.

Strategy

We can use the relationship between flow rate and speed to find both velocities. We will use the subscript 1 for the hose and 2 for the nozzle.

Solution for (a)

First, we solve $Q = Av$ for v_1 and note that the cross-sectional area is $A = \pi r^2$, yielding

Equation:

$$v_1 = \frac{Q}{A_1} = \frac{Q}{\pi r_1^2}.$$

Substituting known values and making appropriate unit conversions yields

Equation:

$$v_1 = \frac{(0.500\ \mathrm{L/s})(10^{-3}\ \mathrm{m^3/L})}{\pi(9.00 \times 10^{-3}\ \mathrm{m})^2} = 1.96\ \mathrm{m/s}.$$

Solution for (b)

We could repeat this calculation to find the speed in the nozzle v_2, but we will use the equation of continuity to give a somewhat different insight. Using the equation which states

Equation:

$$A_1 v_1 = A_2 v_2,$$

solving for v_2 and substituting πr^2 for the cross-sectional area yields

Equation:

$$v_2 = \frac{A_1}{A_2} v_1 = \frac{\pi r_1^2}{\pi r_2^2} v_1 = \frac{r_{1^2}}{r_{2^2}} v_1.$$

Substituting known values,

Equation:

$$v_2 = \frac{(0.900\ \text{cm})^2}{(0.250\ \text{cm})^2} 1.96\ \text{m/s} = 25.5\ \text{m/s}.$$

Discussion

A speed of 1.96 m/s is about right for water emerging from a nozzleless hose. The nozzle produces a considerably faster stream merely by constricting the flow to a narrower tube.

The solution to the last part of the example shows that speed is inversely proportional to the *square* of the radius of the tube, making for large effects when radius varies. We can blow out a candle at quite a distance, for example, by pursing our lips, whereas blowing on a candle with our mouth wide open is quite ineffective.

In many situations, including in the cardiovascular system, branching of the flow occurs. The blood is pumped from the heart into arteries that subdivide into smaller arteries (arterioles) which branch into very fine vessels called capillaries. In this situation, continuity of flow is maintained but it is the *sum* of the flow rates in each of the branches in any portion along the tube that is maintained. The equation of continuity in a more general form becomes

Equation:

$$n_1 A_1 v_1 = n_2 A_2 v_2,$$

where n_1 and n_2 are the number of branches in each of the sections along the tube.

Example:

Calculating Flow Speed and Vessel Diameter: Branching in the Cardiovascular System

The aorta is the principal blood vessel through which blood leaves the heart in order to circulate around the body. (a) Calculate the average speed of the blood in the aorta if the flow rate is 5.0 L/min. The aorta has a radius of 10 mm. (b) Blood also flows through smaller blood vessels known as capillaries. When the rate of blood flow in the aorta is 5.0 L/min, the speed of blood in the capillaries is about 0.33 mm/s. Given that the average diameter of a capillary is 8.0 μm, calculate the number of capillaries in the blood circulatory system.

Strategy

We can use $Q = Av$ to calculate the speed of flow in the aorta and then use the general form of the equation of continuity to calculate the number of capillaries as all of the other variables are known.

Solution for (a)

The flow rate is given by $Q = Av$ or $v = \frac{Q}{\pi r^2}$ for a cylindrical vessel. Substituting the known values (converted to units of meters and seconds) gives

Equation:

$$v = \frac{(5.0\text{ L/min})\left(10^{-3}\text{ m}^3/\text{L}\right)(1\text{ min}/60\text{ s})}{\pi(0.010\text{ m})^2} = 0.27\text{ m/s}.$$

Solution for (b)

Using $n_1 A_1 v_1 = n_2 A_2 v_1$, assigning the subscript 1 to the aorta and 2 to the capillaries, and solving for n_2 (the number of capillaries) gives $n_2 = \frac{n_1 A_1 v_1}{A_2 v_2}$. Converting all quantities to units of meters and seconds and substituting into the equation above gives

Equation:

$$n_2 = \frac{(1)(\pi)\left(10\times10^{-3}\text{ m}\right)^2(0.27\text{ m/s})}{(\pi)\left(4.0\times10^{-6}\text{ m}\right)^2\left(0.33\times10^{-3}\text{ m/s}\right)} = 5.0\times10^{9}\text{ capillaries.}$$

Discussion

Note that the speed of flow in the capillaries is considerably reduced relative to the speed in the aorta due to the significant increase in the total cross-sectional area at the capillaries. This low speed is to allow sufficient time for effective exchange to occur although it is equally important for the flow not to become stationary in order to avoid the possibility of clotting. Does this large number of capillaries in the body seem reasonable? In active muscle, one finds about 200 capillaries per mm^3, or about 200×10^6 per 1 kg of muscle. For 20 kg of muscle, this amounts to about 4×10^9 capillaries.

Section Summary

- Flow rate Q is defined to be the volume V flowing past a point in time t, or $Q = \frac{V}{t}$ where V is volume and t is time.
- The SI unit of volume is m^3.
- Another common unit is the liter (L), which is $10^{-3}\ \text{m}^3$.
- Flow rate and velocity are related by $Q = Av$ where A is the cross-sectional area of the flow and v is its average velocity.
- For incompressible fluids, flow rate at various points is constant. That is,

  Equation:

$$\begin{aligned} Q_1 &= Q_2 \\ A_1 v_1 &= A_2 v_2 \qquad . \\ n_1 A_1 v_1 &= n_2 A_2 v_2 \end{aligned}$$

Conceptual Questions

Exercise:

Problem:

What is the difference between flow rate and fluid velocity? How are they related?

Exercise:

Problem:

Many figures in the text show streamlines. Explain why fluid velocity is greatest where streamlines are closest together. (Hint: Consider the relationship between fluid velocity and the cross-sectional area through which it flows.)

Exercise:

Problem:

Identify some substances that are incompressible and some that are not.

Problems & Exercises

Exercise:

Problem:

What is the average flow rate in $\text{cm}^3\text{/s}$ of gasoline to the engine of a car traveling at 100 km/h if it averages 10.0 km/L?

Solution:

$2.78\text{ cm}^3\text{/s}$

Exercise:

Problem:

The heart of a resting adult pumps blood at a rate of 5.00 L/min. (a) Convert this to $\text{cm}^3\text{/s}$. (b) What is this rate in $\text{m}^3\text{/s}$?

Exercise:

Problem:

Blood is pumped from the heart at a rate of 5.0 L/min into the aorta (of radius 1.0 cm). Determine the speed of blood through the aorta.

Solution:

27 cm/s

Exercise:

Problem:

Blood is flowing through an artery of radius 2 mm at a rate of 40 cm/s. Determine the flow rate and the volume that passes through the artery in a period of 30 s.

Exercise:

Problem:

The Huka Falls on the Waikato River is one of New Zealand’s most visited natural tourist attractions (see [link]). On average the river has a flow rate of about 300,000 L/s. At the gorge, the river narrows to 20 m wide and averages 20 m deep. (a) What is the average speed of the river in the gorge? (b) What is the average speed of the water in the river downstream of the falls when it widens to 60 m and its depth increases to an average of 40 m?

The Huka Falls in Taupo, New Zealand, demonstrate flow rate. (credit: RaviGogna, Flickr)

Solution:

(a) 0.75 m/s

(b) 0.13 m/s

Exercise:

Problem:

A major artery with a cross-sectional area of $1.00\ \text{cm}^2$ branches into 18 smaller arteries, each with an average cross-sectional area of $0.400\ \text{cm}^2$. By what factor is the average velocity of the blood reduced when it passes into these branches?

Exercise:

Problem:

(a) As blood passes through the capillary bed in an organ, the capillaries join to form venules (small veins). If the blood speed increases by a factor of 4.00 and the total cross-sectional area of the venules is $10.0\ \text{cm}^2$, what is the total cross-sectional area of the capillaries feeding these venules? (b) How many capillaries are involved if their average diameter is $10.0\ \mu\text{m}$?

Solution:

(a) $40.0\ \text{cm}^2$

(b) 5.09×10^7

Exercise:

Problem:

The human circulation system has approximately 1×10^9 capillary vessels. Each vessel has a diameter of about $8\ \mu\text{m}$. Assuming cardiac output is 5 L/min, determine the average velocity of blood flow through each capillary vessel.

Exercise:

Problem:

(a) Estimate the time it would take to fill a private swimming pool with a capacity of 80,000 L using a garden hose delivering 60 L/min. (b) How long would it take to fill if you could divert a moderate size river, flowing at $5000 \text{ m}^3/\text{s}$, into it?

Solution:

(a) 22 h

(b) 0.016 s

Exercise:

Problem:

The flow rate of blood through a $2.00 \times 10^{-6}\text{-m}$ -radius capillary is $3.80 \times 10^{-9} \text{ cm}^3/\text{s}$. (a) What is the speed of the blood flow? (This small speed allows time for diffusion of materials to and from the blood.) (b) Assuming all the blood in the body passes through capillaries, how many of them must there be to carry a total flow of $90.0 \text{ cm}^3/\text{s}$? (The large number obtained is an overestimate, but it is still reasonable.)

Exercise:

Problem:

(a) What is the fluid speed in a fire hose with a 9.00-cm diameter carrying 80.0 L of water per second? (b) What is the flow rate in cubic meters per second? (c) Would your answers be different if salt water replaced the fresh water in the fire hose?

Solution:

(a) 12.6 m/s

(b) $0.0800\ \text{m}^3/\text{s}$

(c) No, independent of density.

Exercise:

Problem:

The main uptake air duct of a forced air gas heater is 0.300 m in diameter. What is the average speed of air in the duct if it carries a volume equal to that of the house's interior every 15 min? The inside volume of the house is equivalent to a rectangular solid 13.0 m wide by 20.0 m long by 2.75 m high.

Exercise:

Problem:

Water is moving at a velocity of 2.00 m/s through a hose with an internal diameter of 1.60 cm. (a) What is the flow rate in liters per second? (b) The fluid velocity in this hose's nozzle is 15.0 m/s. What is the nozzle's inside diameter?

Solution:

(a) 0.402 L/s

(b) 0.584 cm

Exercise:

Problem:

Prove that the speed of an incompressible fluid through a constriction, such as in a Venturi tube, increases by a factor equal to the square of the factor by which the diameter decreases. (The converse applies for flow out of a constriction into a larger-diameter region.)

Exercise:

Problem:

Water emerges straight down from a faucet with a 1.80-cm diameter at a speed of 0.500 m/s. (Because of the construction of the faucet, there is no variation in speed across the stream.) (a) What is the flow rate in cm^3/s? (b) What is the diameter of the stream 0.200 m below the faucet? Neglect any effects due to surface tension.

Solution:

(a) $127\ \text{cm}^3/\text{s}$

(b) 0.890 cm

Exercise:

Problem: **Unreasonable Results**

A mountain stream is 10.0 m wide and averages 2.00 m in depth. During the spring runoff, the flow in the stream reaches $100,000\ \text{m}^3/\text{s}$. (a) What is the average velocity of the stream under these conditions? (b) What is unreasonable about this velocity? (c) What is unreasonable or inconsistent about the premises?

Glossary

flow rate
: abbreviated *Q*, it is the volume *V* that flows past a particular point during a time *t*, or *Q* = *V/t*

liter
: a unit of volume, equal to $10^{-3}\ \text{m}^3$

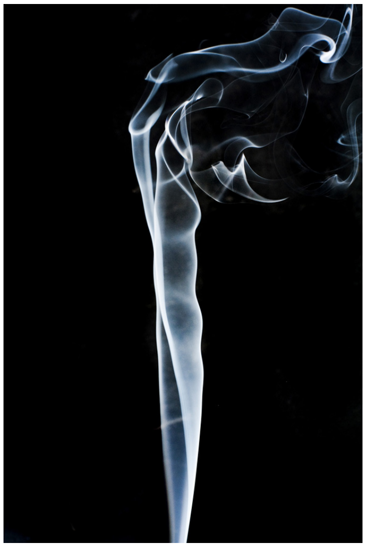
Viscosity and Laminar Flow; Poiseuille’s Law

- Define laminar flow and turbulent flow.
- Explain what viscosity is.
- Calculate flow and resistance with Poiseuille’s law.
- Explain how pressure drops due to resistance.

Laminar Flow and Viscosity

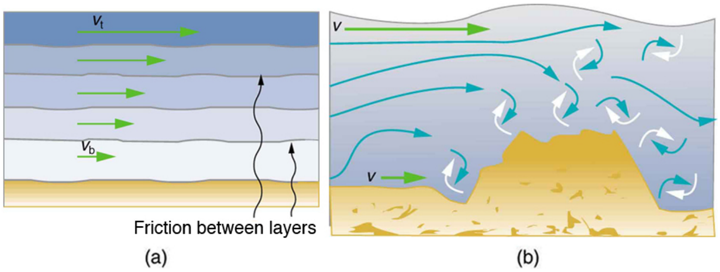
When you pour yourself a glass of juice, the liquid flows freely and quickly. But when you pour syrup on your pancakes, that liquid flows slowly and sticks to the pitcher. The difference is fluid friction, both within the fluid itself and between the fluid and its surroundings. We call this property of fluids *viscosity*. Juice has low viscosity, whereas syrup has high viscosity. In the previous sections we have considered ideal fluids with little or no viscosity. In this section, we will investigate what factors, including viscosity, affect the rate of fluid flow.

The precise definition of viscosity is based on *laminar*, or nonturbulent, flow. Before we can define viscosity, then, we need to define laminar flow and turbulent flow. [link] shows both types of flow. **Laminar** flow is characterized by the smooth flow of the fluid in layers that do not mix. Turbulent flow, or **turbulence**, is characterized by eddies and swirls that mix layers of fluid together.



Smoke rises smoothly for a while and then begins to form swirls and eddies. The smooth flow is called laminar flow, whereas the swirls and eddies typify turbulent flow. If you watch the smoke (being careful not to breathe on it), you will notice that it rises more rapidly when flowing smoothly than after it becomes turbulent, implying that turbulence poses more resistance to flow. (credit: Creativity103)

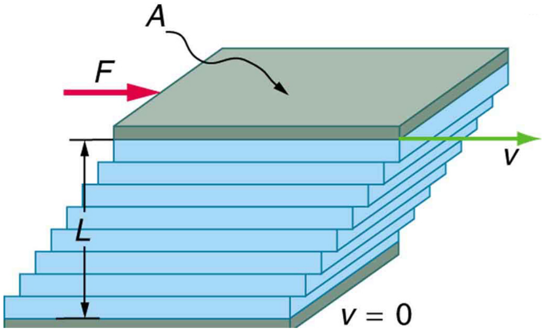
[link] shows schematically how laminar and turbulent flow differ. Layers flow without mixing when flow is laminar. When there is turbulence, the layers mix, and there are significant velocities in directions other than the overall direction of flow. The lines that are shown in many illustrations are the paths followed by small volumes of fluids. These are called *streamlines*. Streamlines are smooth and continuous when flow is laminar, but break up and mix when flow is turbulent. Turbulence has two main causes. First, any obstruction or sharp corner, such as in a faucet, creates turbulence by imparting velocities perpendicular to the flow. Second, high speeds cause turbulence. The drag both between adjacent layers of fluid and between the fluid and its surroundings forms swirls and eddies, if the speed is great enough. We shall concentrate on laminar flow for the remainder of this section, leaving certain aspects of turbulence for later sections.



(a) Laminar flow occurs in layers without mixing. Notice that viscosity causes drag between layers as well as with the fixed surface. (b) An obstruction in the vessel produces turbulence. Turbulent flow mixes the fluid. There is more interaction, greater heating, and more resistance than in laminar flow.

Note:
Making Connections: Take-Home Experiment: Go Down to the River
Try dropping simultaneously two sticks into a flowing river, one near the edge of the river and one near the middle. Which one travels faster? Why?

[link] shows how viscosity is measured for a fluid. Two parallel plates have the specific fluid between them. The bottom plate is held fixed, while the top plate is moved to the right, dragging fluid with it. The layer (or lamina) of fluid in contact with either plate does not move relative to the plate, and so the top layer moves at v while the bottom layer remains at rest. Each successive layer from the top down exerts a force on the one below it, trying to drag it along, producing a continuous variation in speed from v to 0 as shown. Care is taken to insure that the flow is laminar; that is, the layers do not mix. The motion in [link] is like a continuous shearing motion. Fluids have zero shear strength, but the *rate* at which they are sheared is related to the same geometrical factors A and L as is shear deformation for solids.



The graphic shows laminar flow of fluid between two plates of area A. The bottom plate is fixed. When the top plate is pushed to the right, it drags the fluid along with it.

A force F is required to keep the top plate in [link] moving at a constant velocity v, and experiments have shown that this force depends on four factors. First, F is directly proportional to v (until the speed is so high that turbulence occurs—then a much larger force is needed, and it has a more complicated dependence on v). Second, F is proportional to the area A of the plate. This relationship seems reasonable, since A is directly proportional to the amount of fluid being moved. Third, F is inversely proportional to the distance between the plates L. This relationship is also reasonable; L is like a lever arm, and the greater the lever arm, the less force that is needed. Fourth, F is directly proportional to *the coefficient of viscosity*, η. The greater the viscosity, the greater the force required. These dependencies are combined into the equation

Equation:

$$F=\eta\frac{\text{vA}}{L},$$

which gives us a working definition of fluid **viscosity** η. Solving for η gives

Equation:

$$\eta=\frac{\text{FL}}{\text{vA}},$$

which defines viscosity in terms of how it is measured. The SI unit of viscosity is $\text{N}\cdot\text{m/}\left[\left(\text{m/s}\right){\text{m}}^{2}\right]=\left({\text{N/m}}^{2}\right)\text{s or Pa}\cdot\text{s}$. [link] lists the coefficients of viscosity for various fluids.

Viscosity varies from one fluid to another by several orders of magnitude. As you might expect, the viscosities of gases are much less than those of liquids, and these viscosities are often temperature dependent. The viscosity of blood can be reduced by aspirin consumption, allowing it to flow more easily around the body. (When used over the long term in low doses, aspirin can help prevent heart attacks, and reduce the risk of blood clotting.)

Laminar Flow Confined to Tubes—Poiseuille’s Law

What causes flow? The answer, not surprisingly, is pressure difference. In fact, there is a very simple relationship between horizontal flow and pressure. Flow rate Q is in the direction from high to low pressure. The greater the pressure differential between two points, the greater the flow rate. This relationship can be stated as
Equation:

$$Q=\frac{{P}_{2}-{P}_{1}}{R},$$

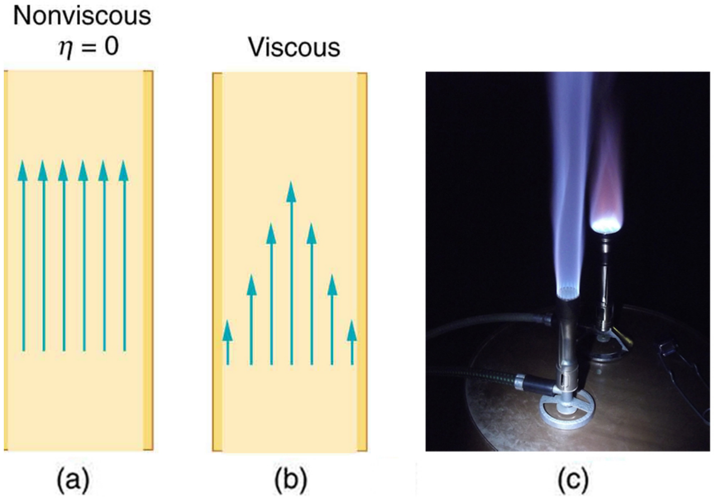
where P_1 and P_2 are the pressures at two points, such as at either end of a tube, and R is the resistance to flow. The resistance R includes everything, except pressure, that affects flow rate. For example, R is greater for a long tube than for a short one. The greater the viscosity of a fluid, the greater the value of R. Turbulence greatly increases R, whereas increasing the diameter of a tube decreases R.

If viscosity is zero, the fluid is frictionless and the resistance to flow is also zero. Comparing frictionless flow in a tube to viscous flow, as in [link], we see that for a viscous fluid, speed is greatest at midstream because of drag at the boundaries. We can see the effect of viscosity in a Bunsen burner flame, even though the viscosity of natural gas is small.

The resistance R to laminar flow of an incompressible fluid having viscosity η through a horizontal tube of uniform radius r and length l, such as the one in [link], is given by
Equation:

$$R=\frac{8\eta l}{\pi {r}^{4}}.$$

This equation is called **Poiseuille’s law for resistance** after the French scientist J. L. Poiseuille (1799–1869), who derived it in an attempt to understand the flow of blood, an often turbulent fluid.



(a) If fluid flow in a tube has negligible resistance, the speed is the same all across the tube. (b) When a viscous fluid flows through a tube, its speed at the walls is zero, increasing steadily to its maximum at the center of the tube. (c) The shape of the Bunsen burner flame is due to the velocity profile across the tube. (credit: Jason Woodhead)

Let us examine Poiseuille’s expression for R to see if it makes good intuitive sense. We see that resistance is directly proportional to both fluid viscosity η and the length l of a tube. After all, both of these directly affect the amount of friction encountered—the greater either is, the greater the resistance and the smaller the flow. The radius r of a tube affects the resistance, which again makes sense, because the greater the radius, the greater the flow (all other factors remaining the same). But it is surprising that r is raised to the *fourth* power in Poiseuille’s law. This exponent means that any change in the radius of a tube has a very large effect on resistance. For example, doubling the radius of a tube decreases resistance by a factor of $2^4 = 16$.

Taken together, $Q = \frac{P_2 - P_1}{R}$ and $R = \frac{8\eta l}{\pi r^4}$ give the following expression for flow rate:

Equation:

$$Q = \frac{(P_2 - P_1)\pi r^4}{8\eta l}.$$

This equation describes laminar flow through a tube. It is sometimes called Poiseuille’s law for laminar flow, or simply **Poiseuille’s law**.

Example:

Using Flow Rate: Plaque Deposits Reduce Blood Flow

Suppose the flow rate of blood in a coronary artery has been reduced to half its normal value by plaque deposits. By what factor has the radius of the artery been reduced, assuming no turbulence occurs?

Strategy

Assuming laminar flow, Poiseuille’s law states that

Equation:

$$Q = \frac{(P_2 - P_1)\pi r^4}{8\eta l}.$$

We need to compare the artery radius before and after the flow rate reduction.

Solution

With a constant pressure difference assumed and the same length and viscosity, along the artery we have

Equation:

$$\frac{Q_1}{r_1^4} = \frac{Q_2}{r_2^4}.$$

So, given that $Q_2 = 0.5Q_1$, we find that $r_2^4 = 0.5r_1^4$.

Therefore, $r_2 = (0.5)^{0.25} r_1 = 0.841 r_1$, a decrease in the artery radius of 16%.

Discussion

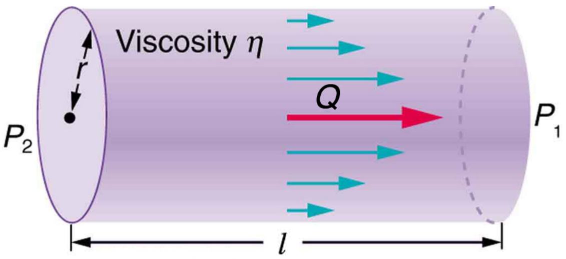
This decrease in radius is surprisingly small for this situation. To restore the blood flow in spite of this buildup would require an increase in the pressure difference $(P_2 - P_1)$ of a factor of two, with subsequent strain on the heart.

Fluid	**Temperature (ºC)**	**Viscosity** η (mPa·s)
Gases		
Air	0	0.0171
	20	0.0181
	40	0.0190
	100	0.0218
Ammonia	20	0.00974
Carbon dioxide	20	0.0147
Helium	20	0.0196
Hydrogen	0	0.0090
Mercury	20	0.0450
Oxygen	20	0.0203
Steam	100	0.0130
Liquids		
Water	0	1.792
	20	1.002
	37	0.6947
	40	0.653
	100	0.282
Whole blood[footnote]	20	3.015

Fluid	**Temperature (ºC)**	**Viscosity** η (mPa·s)
The ratios of the viscosities of blood to water are nearly constant between 0°C and 37°C.	37	2.084
Blood plasma[footnote] See note on Whole Blood.	20	1.810
	37	1.257
Ethyl alcohol	20	1.20
Methanol	20	0.584
Oil (heavy machine)	20	660
Oil (motor, SAE 10)	30	200
Oil (olive)	20	138
Glycerin	20	1500
Honey	20	2000–10000
Maple Syrup	20	2000–3000
Milk	20	3.0
Oil (Corn)	20	65

Coefficients of Viscosity of Various Fluids

The circulatory system provides many examples of Poiseuille’s law in action—with blood flow regulated by changes in vessel size and blood pressure. Blood vessels are not rigid but elastic. Adjustments to blood flow are primarily made by varying the size of the vessels, since the resistance is so sensitive to the radius. During vigorous exercise, blood vessels are selectively dilated to important muscles and organs and blood pressure increases. This creates both greater overall blood flow and increased flow to specific areas. Conversely, decreases in vessel radii, perhaps from plaques in the arteries, can greatly reduce blood flow. If a vessel's radius is reduced by only 5% (to 0.95 of its original value), the flow rate is reduced to about $(0.95)^4 = 0.81$ of its original value. A 19% decrease in flow is caused by a 5% decrease in radius. The body may compensate by increasing blood pressure by 19%, but this presents hazards to the heart and any vessel that has weakened walls. Another example comes from automobile engine oil. If you have a car with an oil pressure gauge, you may notice that oil pressure is high when the engine is cold. Motor oil has greater viscosity when cold than when warm, and so pressure must be greater to pump the same amount of cold oil.



Poiseuille's law applies to laminar flow of an incompressible fluid of viscosity η through a tube of length l and radius r. The direction of flow is from greater to lower pressure. Flow rate Q is directly proportional to the pressure difference $P_2 - P_1$, and inversely proportional to the length l of the tube and viscosity η of the fluid. Flow rate increases with r^4, the fourth power of the radius.

Example:
What Pressure Produces This Flow Rate?

An intravenous (IV) system is supplying saline solution to a patient at the rate of $0.120\ \text{cm}^3/\text{s}$ through a needle of radius 0.150 mm and length 2.50 cm. What pressure is needed at the entrance of the needle to cause this flow, assuming the viscosity of the saline solution to be the same as that of water? The gauge pressure of the blood in the patient's vein is 8.00 mm Hg. (Assume that the temperature is 20ºC .)

Strategy

Assuming laminar flow, Poiseuille's law applies. This is given by

Equation:

$$Q = \frac{(P_2 - P_1)\pi r^4}{8\eta l},$$

where P_2 is the pressure at the entrance of the needle and P_1 is the pressure in the vein. The only unknown is P_2.

Solution

Solving for P_2 yields

Equation:

$$P_2 = \frac{8\eta l}{\pi r^4}Q + P_{1.}$$

P_1 is given as 8.00 mm Hg, which converts to $1.066 \times 10^3\ \text{N/m}^2$. Substituting this and the other known values yields

Equation:

$$\begin{aligned} P_2 &= \left[\frac{8(1.00\times10^{-3}\ \text{N}\cdot\text{s/m}^2)(2.50\times10^{-2}\ \text{m})}{\pi(0.150\times10^{-3}\ \text{m})^4}\right](1.20\times10^{-7}\ \text{m}^3/\text{s}) + 1.066\times10^3\ \text{N/m}^2 \\ &= 1.62\times10^4\ \text{N/m}^2. \end{aligned}$$

Discussion

This pressure could be supplied by an IV bottle with the surface of the saline solution 1.61 m above the entrance to the needle (this is left for you to solve in this chapter's Problems and Exercises), assuming that there is negligible pressure drop in the tubing leading to the needle.

Flow and Resistance as Causes of Pressure Drops

You may have noticed that water pressure in your home might be lower than normal on hot summer days when there is more use. This pressure drop occurs in the water main before it reaches your home. Let us consider flow through the water main as illustrated in [link]. We can understand why the pressure P_1 to the home drops during times of heavy use by rearranging

Equation:

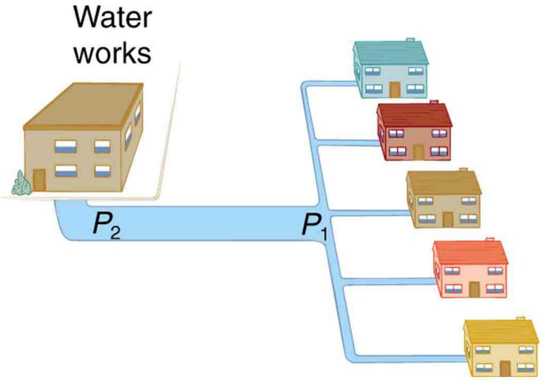
$$Q = \frac{P_2 - P_1}{R}$$

to

Equation:

$$P_2 - P_1 = RQ,$$

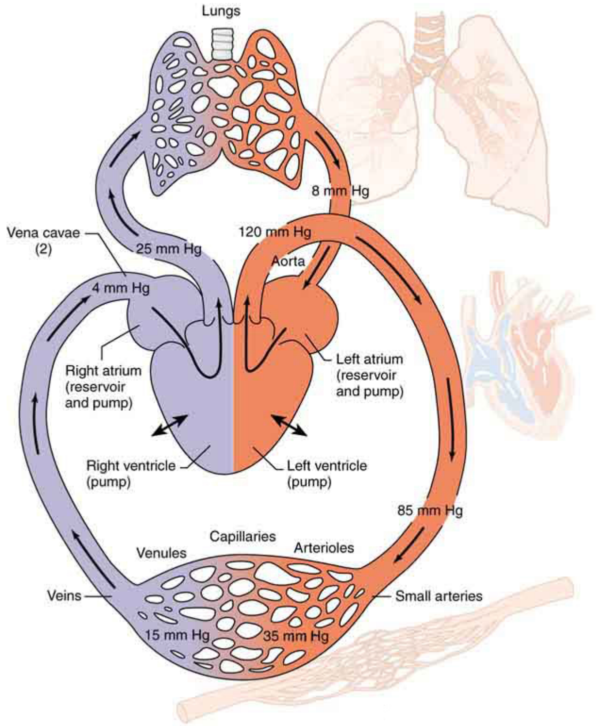
where, in this case, P_2 is the pressure at the water works and R is the resistance of the water main. During times of heavy use, the flow rate Q is large. This means that $P_2 - P_1$ must also be large. Thus P_1 must decrease. It is correct to think of flow and resistance as causing the pressure to drop from P_2 to P_1. $P_2 - P_1 = RQ$ is valid for both laminar and turbulent flows.



During times of heavy use, there is a significant pressure drop in a water main, and P_1 supplied to users is significantly less than P_2 created at the water works. If the flow is very small, then the pressure drop is negligible, and $P_2 \approx P_1$.

We can use $P_2 - P_1 = RQ$ to analyze pressure drops occurring in more complex systems in which the tube radius is not the same everywhere. Resistance will be much greater in narrow places, such as an obstructed coronary artery. For a given flow rate Q, the pressure drop will be greatest where the tube is most narrow. This is how water faucets control flow. Additionally, R is greatly increased by turbulence, and a constriction that creates turbulence greatly reduces the pressure downstream. Plaque in an artery reduces pressure and hence flow, both by its resistance and by the turbulence it creates.

[link] is a schematic of the human circulatory system, showing average blood pressures in its major parts for an adult at rest. Pressure created by the heart’s two pumps, the right and left ventricles, is reduced by the resistance of the blood vessels as the blood flows through them. The left ventricle increases arterial blood pressure that drives the flow of blood through all parts of the body except the lungs. The right ventricle receives the lower pressure blood from two major veins and pumps it through the lungs for gas exchange with atmospheric gases – the disposal of carbon dioxide from the blood and the replenishment of oxygen. Only one major organ is shown schematically, with typical branching of arteries to ever smaller vessels, the smallest of which are the capillaries, and rejoining of small veins into larger ones. Similar branching takes place in a variety of organs in the body, and the circulatory system has considerable flexibility in flow regulation to these organs by the dilation and constriction of the arteries leading to them and the capillaries within them. The sensitivity of flow to tube radius makes this flexibility possible over a large range of flow rates.



Schematic of the circulatory system. Pressure difference is created by the two pumps in the heart and is reduced by resistance in the vessels. Branching of vessels into capillaries allows blood to reach individual cells and exchange substances, such as oxygen and waste products, with them. The system has an impressive ability to regulate flow to individual organs, accomplished largely by varying vessel diameters.

Each branching of larger vessels into smaller vessels increases the total cross-sectional area of the tubes through which the blood flows. For example, an artery with a cross section of $1\ \mathrm{cm}^2$ may branch into 20 smaller arteries, each with cross sections of $0.5\ \mathrm{cm}^2$, with a total of $10\ \mathrm{cm}^2$. In that manner, the resistance of the branchings is reduced so that pressure is not entirely lost. Moreover, because $Q = Av$ and A increases through branching, the average velocity of the blood in the smaller vessels is reduced. The blood velocity in the aorta ($\mathrm{diameter} = 1\ \mathrm{cm}$) is about 25 cm/s, while in the capillaries ($20 \mu\mathrm{m}$ in diameter) the velocity is about 1 mm/s. This reduced velocity allows the blood to exchange substances with the cells in the capillaries and alveoli in particular.

Section Summary

- Laminar flow is characterized by smooth flow of the fluid in layers that do not mix.
- Turbulence is characterized by eddies and swirls that mix layers of fluid together.
- Fluid viscosity η is due to friction within a fluid. Representative values are given in [link]. Viscosity has units of $(\mathrm{N/m^2})\mathrm{s}$ or $\mathrm{Pa \cdot s}$.
- Flow is proportional to pressure difference and inversely proportional to resistance:
  Equation:
  $$Q = \frac{P_2 - P_1}{R}.$$
- For laminar flow in a tube, Poiseuille’s law for resistance states that
  Equation:
  $$R = \frac{8\eta l}{\pi r^4}.$$
- Poiseuille’s law for flow in a tube is
  Equation:
  $$Q = \frac{(P_2 - P_1)\pi r^4}{8\eta l}.$$
- The pressure drop caused by flow and resistance is given by
  Equation:
  $$P_2 - P_1 = RQ.$$

Conceptual Questions

Exercise:

Problem:

Explain why the viscosity of a liquid decreases with temperature—that is, how might increased temperature reduce the effects of cohesive forces in a liquid? Also explain why the viscosity of a gas increases with temperature—that is, how does increased gas temperature create more collisions between atoms and molecules?

Exercise:

Problem:

When paddling a canoe upstream, it is wisest to travel as near to the shore as possible. When canoeing downstream, it may be best to stay near the middle. Explain why.

Exercise:

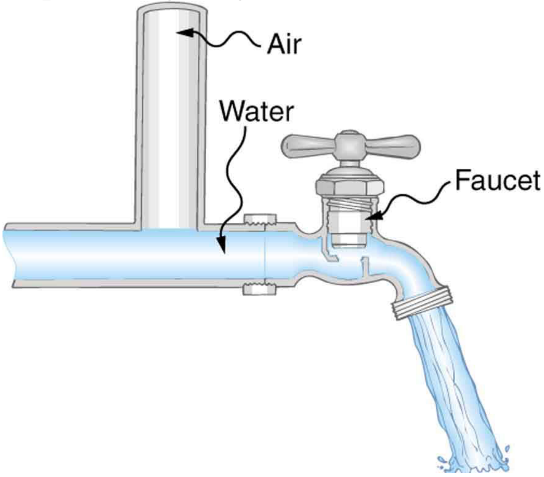
Problem:

Why does flow decrease in your shower when someone flushes the toilet?

Exercise:

Problem:

Plumbing usually includes air-filled tubes near water faucets, as shown in [link]. Explain why they are needed and how they work.



The vertical tube near the water tap remains full of air and serves a useful purpose.

Problems & Exercises

Exercise:

Problem:

(a) Calculate the retarding force due to the viscosity of the air layer between a cart and a level air track given the following information—air temperature is 20^{o} C, the cart is moving at 0.400 m/s, its surface area is $2.50 \times 10^{-2}\text{ m}^2$, and the thickness of the air layer is $6.00 \times 10^{-5}\text{ m}$. (b) What is the ratio of this force to the weight of the 0.300-kg cart?

Solution:

(a) $3.02 \times 10^{-3}\text{ N}$

(b) 1.03×10^{-3}

Exercise:

Problem:

What force is needed to pull one microscope slide over another at a speed of 1.00 cm/s, if there is a 0.500-mm-thick layer of 20^{o} C water between them and the contact area is 8.00 cm^2?

Exercise:

Problem:

A glucose solution being administered with an IV has a flow rate of $4.00\text{ cm}^3/\text{min}$. What will the new flow rate be if the glucose is replaced by whole blood having the same density but a viscosity 2.50 times that of the glucose? All other factors remain constant.

Solution:

$1.60\text{ cm}^3/\text{min}$

Exercise:

Problem:

The pressure drop along a length of artery is 100 Pa, the radius is 10 mm, and the flow is laminar. The average speed of the blood is 15 mm/s. (a) What is the net force on the blood in this section of artery? (b) What is the power expended maintaining the flow?

Exercise:

Problem:

A small artery has a length of $1.1 \times 10^{-3}\ \text{m}$ and a radius of $2.5 \times 10^{-5}\ \text{m}$. If the pressure drop across the artery is 1.3 kPa, what is the flow rate through the artery? (Assume that the temperature is $37^{\text{o}}\ \text{C}$.)

Solution:

$8.7 \times 10^{-11}\ \text{m}^3/\text{s}$

Exercise:

Problem:

Fluid originally flows through a tube at a rate of $100\ \text{cm}^3/\text{s}$. To illustrate the sensitivity of flow rate to various factors, calculate the new flow rate for the following changes with all other factors remaining the same as in the original conditions. (a) Pressure difference increases by a factor of 1.50. (b) A new fluid with 3.00 times greater viscosity is substituted. (c) The tube is replaced by one having 4.00 times the length. (d) Another tube is used with a radius 0.100 times the original. (e) Yet another tube is substituted with a radius 0.100 times the original and half the length, *and* the pressure difference is increased by a factor of 1.50.

Exercise:

Problem:

The arterioles (small arteries) leading to an organ, constrict in order to decrease flow to the organ. To shut down an organ, blood flow is reduced naturally to 1.00% of its original value. By what factor did the radii of the arterioles constrict? Penguins do this when they stand on ice to reduce the blood flow to their feet.

Solution:

0.316

Exercise:

Problem:

Angioplasty is a technique in which arteries partially blocked with plaque are dilated to increase blood flow. By what factor must the radius of an artery be increased in order to increase blood flow by a factor of 10?

Exercise:

Problem:

(a) Suppose a blood vessel’s radius is decreased to 90.0% of its original value by plaque deposits and the body compensates by increasing the pressure difference along the vessel to keep the flow rate constant. By what factor must the pressure difference increase? (b) If turbulence is created by the obstruction, what additional effect would it have on the flow rate?

Solution:

(a) 1.52

(b) Turbulence will decrease the flow rate of the blood, which would require an even larger increase in the pressure difference, leading to higher blood pressure.

Exercise:

Problem:

A spherical particle falling at a terminal speed in a liquid must have the gravitational force balanced by the drag force and the buoyant force. The buoyant force is equal to the weight of the displaced fluid, while the drag force is assumed to be given by Stokes Law, $F_s = 6\pi r\eta v$. Show that the terminal speed is given by

Equation:

$$v = \frac{2R^2 g}{9\eta}(\rho_\text{s} - \rho_1),$$

where R is the radius of the sphere, ρ_s is its density, and ρ_1 is the density of the fluid and η the coefficient of viscosity.

Exercise:

Problem:

Using the equation of the previous problem, find the viscosity of motor oil in which a steel ball of radius 0.8 mm falls with a terminal speed of 4.32 cm/s. The densities of the ball and the oil are 7.86 and 0.88 g/mL, respectively.

Solution:
Equation:

$$225\ \mathrm{mPa\cdot s}$$

Exercise:

Problem:

A skydiver will reach a terminal velocity when the air drag equals their weight. For a skydiver with high speed and a large body, turbulence is a factor. The drag force then is approximately proportional to the square of the velocity. Taking the drag force to be $F_{\mathrm{D}}=\frac{1}{2}\rho Av^2$ and setting this equal to the person's weight, find the terminal speed for a person falling "spread eagle." Find both a formula and a number for v_{t}, with assumptions as to size.

Exercise:

Problem:

A layer of oil 1.50 mm thick is placed between two microscope slides. Researchers find that a force of $5.50\times10^{-4}\ \mathrm{N}$ is required to glide one over the other at a speed of 1.00 cm/s when their contact area is $6.00\ \mathrm{cm}^2$. What is the oil's viscosity? What type of oil might it be?

Solution:
Equation:

$$0.138\ \mathrm{Pa\cdot s},$$

or

Olive oil.

Exercise:

Problem:

(a) Verify that a 19.0% decrease in laminar flow through a tube is caused by a 5.00% decrease in radius, assuming that all other factors remain constant, as stated in the text. (b) What increase in flow is obtained from a 5.00% increase in radius, again assuming all other factors remain constant?

Exercise:

Problem:

[link] dealt with the flow of saline solution in an IV system. (a) Verify that a pressure of $1.62 \times 10^4 \text{ N/m}^2$ is created at a depth of 1.61 m in a saline solution, assuming its density to be that of sea water. (b) Calculate the new flow rate if the height of the saline solution is decreased to 1.50 m. (c) At what height would the direction of flow be reversed? (This reversal can be a problem when patients stand up.)

Solution:

(a) $1.62 \times 10^4 \text{ N/m}^2$

(b) $0.111 \text{ cm}^3\text{/s}$

(c)10.6 cm

Exercise:

Problem:

When physicians diagnose arterial blockages, they quote the reduction in flow rate. If the flow rate in an artery has been reduced to 10.0% of its normal value by a blood clot and the average pressure difference has increased by 20.0%, by what factor has the clot reduced the radius of the artery?

Exercise:

Problem:

During a marathon race, a runner’s blood flow increases to 10.0 times her resting rate. Her blood’s viscosity has dropped to 95.0% of its normal value, and the blood pressure difference across the circulatory system has increased by 50.0%. By what factor has the average radii of her blood vessels increased?

Solution:

1.59

Exercise:

Problem:

Water supplied to a house by a water main has a pressure of $3.00 \times 10^5\ \mathrm{N/m^2}$ early on a summer day when neighborhood use is low. This pressure produces a flow of 20.0 L/min through a garden hose. Later in the day, pressure at the exit of the water main and entrance to the house drops, and a flow of only 8.00 L/min is obtained through the same hose. (a) What pressure is now being supplied to the house, assuming resistance is constant? (b) By what factor did the flow rate in the water main increase in order to cause this decrease in delivered pressure? The pressure at the entrance of the water main is $5.00 \times 10^5\ \mathrm{N/m^2}$, and the original flow rate was 200 L/min. (c) How many more users are there, assuming each would consume 20.0 L/min in the morning?

Exercise:

Problem:

An oil gusher shoots crude oil 25.0 m into the air through a pipe with a 0.100-m diameter. Neglecting air resistance but not the resistance of the pipe, and assuming laminar flow, calculate the gauge pressure at the entrance of the 50.0-m-long vertical pipe. Take the density of the oil to be $900\ \mathrm{kg/m^3}$ and its viscosity to be $1.00\ (\mathrm{N/m^2}) \cdot \mathrm{s}$ (or $1.00\ \mathrm{Pa} \cdot \mathrm{s}$). Note that you must take into account the pressure due to the 50.0-m column of oil in the pipe.

Solution:

$2.95 \times 10^6\ \mathrm{N/m^2}$(gauge pressure)

Exercise:

Problem:

Concrete is pumped from a cement mixer to the place it is being laid, instead of being carried in wheelbarrows. The flow rate is 200.0 L/min through a 50.0-m-long, 8.00-cm-diameter hose, and the pressure at the pump is $8.00 \times 10^6\ \mathrm{N/m^2}$. (a) Calculate the resistance of the hose. (b) What is the viscosity of the concrete, assuming the flow is laminar? (c) How much power is being supplied, assuming the point of use is at the same level as the pump? You may neglect the power supplied to increase the concrete’s velocity.

Exercise:

Problem: Construct Your Own Problem

Consider a coronary artery constricted by arteriosclerosis. Construct a problem in which you calculate the amount by which the diameter of the artery is decreased, based on an assessment of the decrease in flow rate.

Exercise:

Problem:

Consider a river that spreads out in a delta region on its way to the sea. Construct a problem in which you calculate the average speed at which water moves in the delta region, based on the speed at which it was moving up river. Among the things to consider are the size and flow rate of the river before it spreads out and its size once it has spread out. You can construct the problem for the river spreading out into one large river or into multiple smaller rivers.

Glossary

laminar
: a type of fluid flow in which layers do not mix

turbulence
: fluid flow in which layers mix together via eddies and swirls

viscosity
: the friction in a fluid, defined in terms of the friction between layers

Poiseuille’s law for resistance
: the resistance to laminar flow of an incompressible fluid in a tube: $R = 8\eta l/\pi r^4$

Poiseuille’s law
: the rate of laminar flow of an incompressible fluid in a tube: $Q = (P_2 - P_1)\pi r^4/8\eta l$

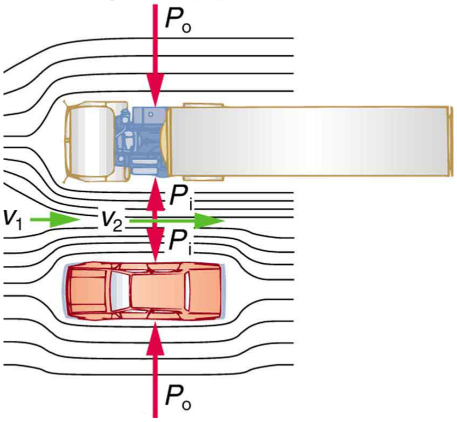
Bernoulli Effect

- Explain the terms in Bernoulli’s equation.
- Explain how Bernoulli’s equation is related to conservation of energy.
- Explain how to derive Bernoulli’s principle from Bernoulli’s equation.
- Calculate with Bernoulli’s principle.
- List some applications of Bernoulli’s principle.

When a fluid flows into a narrower channel, its speed increases and the pressure decreases.

There is a pressure difference when the channel narrows. This pressure difference results in a net force on the fluid.

There are a number of common examples of pressure dropping in rapidly-moving fluids. Shower curtains have a disagreeable habit of bulging into the shower stall when the shower is on. The high-velocity stream of water and air creates a region of lower pressure inside the shower, and standard atmospheric pressure on the other side. The pressure difference results in a net force inward pushing the curtain in. You may also have noticed that when passing a truck on the highway, your car tends to veer toward it. The reason is the same—the high velocity of the air between the car and the truck creates a region of lower pressure, and the vehicles are pushed together by greater pressure on the outside. (See [link].) This effect was observed as far back as the mid-1800s, when it was found that trains passing in opposite directions tipped precariously toward one another.



An overhead view of a

car passing a truck on a highway. Air passing between the vehicles flows in a narrower channel and must increase its speed (v_2 is greater than v_1), causing the pressure between them to drop (P_i is less than P_o). Greater pressure on the outside pushes the car and truck together.

Note:

Making Connections: Take-Home Investigation with a Sheet of Paper

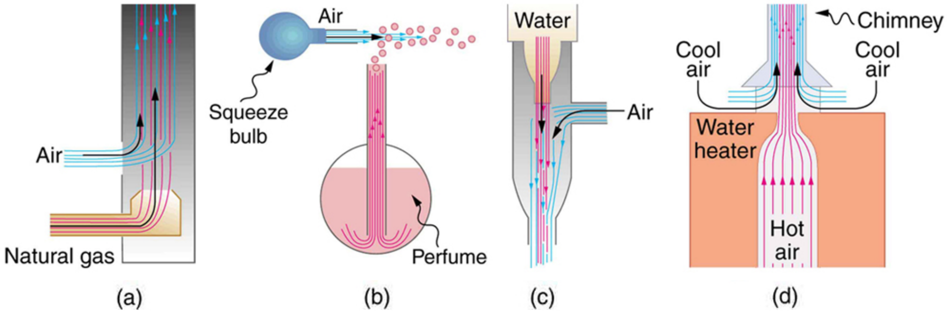
Hold the short edge of a sheet of paper parallel to your mouth with one hand on each side of your mouth. The page should slant downward over your hands. Blow over the top of the page. Describe what happens and explain the reason for this behavior.

Applications of Bernoulli’s Principle

There are a number of devices and situations in which fluid flows at a constant height and, thus, can be analyzed with Bernoulli’s principle.

Entrainment

People have long put the Bernoulli principle to work by using reduced pressure in high-velocity fluids to move things about. With a higher pressure on the outside, the high-velocity fluid forces other fluids into the stream. This process is called *entrainment*. Entrainment devices have been in use since ancient times, particularly as pumps to raise water small heights, as in draining swamps, fields, or other low-lying areas. Some other devices that use the concept of entrainment are shown in [link].



Examples of entrainment devices that use increased fluid speed to create low pressures, which then entrain one fluid into another. (a) A Bunsen burner uses an adjustable gas nozzle, entraining air for proper combustion. (b) An atomizer uses a squeeze bulb to create a jet of air that entrains drops of perfume. Paint sprayers and carburetors use very similar techniques to move their respective liquids. (c) A common aspirator uses a high-speed stream of water to create a region of lower pressure. Aspirators may be used as suction pumps in dental and surgical situations or for draining a flooded basement or producing a reduced pressure in a vessel. (d) The chimney of a water heater is designed to entrain air into the pipe leading through the ceiling.

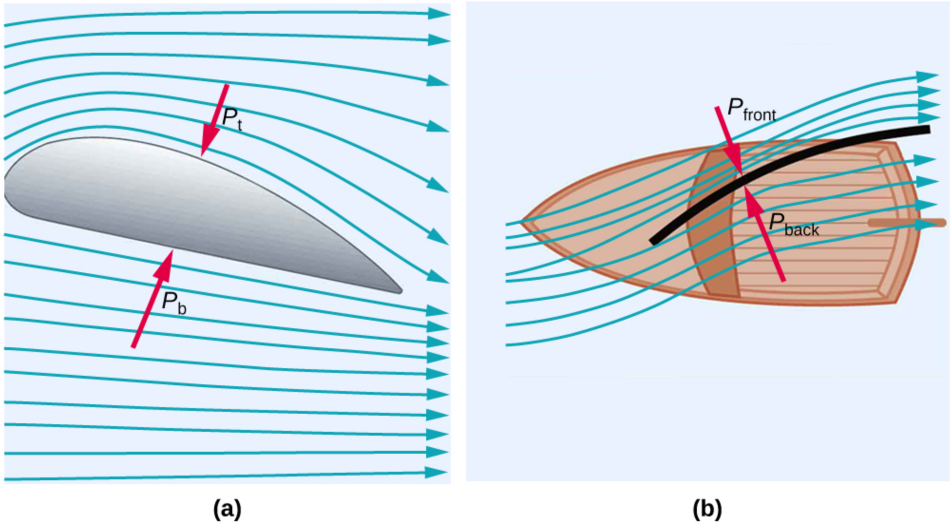
Wings and Sails

The airplane wing is a beautiful example of Bernoulli’s principle in action. [link](a) shows the characteristic shape of a wing. The wing is tilted upward at a small angle and the upper surface is longer, causing air to flow faster over it. The pressure on top of the wing is therefore reduced, creating a net upward force or lift. (Wings can also gain lift by pushing air downward, utilizing the conservation of momentum principle. The deflected air molecules result in an upward force on the wing — Newton’s third law.) Sails also have the characteristic shape of a wing. (See [link](b).) The pressure on the front side of the sail, P_{front}, is lower than the pressure on the back of the sail, P_{back}. This results in a forward force and even allows you to sail into the wind.

Note:

Making Connections: Take-Home Investigation with Two Strips of Paper

For a good illustration of Bernoulli’s principle, make two strips of paper, each about 15 cm long and 4 cm wide. Hold the small end of one strip up to your lips and let it drape over your finger. Blow across the paper. What happens? Now hold two strips of paper up to your lips, separated by your fingers. Blow between the strips. What happens?



(a) The Bernoulli principle helps explain lift generated by a wing. (b) Sails use the same technique to generate part of their thrust.

Summary

- Bernoulli's equation states that the sum on each side of the following equation is constant, or the same at any two points in an incompressible frictionless fluid:
  Equation:
  $$P_1 + \frac{1}{2}\rho v_1^2 + \rho\, gh_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho \mathrm{gh}_2.$$
- Bernoulli's principle is Bernoulli's equation applied to situations in which depth is constant. The terms involving depth (or height h) subtract out, yielding
  Equation:
  $$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2.$$
- Bernoulli's principle has many applications, including entrainment, wings and sails, and velocity measurement.

Conceptual Questions

Exercise:

Problem:

You can squirt water a considerably greater distance by placing your thumb over the end of a garden hose and then releasing, than by leaving it completely uncovered. Explain how this works.

Exercise:

Problem:

Water is shot nearly vertically upward in a decorative fountain and the stream is observed to broaden as it rises. Conversely, a stream of water falling straight down from a faucet narrows. Explain why, and discuss whether surface tension enhances or reduces the effect in each case.

Exercise:

Problem:

Look back to [link]. Answer the following two questions. Why is P_o less than atmospheric? Why is P_o greater than P_i?

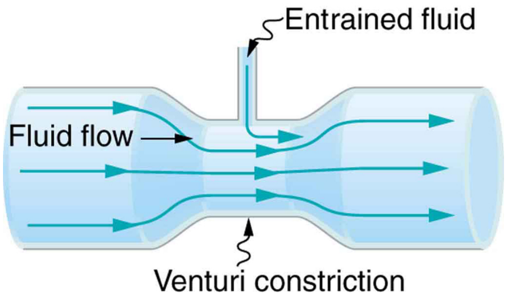
Exercise:

Problem: Give an example of entrainment not mentioned in the text.

Exercise:

Problem:

Many entrainment devices have a constriction, called a Venturi, such as shown in [link]. How does this bolster entrainment?



A tube with a narrow segment designed to enhance entrainment is called a Venturi. These are very commonly used

in carburetors and
aspirators.

Exercise:

Problem:

Some chimney pipes have a T-shape, with a crosspiece on top that helps draw up gases whenever there is even a slight breeze. Explain how this works in terms of Bernoulli’s principle.

Exercise:

Problem:

Is there a limit to the height to which an entrainment device can raise a fluid? Explain your answer.

Exercise:

Problem:

Why is it preferable for airplanes to take off into the wind rather than with the wind?

Exercise:

Problem:

Roofs are sometimes pushed off vertically during a tropical cyclone, and buildings sometimes explode outward when hit by a tornado. Use Bernoulli’s principle to explain these phenomena.

Exercise:

Problem: Why does a sailboat need a keel?

Exercise:

Problem:

It is dangerous to stand close to railroad tracks when a rapidly moving commuter train passes. Explain why atmospheric pressure would push you toward the moving train.

Exercise:

Problem:

Water pressure inside a hose nozzle can be less than atmospheric pressure due to the Bernoulli effect. Explain in terms of energy how the water can emerge from the nozzle against the opposing atmospheric pressure.

Exercise:

Problem:

A perfume bottle or atomizer sprays a fluid that is in the bottle. ([link].) How does the fluid rise up in the vertical tube in the bottle?

Atomizer: perfume bottle with tube to carry perfume up through the bottle.

(credit:
Antonia Foy,
Flickr)

Exercise:

Problem:

If you lower the window on a car while moving, an empty plastic bag can sometimes fly out the window. Why does this happen?

Problems & Exercises

Exercise:

Problem: Verify that pressure has units of energy per unit volume.

Solution:

$$\begin{aligned} P &= \frac{\text{Force}}{\text{Area}}, \\ (P)_{\text{units}} &= \text{N/m}^2 = \text{N}\cdot\text{m/m}^3 = \text{J/m}^3 \\ &= \text{energy/volume} \end{aligned}$$

Exercise:

Problem:

Suppose you have a wind speed gauge like the pitot tube shown in [link](b). By what factor must wind speed increase to double the value of h in the manometer? Is this independent of the moving fluid and the fluid in the manometer?

Exercise:

Problem:

If the pressure reading of your pitot tube is 15.0 mm Hg at a speed of 200 km/h, what will it be at 700 km/h at the same altitude?

Solution:

184 mm Hg

Exercise:

Problem:

Calculate the maximum height to which water could be squirted with the hose in [link] example if it: (a) Emerges from the nozzle. (b) Emerges with the nozzle removed, assuming the same flow rate.

Exercise:

Problem:

Every few years, winds in Boulder, Colorado, attain sustained speeds of 45.0 m/s (about 100 mi/h) when the jet stream descends during early spring. Approximately what is the force due to the Bernoulli effect on a roof having an area of $220\ \mathrm{m}^2$? Typical air density in Boulder is $1.14\ \mathrm{kg/m}^3$, and the corresponding atmospheric pressure is $8.89 \times 10^4\ \mathrm{N/m}^2$. (Bernoulli’s principle as stated in the text assumes laminar flow. Using the principle here produces only an approximate result, because there is significant turbulence.)

Solution:

$2.54 \times 10^5\ \mathrm{N}$

Exercise:

Problem:

(a) Calculate the approximate force on a square meter of sail, given the horizontal velocity of the wind is 6.00 m/s parallel to its front surface and 3.50 m/s along its back surface. Take the density of air to be $1.29\ \text{kg/m}^3$. (The calculation, based on Bernoulli's principle, is approximate due to the effects of turbulence.) (b) Discuss whether this force is great enough to be effective for propelling a sailboat.

Exercise:

Problem:

(a) What is the pressure drop due to the Bernoulli effect as water goes into a 3.00-cm-diameter nozzle from a 9.00-cm-diameter fire hose while carrying a flow of 40.0 L/s? (b) To what maximum height above the nozzle can this water rise? (The actual height will be significantly smaller due to air resistance.)

Solution:

(a) $1.58 \times 10^6\ \text{N/m}^2$

(b) 163 m

Exercise:

Problem:

(a) Using Bernoulli's equation, show that the measured fluid speed v for a pitot tube, like the one in [link](b), is given by

Equation:

$$v = \left(\frac{2\rho\prime gh}{\rho}\right)^{1/2},$$

where h is the height of the manometer fluid, $\rho\prime$ is the density of the manometer fluid, ρ is the density of the moving fluid, and g is the acceleration due to gravity. (Note that v is indeed proportional to the square root of h, as stated in the text.) (b) Calculate v for moving air if a mercury manometer's h is 0.200 m.

Glossary

Bernoulli's equation
the equation resulting from applying conservation of energy to an incompressible frictionless fluid: $P + 1/2pv^2 + pgh$ = constant , through the fluid

Bernoulli's principle
Bernoulli's equation applied at constant depth: $P_1 + 1/2pv_1^2 = P_2 + 1/2pv_2^2$

Derived copy of Cohesion and Adhesion in Liquids: Surface Tension and Capillary Action

- Understand cohesive and adhesive forces.
- Define surface tension.
- Understand capillary action.

Cohesion and Adhesion in Liquids

Children blow soap bubbles and play in the spray of a sprinkler on a hot summer day. (See [link].) An underwater spider keeps his air supply in a shiny bubble he carries wrapped around him. A technician draws blood into a small-diameter tube just by touching it to a drop on a pricked finger. A premature infant struggles to inflate her lungs. What is the common thread? All these activities are dominated by the attractive forces between atoms and molecules in liquids—both within a liquid and between the liquid and its surroundings.

Attractive forces between molecules of the same type are called **cohesive forces**. Liquids can, for example, be held in open containers because cohesive forces hold the molecules together. Attractive forces between molecules of different types are called **adhesive forces**. Such forces cause liquid drops to cling to window panes, for example. In this section we examine effects directly attributable to cohesive and adhesive forces in liquids.

Note:
Cohesive Forces
Attractive forces between molecules of the same type are called cohesive forces.

Note:
Adhesive Forces

Attractive forces between molecules of different types are called adhesive forces.

The soap bubbles in this photograph are caused by cohesive forces among molecules in liquids. (credit: Steve Ford Elliott)

Surface Tension

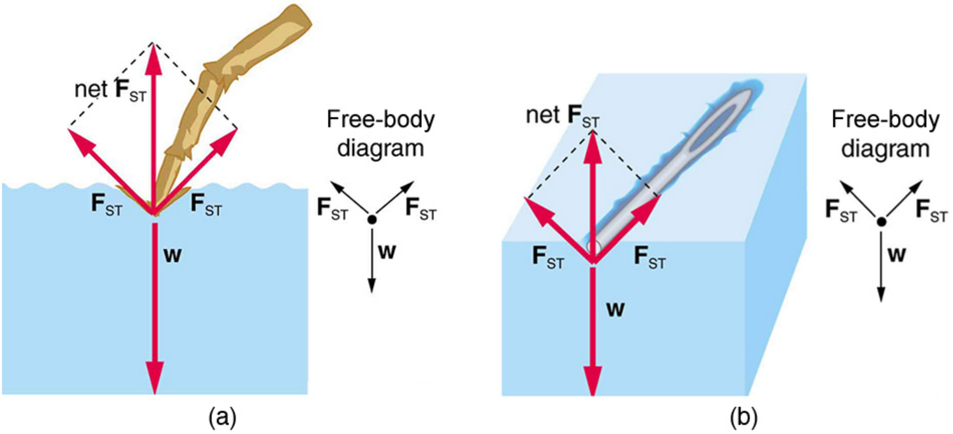
Cohesive forces between molecules cause the surface of a liquid to contract to the smallest possible surface area. This general effect is called **surface tension**. Molecules on the surface are pulled inward by cohesive forces, reducing the surface area. Molecules inside the liquid experience zero net force, since they have neighbors on all sides.

Note:
Surface Tension

Cohesive forces between molecules cause the surface of a liquid to contract to the smallest possible surface area. This general effect is called surface tension.

Note:
Making Connections: Surface Tension
Forces between atoms and molecules underlie the macroscopic effect called surface tension. These attractive forces pull the molecules closer together and tend to minimize the surface area. This is another example of a submicroscopic explanation for a macroscopic phenomenon.

The model of a liquid surface acting like a stretched elastic sheet can effectively explain surface tension effects. For example, some insects can walk on water (as opposed to floating in it) as we would walk on a trampoline—they dent the surface as shown in [link](a). [link](b) shows another example, where a needle rests on a water surface. The iron needle cannot, and does not, float, because its density is greater than that of water. Rather, its weight is supported by forces in the stretched surface that try to make the surface smaller or flatter. If the needle were placed point down on the surface, its weight acting on a smaller area would break the surface, and it would sink.



Surface tension supporting the weight of an insect and an iron needle, both of which rest on the surface without penetrating it. They are not floating; rather, they are supported by the surface of the liquid. (a) An insect leg dents the water surface. F_{ST} is a restoring force (surface tension) parallel to the surface. (b) An iron needle similarly dents a water surface until the restoring force (surface tension) grows to equal its weight.

Surface tension is proportional to the strength of the cohesive force, which varies with the type of liquid. Surface tension γ is defined to be the force F per unit length L exerted by a stretched liquid membrane:

Equation:

$$\gamma = \frac{F}{L}.$$

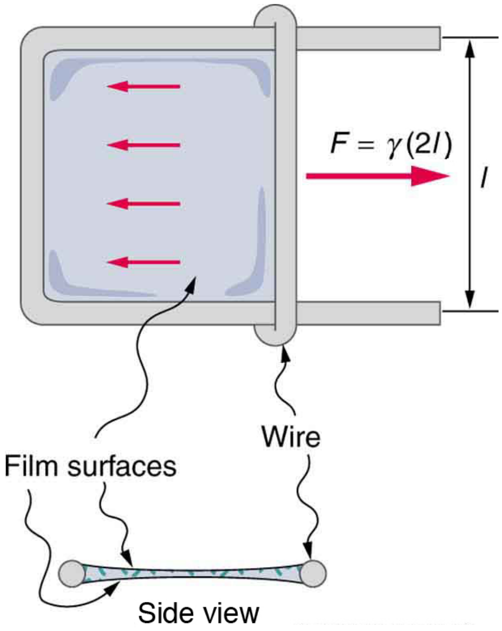
[link] lists values of γ for some liquids. For the insect of [link](a), its weight w is supported by the upward components of the surface tension force: $w = \gamma L \sin\theta$, where L is the circumference of the insect's foot in contact with the water. [link] shows one way to measure surface tension. The liquid film exerts a force on the movable wire in an attempt to reduce its surface area. The magnitude of this force depends on the surface tension of the liquid and can be measured accurately.

Surface tension is the reason why liquids form bubbles and droplets. The inward surface tension force causes bubbles to be approximately spherical and raises the pressure of the gas trapped inside relative to atmospheric pressure outside. It can be shown that the gauge pressure P inside a spherical bubble is given by

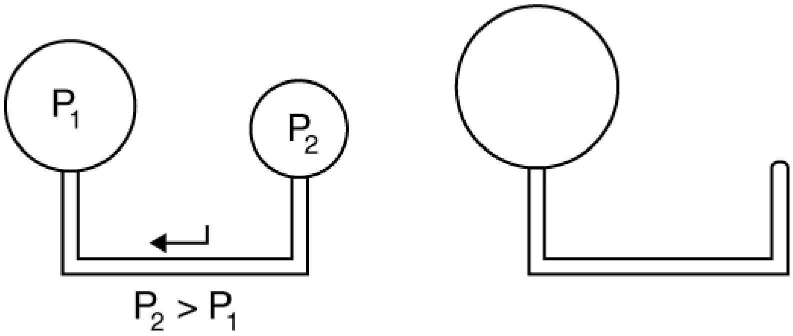
Equation:

$$P = \frac{4\gamma}{r},$$

where r is the radius of the bubble. Thus the pressure inside a bubble is greatest when the bubble is the smallest. Another bit of evidence for this is illustrated in [link]. When air is allowed to flow between two balloons of unequal size, the smaller balloon tends to collapse, filling the larger balloon.



Sliding wire device used for measuring surface tension; the device exerts a force to reduce the film’s surface area. The force needed to hold the wire in place is $F = \gamma L = \gamma(2l)$, since there are *two* liquid surfaces attached to the wire. This force remains nearly constant as the film is stretched, until the film approaches its breaking point.



With the valve closed, two balloons of different sizes are attached to each end of a tube. Upon opening the valve, the smaller balloon decreases in size with the air moving to fill the larger balloon. The pressure in a spherical balloon is inversely proportional to its radius, so that the smaller balloon has a greater internal pressure than the larger balloon, resulting in this flow.

Liquid	**Surface tension γ(N/m)**
Water at 0ºC	0.0756

Liquid	**Surface tension γ(N/m)**
Water at 20ºC	0.0728
Water at 100ºC	0.0589
Soapy water (typical)	0.0370
Ethyl alcohol	0.0223
Glycerin	0.0631
Mercury	0.465
Olive oil	0.032
Tissue fluids (typical)	0.050
Blood, whole at 37ºC	0.058
Blood plasma at 37ºC	0.073
Gold at 1070ºC	1.000
Oxygen at -193ºC	0.0157
Helium at -269ºC	0.00012

Surface Tension of Some Liquids[footnote]
At 20ºC unless otherwise stated.

Example:
Surface Tension: Pressure Inside a Bubble
Calculate the gauge pressure inside a soap bubble 2.00×10^{-4} m in radius using the surface tension for soapy water in [link]. Convert this pressure to

mm Hg.

Strategy

The radius is given and the surface tension can be found in [link], and so P can be found directly from the equation $P = \frac{4\gamma}{r}$.

Solution

Substituting r and γ into the equation $P = \frac{4\gamma}{r}$, we obtain

Equation:

$$P = \frac{4\gamma}{r} = \frac{4(0.037\ \text{N/m})}{2.00 \times 10^{-4}\ \text{m}} = 740\ \text{N/m}^2 = 740\ \text{Pa}.$$

We use a conversion factor to get this into units of mm Hg:

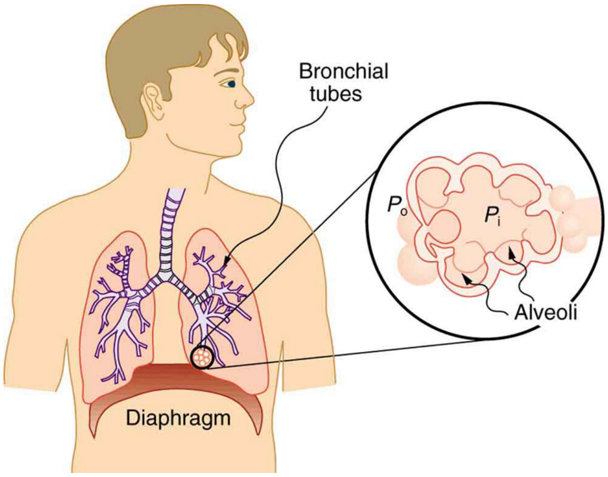
Equation:

$$P = \left(740\ \text{N/m}^2\right)\frac{1.00\ \text{mm Hg}}{133\ \text{N/m}^2} = 5.56\ \text{mm Hg}.$$

Discussion

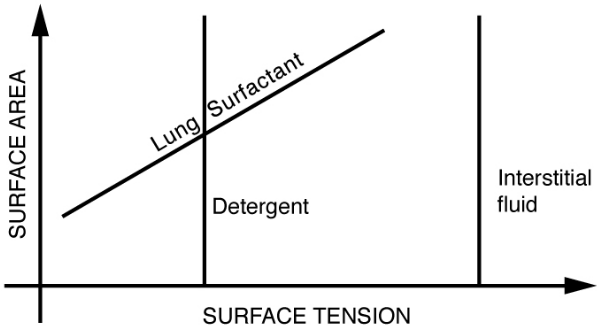
Note that if a hole were to be made in the bubble, the air would be forced out, the bubble would decrease in radius, and the pressure inside would *increase* to atmospheric pressure (760 mm Hg).

Our lungs contain hundreds of millions of mucus-lined sacs called *alveoli*, which are very similar in size, and about 0.1 mm in diameter. (See [link].) You can exhale without muscle action by allowing surface tension to contract these sacs. Medical patients whose breathing is aided by a positive pressure respirator have air blown into the lungs, but are generally allowed to exhale on their own. Even if there is paralysis, surface tension in the alveoli will expel air from the lungs. Since pressure increases as the radii of the alveoli decrease, an occasional deep cleansing breath is needed to fully reinflate the alveoli. Respirators are programmed to do this and we find it natural, as do our companion dogs and cats, to take a cleansing breath before settling into a nap.



Bronchial tubes in the lungs branch into ever-smaller structures, finally ending in alveoli. The alveoli act like tiny bubbles. The surface tension of their mucous lining aids in exhalation and can prevent inhalation if too great.

The tension in the walls of the alveoli results from the membrane tissue and a liquid on the walls of the alveoli containing a long lipoprotein that acts as a surfactant (a surface-tension reducing substance). The need for the surfactant results from the tendency of small alveoli to collapse and the air to fill into the larger alveoli making them even larger (as demonstrated in [link]). During inhalation, the lipoprotein molecules are pulled apart and the wall tension increases as the radius increases (increased surface tension). During exhalation, the molecules slide back together and the surface tension decreases, helping to prevent a collapse of the alveoli. The surfactant therefore serves to change the wall tension so that small alveoli don’t collapse and large alveoli are prevented from expanding too much. This tension change is a unique property of these surfactants, and is not shared by detergents (which simply lower surface tension). (See [link].)



Surface tension as a function of surface area. The surface tension for lung surfactant decreases with decreasing area. This ensures that small alveoli don’t collapse and large alveoli are not able to over expand.

If water gets into the lungs, the surface tension is too great and you cannot inhale. This is a severe problem in resuscitating drowning victims. A similar problem occurs in newborn infants who are born without this surfactant—their lungs are very difficult to inflate. This condition is known as *hyaline membrane disease* and is a leading cause of death for infants, particularly in premature births. Some success has been achieved in treating hyaline membrane disease by spraying a surfactant into the infant’s breathing passages. Emphysema produces the opposite problem with alveoli. Alveolar walls of emphysema victims deteriorate, and the sacs combine to form larger sacs. Because pressure produced by surface tension decreases with increasing radius, these larger sacs produce smaller pressure, reducing the ability of emphysema victims to exhale. A common test for emphysema is to measure the pressure and volume of air that can be exhaled.

Note:
Making Connections: Take-Home Investigation

(1) Try floating a sewing needle on water. In order for this activity to work, the needle needs to be very clean as even the oil from your fingers can be sufficient to affect the surface properties of the needle. (2) Place the bristles of a paint brush into water. Pull the brush out and notice that for a short while, the bristles will stick together. The surface tension of the water surrounding the bristles is sufficient to hold the bristles together. As the bristles dry out, the surface tension effect dissipates. (3) Place a loop of thread on the surface of still water in such a way that all of the thread is in contact with the water. Note the shape of the loop. Now place a drop of detergent into the middle of the loop. What happens to the shape of the loop? Why? (4) Sprinkle pepper onto the surface of water. Add a drop of detergent. What happens? Why? (5) Float two matches parallel to each other and add a drop of detergent between them. What happens? Note: For each new experiment, the water needs to be replaced and the bowl washed to free it of any residual detergent.

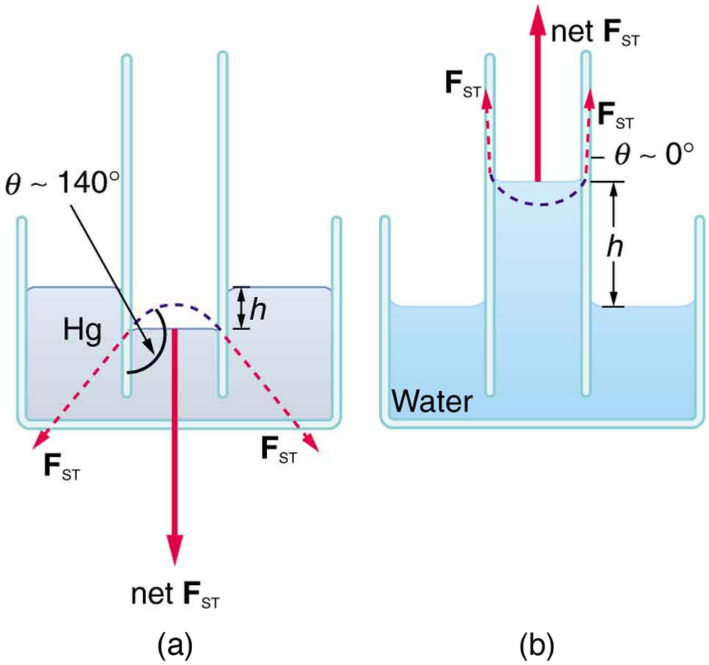
Adhesion and Capillary Action

One important phenomenon related to the relative strength of cohesive and adhesive forces is **capillary action**—the tendency of a fluid to be raised or suppressed in a narrow tube, or *capillary tube*. This action causes blood to be drawn into a small-diameter tube when the tube touches a drop.

Note:
Capillary Action
The tendency of a fluid to be raised or suppressed in a narrow tube, or capillary tube, is called capillary action.

If a capillary tube is placed vertically into a liquid, as shown in [link], capillary action will raise or suppress the liquid inside the tube depending on the combination of substances. The actual effect depends on the relative strength of the cohesive and adhesive forces and, thus, the contact angle θ

given in the table. If θ is less than 90°, then the fluid will be raised; if θ is greater than 90°, it will be suppressed. Mercury, for example, has a very large surface tension and a large contact angle with glass. When placed in a tube, the surface of a column of mercury curves downward, somewhat like a drop. The curved surface of a fluid in a tube is called a **meniscus**. The tendency of surface tension is always to reduce the surface area. Surface tension thus flattens the curved liquid surface in a capillary tube. This results in a downward force in mercury and an upward force in water, as seen in [link].



(a) Mercury is suppressed in a glass tube because its contact angle is greater than 90°. Surface tension exerts a downward force as it flattens the mercury, suppressing it in the tube. The dashed line shows the shape the mercury surface would have without the flattening effect of surface tension. (b) Water is raised in a glass tube because its contact angle is nearly 0°.

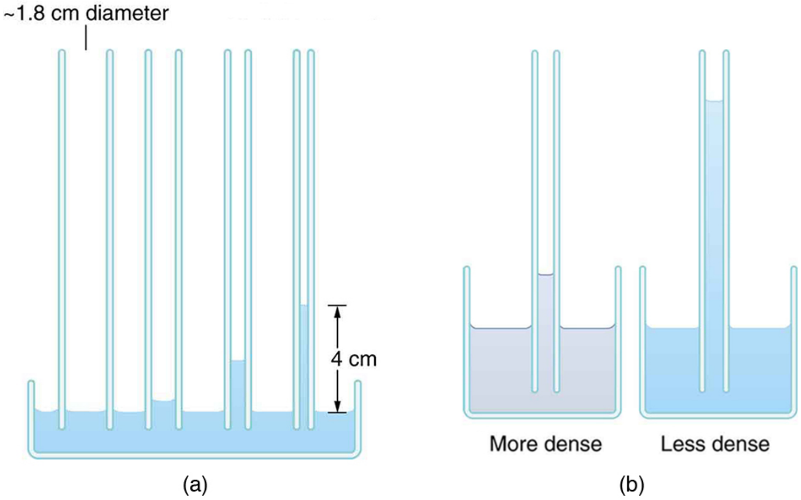
Surface tension therefore exerts an upward force when it flattens the surface to reduce its area.

Interface	**Contact angle Θ**
Mercury–glass	140°
Water–glass	0°
Water–paraffin	107°
Water–silver	90°
Organic liquids (most)–glass	0°
Ethyl alcohol–glass	0°

Interface	**Contact angle Θ**
Kerosene–glass	26º

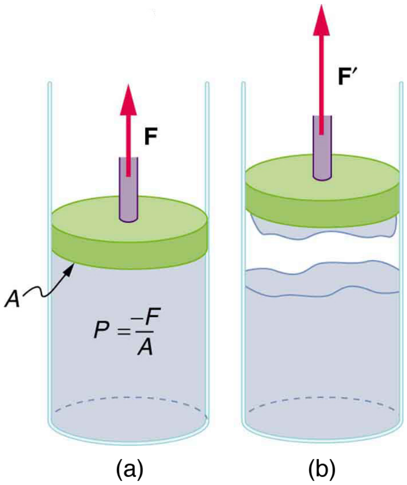
Contact Angles of Some Substances

If we look at the different factors in this expression, we might see how it makes good sense. The height is directly proportional to the surface tension γ, which is its direct cause. Furthermore, the height is inversely proportional to tube radius—the smaller the radius r, the higher the fluid can be raised, since a smaller tube holds less mass. The height is also inversely proportional to fluid density ρ, since a larger density means a greater mass in the same volume. (See [link].)



(a) Capillary action depends on the radius of a tube. The smaller the tube, the greater the height reached. The height is negligible for large-radius tubes. (b) A denser fluid in the same tube rises to a smaller height, all other factors being the same.

How *does* sap get to the tops of tall trees? (Recall that a column of water can only rise to a height of 10 m when there is a vacuum at the top—see [link].) The question has not been completely resolved, but it appears that it is pulled up like a chain held together by cohesive forces. As each molecule of sap enters a leaf and evaporates (a process called transpiration), the entire chain is pulled up a notch. So a negative pressure created by water evaporation must be present to pull the sap up through the xylem vessels. In most situations, *fluids can push but can exert only negligible pull*, because the cohesive forces seem to be too small to hold the molecules tightly together. But in this case, the cohesive force of water molecules provides a very strong pull. [link] shows one device for studying negative pressure. Some experiments have demonstrated that negative pressures sufficient to pull sap to the tops of the tallest trees *can* be achieved.



(a) When the piston is raised, it stretches the liquid slightly, putting it under tension and creating a negative absolute pressure $P = -F/A$. (b) The liquid eventually separates, giving an experimental limit to negative pressure in this liquid.

Section Summary

- Attractive forces between molecules of the same type are called cohesive forces.
- Attractive forces between molecules of different types are called adhesive forces.
- Cohesive forces between molecules cause the surface of a liquid to contract to the smallest possible surface area. This general effect is called surface tension.
- Capillary action is the tendency of a fluid to be raised or suppressed in a narrow tube, or capillary tube which is due to the relative strength of cohesive and adhesive forces.

Conceptual Questions

Exercise:

Problem:

The density of oil is less than that of water, yet a loaded oil tanker sits lower in the water than an empty one. Why?

Exercise:

Problem:

Is surface tension due to cohesive or adhesive forces, or both?

Exercise:

Problem:

Is capillary action due to cohesive or adhesive forces, or both?

Exercise:

Problem:

Birds such as ducks, geese, and swans have greater densities than water, yet they are able to sit on its surface. Explain this ability, noting that water does not wet their feathers and that they cannot sit on soapy water.

Exercise:

Problem:

Water beads up on an oily sunbather, but not on her neighbor, whose skin is not oiled. Explain in terms of cohesive and adhesive forces.

Exercise:

Problem:

Could capillary action be used to move fluids in a “weightless” environment, such as in an orbiting space probe?

Exercise:

Problem:

What effect does capillary action have on the reading of a manometer with uniform diameter? Explain your answer.

Exercise:

Problem:

Pressure between the inside chest wall and the outside of the lungs normally remains negative. Explain how pressure inside the lungs can become positive (to cause exhalation) without muscle action.

Problems & Exercises

Exercise:

Problem:

What is the pressure inside an alveolus having a radius of $2.50 \times 10^{-4}\ \text{m}$ if the surface tension of the fluid-lined wall is the same as for soapy water? You may assume the pressure is the same as that created by a spherical bubble.

Solution:

$592\ \text{N/m}^2$

Exercise:

Problem:

(a) The pressure inside an alveolus with a 2.00×10^{-4}-m radius is $1.40 \times 10^3\ \text{Pa}$, due to its fluid-lined walls. Assuming the alveolus acts like a spherical bubble, what is the surface tension of the fluid? (b) Identify the likely fluid. (You may need to extrapolate between values in [link].)

Exercise:

Problem:

What is the gauge pressure in millimeters of mercury inside a soap bubble 0.100 m in diameter?

Solution:

$2.23 \times 10^{-2}\ \text{mm Hg}$

Exercise:

Problem:

Calculate the force on the slide wire in [link] if it is 3.50 cm long and the fluid is ethyl alcohol.

Exercise:

Problem:

[link](a) shows the effect of tube radius on the height to which capillary action can raise a fluid. (a) Calculate the height h for water in a glass tube with a radius of 0.900 cm—a rather large tube like the one on the left. (b) What is the radius of the glass tube on the right if it raises water to 4.00 cm?

Solution:

(a) $1.65 \times 10^{-3}\text{ m}$

(b) $3.71 \times 10^{-4}\text{ m}$

Exercise:

Problem:

We stated in [link] that a xylem tube is of radius $2.50 \times 10^{-5}\text{ m}$. Verify that such a tube raises sap less than a meter by finding h for it, making the same assumptions that sap's density is 1050 kg/m^3, its contact angle is zero, and its surface tension is the same as that of water at 20.0^o C.

Exercise:

Problem:

What fluid is in the device shown in [link] if the force is $3.16 \times 10^{-3}\text{ N}$ and the length of the wire is 2.50 cm? Calculate the surface tension γ and find a likely match from [link].

Solution:

$$6.32 \times 10^{-2}\ \mathrm{N/m}$$

Based on the values in table, the fluid is probably glycerin.

Exercise:

Problem:

If the gauge pressure inside a rubber balloon with a 10.0-cm radius is 1.50 cm of water, what is the effective surface tension of the balloon?

Exercise:

Problem:

Calculate the gauge pressures inside 2.00-cm-radius bubbles of water, alcohol, and soapy water. Which liquid forms the most stable bubbles, neglecting any effects of evaporation?

Solution:

$$\begin{aligned} P_{\mathrm{w}} &= 14.6\ \mathrm{N/m}^2, \\ P_{\mathrm{a}} &= 4.46\ \mathrm{N/m}^2, \\ P_{\mathrm{sw}} &= 7.40\ \mathrm{N/m}^2. \end{aligned}$$

Alcohol forms the most stable bubble, since the absolute pressure inside is closest to atmospheric pressure.

Exercise:

Problem:

Suppose water is raised by capillary action to a height of 5.00 cm in a glass tube. (a) To what height will it be raised in a paraffin tube of the same radius? (b) In a silver tube of the same radius?

Exercise:

Problem:

Calculate the contact angle θ for olive oil if capillary action raises it to a height of 7.07 cm in a glass tube with a radius of 0.100 mm. Is this value consistent with that for most organic liquids?

Solution:

5.1^{o}

This is near the value of $\theta = 0^{\text{o}}$ for most organic liquids.

Exercise:

Problem:

When two soap bubbles touch, the larger is inflated by the smaller until they form a single bubble. (a) What is the gauge pressure inside a soap bubble with a 1.50-cm radius? (b) Inside a 4.00-cm-radius soap bubble? (c) Inside the single bubble they form if no air is lost when they touch?

Exercise:

Problem:

Calculate the ratio of the heights to which water and mercury are raised by capillary action in the same glass tube.

Solution:

-2.78

The ratio is negative because water is raised whereas mercury is lowered.

Exercise:

Problem:

What is the ratio of heights to which ethyl alcohol and water are raised by capillary action in the same glass tube?

Glossary

adhesive forces
the attractive forces between molecules of different types

capillary action
the tendency of a fluid to be raised or lowered in a narrow tube

cohesive forces
the attractive forces between molecules of the same type

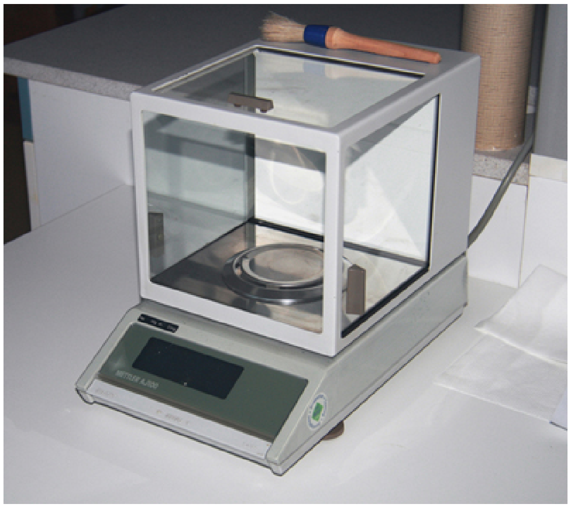
contact angle
the angle θ between the tangent to the liquid surface and the surface

surface tension
the cohesive forces between molecules which cause the surface of a liquid to contract to the smallest possible surface area

Accuracy, Precision, and Significant Figures

- Determine the appropriate number of significant figures in both addition and subtraction, as well as multiplication and division calculations.
- Calculate the percent uncertainty of a measurement.

A double-pan mechanical balance is used to compare different masses. Usually an object with unknown mass is placed in one pan and objects of known mass are placed in the other pan. When the bar that connects the two pans is horizontal, then the masses in both pans are equal. The "known masses" are typically metal cylinders of standard mass such as 1 gram, 10 grams, and 100 grams. (credit: Serge Melki)



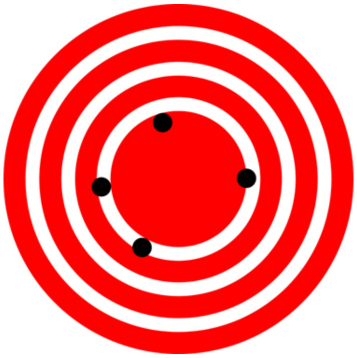
Many mechanical balances, such as double-pan balances, have been replaced by digital scales, which can typically measure the mass of an object more precisely. Whereas a mechanical balance may only read the mass of an object to the nearest tenth of a gram, many digital scales can measure the mass of an object up to the nearest thousandth of a gram. (credit: Karel Jakubec)

Accuracy and Precision of a Measurement

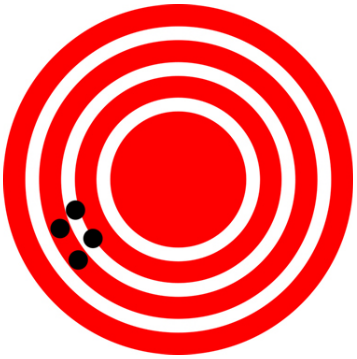
Science is based on observation and experiment—that is, on measurements. **Accuracy** is how close a measurement is to the correct value for that measurement.

The **precision** of a measurement system is refers to how close the agreement is between repeated measurements (which are repeated under the same conditions).

The measurements in the paper example are both accurate and precise, but in some cases, measurements are accurate but not precise, or they are precise but not accurate. Let us consider an example of a GPS system that is attempting to locate the position of a restaurant in a city. Think of the restaurant location as existing at the center of a bull’s-eye target, and think of each GPS attempt to locate the restaurant as a black dot. In [link], you can see that the GPS measurements are spread out far apart from each other, but they are all relatively close to the actual location of the restaurant at the center of the target. This indicates a low precision, high accuracy measuring system. However, in [link], the GPS measurements are concentrated quite closely to one another, but they are far away from the target location. This indicates a high precision, low accuracy measuring system.



A GPS system attempts to locate a restaurant at the center of the bull’s-eye. The black dots represent each attempt to pinpoint the location of the restaurant. The dots are spread out quite far apart from one another, indicating low precision, but they are each rather close to the actual location of the restaurant, indicating high accuracy. (credit: Dark Evil)



In this figure, the dots are concentrated rather closely to one another, indicating high precision, but they are rather far away from the actual location of the restaurant, indicating low accuracy. (credit: Dark Evil)

Accuracy, Precision, and Uncertainty

The degree of accuracy and precision of a measuring system are related to the **uncertainty** in the measurements. Uncertainty is a quantitative measure of how much your measured values deviate from a standard or expected value. If your measurements are not very accurate or precise, then the uncertainty of your values will be very high. The uncertainty in a measurement, A, is often denoted as δA ("delta A"), so the measurement result would be recorded as $A \pm \delta A$. Example, 11 ± 0.2.

In our example, such factors contributing to the uncertainty could be the following: the smallest division on the ruler is 0.1 in., the person using the ruler has bad eyesight, or one side of the paper is slightly longer than the other. At any rate, the uncertainty in a measurement must be based on a careful consideration of all the factors that might contribute and their possible effects.

Note:
Making Connections: Real-World Connections – Fevers or Chills?
Uncertainty is a critical piece of information, both in physics and in many other real-world applications. Imagine you are caring for a sick child. You suspect the child has a fever, so you check his or her temperature with a thermometer. What if the uncertainty of the thermometer were $3.0^{\circ}\mathrm{C}$? If the child's temperature reading was $37.0^{\circ}\mathrm{C}$ (which is normal body temperature), the "true" temperature could be anywhere from a

hypothermic $34.0^{o}C$ to a dangerously high $40.0^{o}C$. A thermometer with an uncertainty of $3.0^{o}C$ would be useless.

Percent Uncertainty

One method of expressing uncertainty is as a percent of the measured value. If a measurement A is expressed with uncertainty, δA, the **percent uncertainty** (%unc) is defined to be

Equation:

$$\%\text{ unc} = \frac{\delta A}{A} \times 100\%.$$

Example:

Calculating Percent Uncertainty: A Bag of Apples

A grocery store sells 5-kg bags of apples. You purchase four bags over the course of a month and weigh the apples each time. You obtain the following measurements:

Week 1 weight: 4.8 kg
Week 2 weight: 5.3 kg
Week 3 weight: 4.9 kg
Week 4 weight: 5.4 kg

You determine that the weight of the 5-kg bag has an uncertainty of ± 0.4 kg. What is the percent uncertainty of the bag's weight?

Strategy

First, observe that the expected value of the bag's weight, A, is 5 kg. The uncertainty in this value, δA, is 0.4 kg. We can use the following equation to determine the percent uncertainty of the weight:

Equation:

$$\% \text{ unc} = \frac{\delta A}{A} \times 100\%.$$

Solution

Plug the known values into the equation:

Equation:

$$\% \text{ unc} = \frac{0.4 \text{ kg}}{5 \text{ kg}} \times 100\% = 8\%.$$

Discussion

We can conclude that the weight of the apple bag is $5 \text{ kg} \pm 8\%$. Consider how this percent uncertainty would change if the bag of apples were half as heavy, but the uncertainty in the weight remained the same. Hint for future calculations: when calculating percent uncertainty, always remember that you must multiply the fraction by 100%. If you do not do this, you will have a decimal quantity, not a percent value.

Precision of Measuring Tools and Significant Figures

An important factor in the accuracy and precision of measurements involves the precision of the measuring tool. In general, a precise measuring tool is one that can measure values in very small increments. For example, a standard ruler can measure length to the nearest millimeter, while a caliper can measure length to the nearest 0.01 millimeter. The caliper is a more precise measuring tool because it can measure extremely small differences in length. The more precise the measuring tool, the more precise and accurate the measurements can be.

When we express measured values, we can only list as many digits as we initially measured with our measuring tool. For example, if you use a standard ruler to measure the length of a stick, you may measure it to be 36.7 cm. You could not express this value as 36.71 cm because your measuring tool was not precise enough to measure a hundredth of a centimeter. It should be noted that the last digit in a measured value has been estimated in some way by the person performing the measurement.

For example, the person measuring the length of a stick with a ruler notices that the stick length seems to be somewhere in between 36.6 cm and 36.7 cm, and he or she must estimate the value of the last digit. Using the method of **significant figures**, the rule is that *the last digit written down in a measurement is the first digit with some uncertainty*. In order to determine the number of significant digits in a value, start with the first measured value at the left and count the number of digits through the last digit written on the right. For example, the measured value 36.7 cm has three digits, or significant figures. Significant figures indicate the precision of a measuring tool that was used to measure a value.

Zeros

Special consideration is given to zeros when counting significant figures. The zeros in 0.053 are not significant, because they are only placekeepers that locate the decimal point. There are two significant figures in 0.053. The zeros in 10.053 are not placekeepers but are significant—this number has five significant figures. The zeros in 1300 may or may not be significant depending on the style of writing numbers. They could mean the number is known to the last digit, or they could be placekeepers. So 1300 could have two, three, or four significant figures. (To avoid this ambiguity, write 1300 in scientific notation.) *Zeros are significant except when they serve only as placekeepers*.

Exercise:

Check Your Understanding

Problem:

Determine the number of significant figures in the following measurements:

a. 0.0009
b. 15,450.0
c. 6×10^3
d. 87.990
e. 30.42

Solution:

(a) 1; the zeros in this number are placekeepers that indicate the decimal point

(b) 6; here, the zeros indicate that a measurement was made to the 0.1 decimal point, so the zeros are significant

(c) 1; the value 10^3 signifies the decimal place, not the number of measured values

(d) 5; the final zero indicates that a measurement was made to the 0.001 decimal point, so it is significant

(e) 4; any zeros located in between significant figures in a number are also significant

Significant Figures in this Text

In this text, most numbers are assumed to have three significant figures. Furthermore, consistent numbers of significant figures are used in all worked examples. You will note that an answer given to three digits is based on input good to at least three digits, for example. If the input has fewer significant figures, the answer will also have fewer significant figures. Care is also taken that the number of significant figures is reasonable for the situation posed. In some topics, particularly in optics, more accurate numbers are needed and more than three significant figures will be used. Finally, if a number is *exact*, such as the two in the formula for the circumference of a circle, $c = 2\pi r$, it does not affect the number of significant figures in a calculation.

Note:
PhET Explorations: Estimation

Explore size estimation in one, two, and three dimensions! Multiple levels of difficulty allow for progressive skill improvement.
https://phet.colorado.edu/sims/estimation/estimation_en.html

Summary

- Accuracy of a measured value refers to how close a measurement is to the correct value. The uncertainty in a measurement is an estimate of the amount by which the measurement result may differ from this value.
- Precision of measured values refers to how close the agreement is between repeated measurements.
- The precision of a *measuring tool* is related to the size of its measurement increments. The smaller the measurement increment, the more precise the tool.
- Significant figures express the precision of a measuring tool.
- When multiplying or dividing measured values, the final answer can contain only as many significant figures as the least precise value.
- When adding or subtracting measured values, the final answer cannot contain more decimal places than the least precise value.

Conceptual Questions

Exercise:

Problem:

What is the relationship between the accuracy and uncertainty of a measurement?

Exercise:

Problem:

Prescriptions for vision correction are given in units called *diopters* (D). Determine the meaning of that unit. Obtain information (perhaps by calling an optometrist or performing an internet search) on the minimum uncertainty with which corrections in diopters are determined and the accuracy with which corrective lenses can be produced. Discuss the sources of uncertainties in both the prescription and accuracy in the manufacture of lenses.

Problems & Exercises

Express your answers to problems in this section to the correct number of significant figures and proper units.

Exercise:

Problem:

Suppose that your bathroom scale reads your mass as 65 kg with a 3% uncertainty. What is the uncertainty in your mass (in kilograms)?

Solution:

2 kg

Exercise:

Problem:

A good-quality measuring tape can be off by 0.50 cm over a distance of 20 m. What is its percent uncertainty?

Exercise:

Problem:

(a) A car speedometer has a 5.0% uncertainty. What is the range of possible speeds when it reads $90\ \text{km/h}$? (b) Convert this range to miles per hour. $(1\ \text{km} = 0.6214\ \text{mi})$

Solution:

a. $85.5 \text{ to } 94.5\text{ km/h}$
b. $53.1 \text{ to } 58.7\text{ mi/h}$

Exercise:

Problem:

An infant's pulse rate is measured to be 130 ± 5 beats/min. What is the percent uncertainty in this measurement?

Exercise:

Problem:

(a) Suppose that a person has an average heart rate of 72.0 beats/min. How many beats does he or she have in 2.0 y? (b) In 2.00 y? (c) In 2.000 y?

Solution:

(a) $7.6 \times 10^7\text{ beats}$

(b) $7.57 \times 10^7\text{ beats}$

(c) $7.57 \times 10^7\text{ beats}$

Exercise:

Problem:

A can contains 375 mL of soda. How much is left after 308 mL is removed?

Exercise:

Problem:

State how many significant figures are proper in the results of the following calculations: (a) $(106.7)(98.2)/(46.210)(1.01)$ (b) $(18.7)^2$ (c) $\left(1.60 \times 10^{-19}\right)(3712)$.

Solution:

a. 3
b. 3
c. 3

Exercise:

Problem:

(a) How many significant figures are in the numbers 99 and 100? (b) If the uncertainty in each number is 1, what is the percent uncertainty in each? (c) Which is a more meaningful way to express the accuracy of these two numbers, significant figures or percent uncertainties?

Exercise:

Problem:

(a) If your speedometer has an uncertainty of 2.0 km/h at a speed of 90 km/h, what is the percent uncertainty? (b) If it has the same percent uncertainty when it reads 60 km/h, what is the range of speeds you could be going?

Solution:

a) 2.2%

(b) $59 \text{ to } 61 \text{ km/h}$

Exercise:

Problem:

(a) A person’s blood pressure is measured to be $120 \pm 2 \text{ mm Hg}$. What is its percent uncertainty? (b) Assuming the same percent uncertainty, what is the uncertainty in a blood pressure measurement of 80 mm Hg?

Exercise:

Problem:

A person measures his or her heart rate by counting the number of beats in 30 s. If 40 ± 1 beats are counted in $30.0 \pm 0.5 \text{ s}$, what is the heart rate and its uncertainty in beats per minute?

Solution:

$80 \pm 3 \text{ beats/min}$

Exercise:

Problem: What is the area of a circle 3.102 cm in diameter?

Exercise:

Problem:

If a marathon runner averages 9.5 mi/h, how long does it take him or her to run a 26.22-mi marathon?

Solution:

2.8 h

Exercise:

Problem:

A marathon runner completes a 42.188-km course in 2 h, 30 min, and 12 s. There is an uncertainty of 25 m in the distance traveled and an uncertainty of 1 s in the elapsed time. (a) Calculate the percent uncertainty in the distance. (b) Calculate the uncertainty in the elapsed time. (c) What is the average speed in meters per second? (d) What is the uncertainty in the average speed?

Exercise:

Problem:

The sides of a small rectangular box are measured to be $1.80 \pm 0.01\text{ cm}$, $2.05 \pm 0.02\text{ cm}$, and $3.1 \pm 0.1\text{ cm}$ long. Calculate its volume and uncertainty in cubic centimeters.

Solution:

$11 \pm 1\text{ cm}^3$

Exercise:

Problem:

When non-metric units were used in the United Kingdom, a unit of mass called the *pound-mass* (lbm) was employed, where $1\text{ lbm} = 0.4539\text{ kg}$. (a) If there is an uncertainty of 0.0001 kg in the pound-mass unit, what is its percent uncertainty? (b) Based on that percent uncertainty, what mass in pound-mass has an uncertainty of 1 kg when converted to kilograms?

Exercise:

Problem:

The length and width of a rectangular room are measured to be $3.955 \pm 0.005\text{ m}$ and $3.050 \pm 0.005\text{ m}$. Calculate the area of the room and its uncertainty in square meters.

Solution:

$12.06 \pm 0.04 \text{ m}^2$

Exercise:

Problem:

A car engine moves a piston with a circular cross section of $7.500 \pm 0.002 \text{ cm}$ diameter a distance of $3.250 \pm 0.001 \text{ cm}$ to compress the gas in the cylinder. (a) By what amount is the gas decreased in volume in cubic centimeters? (b) Find the uncertainty in this volume.

Glossary

accracy

the degree to which a measured value agrees with correct value for that measurement

method of adding percents

the percent uncertainty in a quantity calculated by multiplication or division is the sum of the percent uncertainties in the items used to make the calculation

percent uncertainty

the ratio of the uncertainty of a measurement to the measured value, expressed as a percentage

precision

the degree to which repeated measurements agree with each other

significant figures

express the precision of a measuring tool used to measure a value

uncertainty

a quantitative measure of how much your measured values deviate from a standard or expected value